Electromagnetism
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Lecture – 25
Electric field due to a line charge distribution

Hello. We have discussed the continuous charge density. Now, we will discuss some examples related to that.

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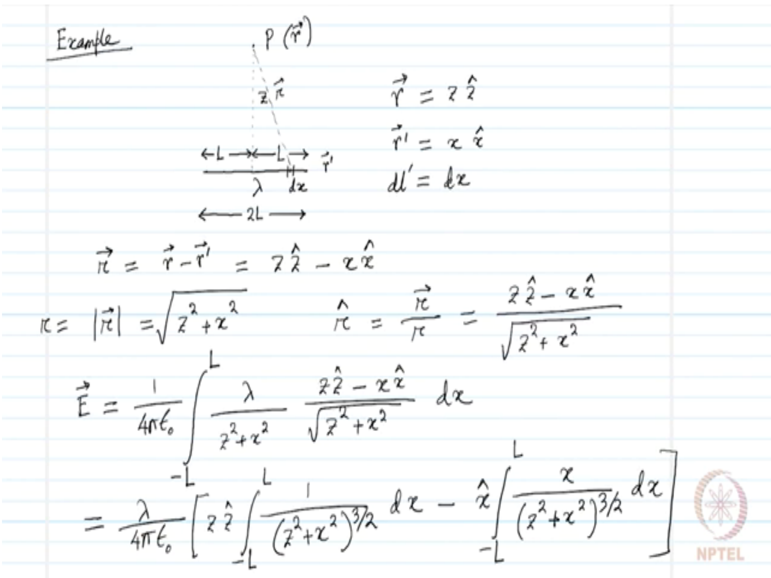

So, the first example that we will discuss is we will consider, we will try to find the electric field at a distance z above from the midpoint of a straight line segment of length 2L that carries a uniform line charge lambda. What does that mean? We have a line here that carries uniform line charge lambda and the length of this line segment is 2L twice of L. So, this much length is twice of L and we will consider a point at a distance z above the midpoint of this line segment.

So, from this point, if we drop a normal on this line, then this part is L, this part is also L like this and this height of that point is z. So, let us consider a line element here dx. So, the position vector of our point of observation P can be given as r; where, r is z z cap and the position vector of this little line element that we have here that can be given as r prime; where, r prime is x x cap and dl prime that is on the source, that is on the line charge distribution, a line element on this line charge distribution this can be given as dx clearly.

So, the distance between the distance vector between the point of observation and the source element is this much. This is curly r that can be given as r minus r prime, the vector subtraction of this which is nothing but z z cap minus x x cap. Given this, we can write the magnitude of this curly r vector the distance vector that is the magnitude of the distance that is z square plus x square square root of that and the direction of this curly r vector that is r cap is r vector over r magnitude is z z cap minus x x cap over z square plus x square square root of this.

So, the electric field for this charge distribution at the point of observation P would be given by 1 over 4 pi epsilon naught. We will have to integrate over dx from the range minus L to L assuming at the point where the normal is dropped the value of x equals 0. We will have the charge density line charge density over z square plus x square that is the distance squared, multi. Then, comes the unit vector along the distance that is z z cap minus x x cap over z square plus x square square root of this times dx.

And this can be evaluated as lambda over 4 pi epsilon naught z z cap integration minus L to L 1 over z square plus x square power 3 over 2 dx minus we will have x cap out. We cannot take x out because x is a function of x we will have to keep it within the integral minus L to L x over z square plus x square power 3 by 2 dx. We need to evaluate this integral. How do we do that?

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$$= \frac{\lambda}{4\pi\epsilon_0}\left[z\hat{z}\left(\frac{x}{z^2\sqrt{z^2+x^2}}\right)\Bigg|_{-L}^{L} - \hat{x}\left(-\frac{1}{\sqrt{z^2+x^2}}\right)\Bigg|_{-L}^{L}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\,\frac{2\lambda L}{z\sqrt{z^2+L^2}}\,\hat{z}$$

If we try to evaluate this integral, we will end up getting lambda over 4 pi epsilon naught z z cap; x over z squared z squared plus x squared square root of this quantity minus L to L minus x cap minus 1 over z squared plus x squared square root. Here, also the limits say minus L to L.

So, we need to evaluate this and if we evaluate if we work out the algebra, we will find 1 over 4 pi epsilon naught 2 lambda L over z z square plus L square and the direction of this electric field would be z cap.