Electromagnetism
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Lecture – 20
Vector calculus in spherical coordinate system Part - 02

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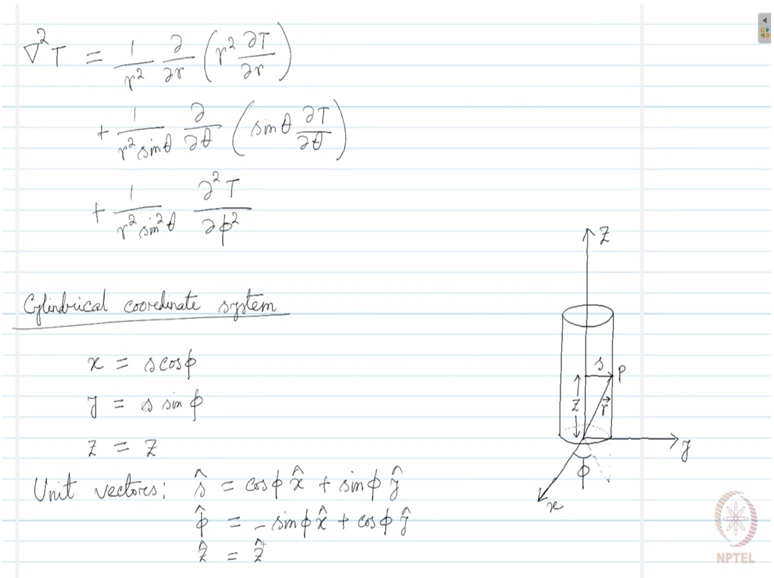

Now, let us move on to the cylindrical coordinate system. In cylindrical coordinate system, we have a cylindrical symmetry. So, let us draw a cylinder to begin with. And the Cartesian coordinate system reference for this kind of a system is given as this. This is our x-axis, this is y-axis and this is the z-axis. So, from, the z-axis is very important here.

From the z-axis if we consider a point P on the surface of the cylinder, so, the distance of that point P from the z-axis we will call that s small s and the position vector of that point P is given as r. We have if we project r on to the x y plane say the projection comes somewhere here, then that projection makes an angle with the x-axis that angle is phi, and the z component of that position vector is called z. So, z is also there in cylindrical coordinate system.

So, we have x equals s cosine of phi, y equals s sin phi, and z equals z. The unit vectors can be given as s cap equals cosine phi x cap plus sin phi y cap phi cap equals minus sin phi x cap plus cosine phi y cap and z cap is certainly equal to z cap in Cartesian coordinate system. So, these are the unit vectors.

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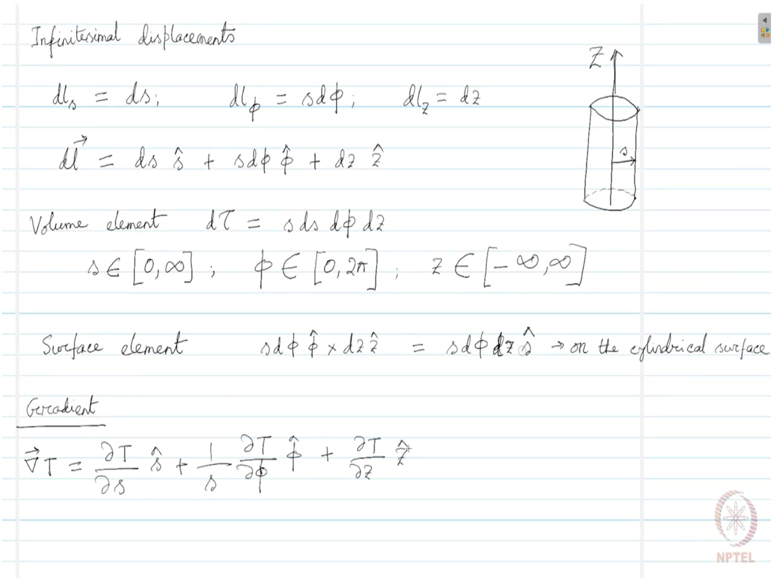

Now, let us consider the infinitesimal displacement for line element. Along s direction an infinitesimal displacement dl s will be given as d s; along the phi direction dl phi can be given because this is an angle as s d phi and along the z direction is simple d z. So, the line element dl can be expressed as ds s cap plus s d phi phi cap plus dz z cap, it is simple.

Let us consider the volume element volume element d tau would be given as multiplication of all these components that is s ds d phi d z. We have to consider that s belongs to the range 0 to infinity, phi belongs to the range 0 to 2 pi, and z just like the Cartesian coordinate system belongs to the range minus infinity to infinity.

Now, if we consider a surface element, let us draw a cylinder here, and if we consider a surface element on this cylindrical surface, then the distance is s from this axis that is z-axis, and we will rotate it by an element d phi. So, the distance that will travel that it will travel along the surface is s d phi along the phi direction. And the other component would be z components, so d z z cap and this will give us s d phi d z, and the direction of this surface element would be s cap in this notation. So, this is a surface element on the cylindrical surface.

We can find out other surface elements in a similar way. Let us move onto writing down the expressions for gradient, divergence, curl and Laplacian. The expression for gradient is gradient of a scalar field T is given as del T del s s cap plus 1 over s del T del phi phi cap plus del T del z z cap.

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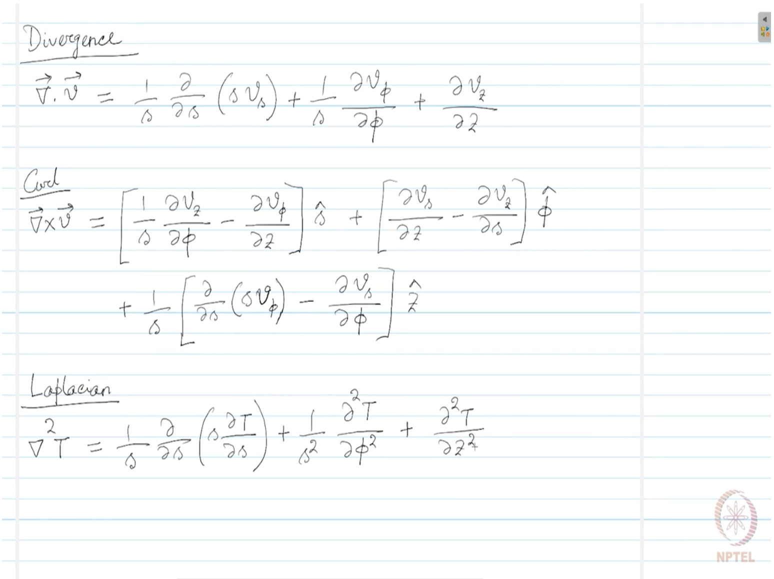

For divergence of a vector field v in cylindrical coordinate system, the expression can be given as 1 over s del del s s times v s plus 1 over s del v phi del phi plus del v z del z. For curl on the expression is curl of a vector field v is given as 1 over s del v z del phi minus del v phi del z s cap plus del v s del z minus del v z del s phi cap plus 1 over s del del s s v phi minus del v s del phi, this is along z cap. The Laplacian in cylindrical coordinate system acting on a scalar field T is given as 1 over s del del s s del T del s plus 1 over s squared del 2 T del phi 2 plus del 2 T del z 2.