Electromagnetism
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Lecture - 02
Vector algebra in component form

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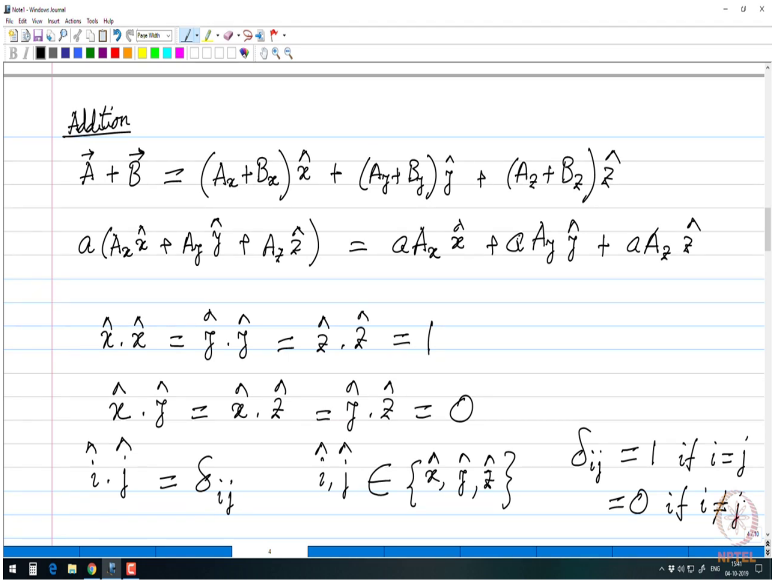

Now, how do we add two vectors in this component form? If we have vector A and vector B we want to add them two and A has x, y and z components A x, A y and A z. B has B x, B y and B z. So, this in the component form can be written as A x plus B x x cap plus A y plus B y y cap plus A z plus B z z cap.

How do we multiply a scalar with a vector in written in its component form? A scalar a is multiplied with a vector A x x cap plus A y y cap plus A z z cap that just gives a this vector. So, we have in Cartesian coordinate system x cap, y cap and z cap they are mutually perpendicular, these are orthogonal vectors and the magnitude of all these vectors that unit vectors is 1.

Therefore, we have x cap dot x cap equals y cap dot y cap it is the same equals z cap dot z cap is 1 and if we take dot product of two different unit vectors that is x cap dot y cap that is equals x cap dot z cap that equals y cap dot z cap that always becomes 0. So, in a more compact notation we can write i cap dot j cap equals Kronecker delta ij.

Now, i and j can take values from x, y and z. So, we can write i, j belongs to the set x, y, z and Kronecker delta function is defined as this is 1 if i equals j is 0, if i not equals j that is the definition of Kronecker delta.

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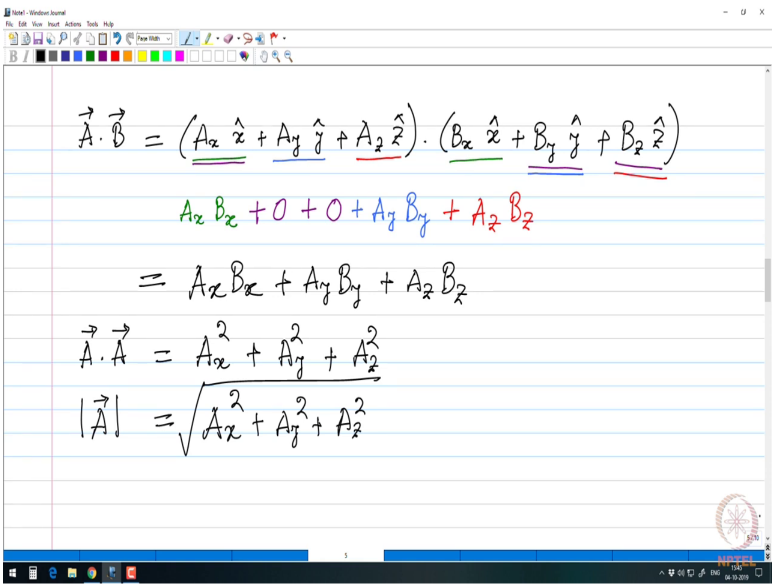

Accordingly, now if we make a dot product of A and B; so, A dot B where A and B are written in component form. So, that is written as A x x cap plus A y y cap plus A z z cap dot B x x cap plus B y y cap plus B z z cap.

Now, if we take a dot product between this part and this part, we obtain A x B x. If we take a dot product between this part and this part because the dot product of x cap and y cap goes to 0, we get 0 here and for the next one also we get. So, the dot product between A x x cap and B z z cap that also becomes 0 then we come to A y y cap it is dot product with B x x cap goes to 0. The dot product that sustains is B y y cap.

So, with for that we get A y B y and the dot product with B z z cap for A y y cap also goes to 0. Similarly, for A z z cap we will have only the dot product sustaining is with B z z cap. So, we get here A z B z; that means, the dot product that we are interested in becomes A x B x plus A y B y plus A z B z.

Now, if we take a dot product of vector A with itself, A dot A what do we get as a result. That will looking at the example earlier we can write that it would become A x squared plus A y squared plus A z squared. If that is the case then we can write that the magnitude of the vector A is actually square root of this part, A x squared plus A y squared plus A z squared square root of this.

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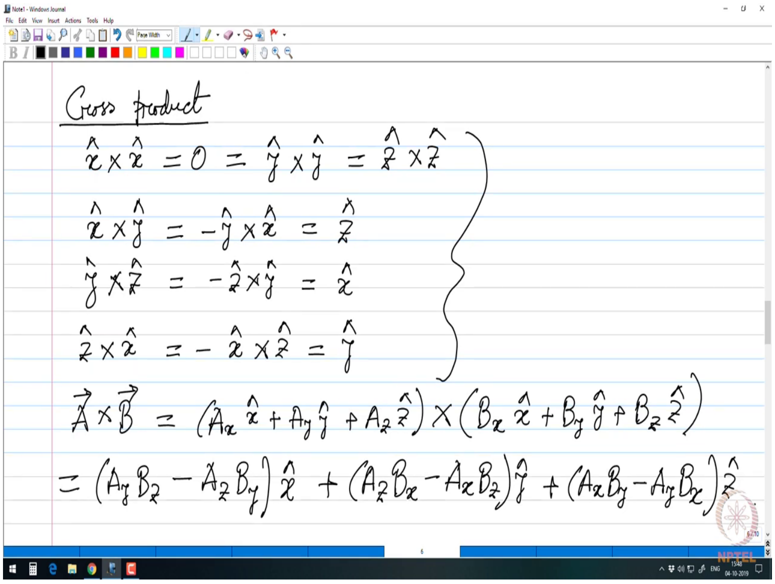

Now let us go for cross product in this component form because with cross product we get a vector perpendicular to the plane therefore, x cap cross x cap becomes 0. Similarly, y cap cross y cap and z cap cross z cap are also 0. If we take x cap cross y cap that is minus y cap cross x cap as we have found out for any two vectors earlier and this equals z cap in orthogonal Cartesian coordinate system.

Similarly, y cap cross z cap equals minus z cap cross y cap equals x cap and z cap cross x cap equals minus x cap cross z cap equals y cap. This will be useful later for performing any cross product. So, if we now perform a cross product between two vectors A and B, we can write it as A x x cap plus A y y cap plus A z z cap cross B x x cap plus B y y cap plus B z z cap and that becomes going by the rules that we have set here. A y B z minus A z B y x cap plus A z B x minus A x B z y cap plus A x B y minus A y B x z cap.

Now, this expression so, the way we have performed cross product is by going through the relations we have set here in above, but there is a better way to remember it and represent it in a compact form that is by making a determinant.

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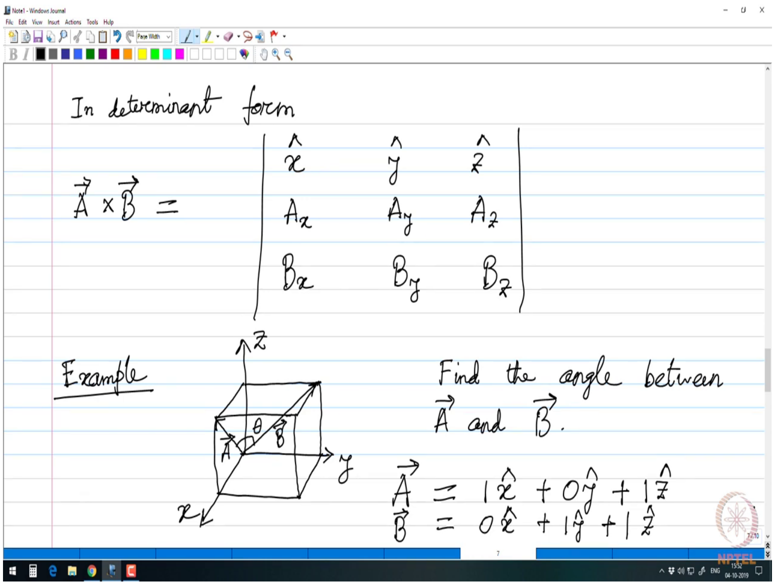

So, the cross product can be written in a determinant form as follows. A cross B with its Cartesian components can be written as a determinant x cap y cap z cap; A x A y A z, B x B y B z this determinant. Let us see an example, let us consider a cube of arm length one in Cartesian coordinate system. Here is our Cartesian coordinate system these are the axis x, y and z. So, this much is arm length one. So, we have the cube like this, here is the cube. And we have two vectors, this is vector A and here is vector B. We want to find the angle between these two vectors; that means, this angle let us call it theta.

So, let us write down A in its coordinate form, if we look at it we can see that A projected onto x axis will give us 1; that means, its x component is 1 1 x cap. Projected onto y axis there is no projection it is 0 and if we project it on to z axis that is also 1. So, this is the expression, this is the component form for A vector. For B vector the component form would look like its projection onto x axis is 0. So, 0 x cap plus projection onto y is 1, so 1 y cap and projection on to z is also 1. So, 1 z cap this is the form of B vector.

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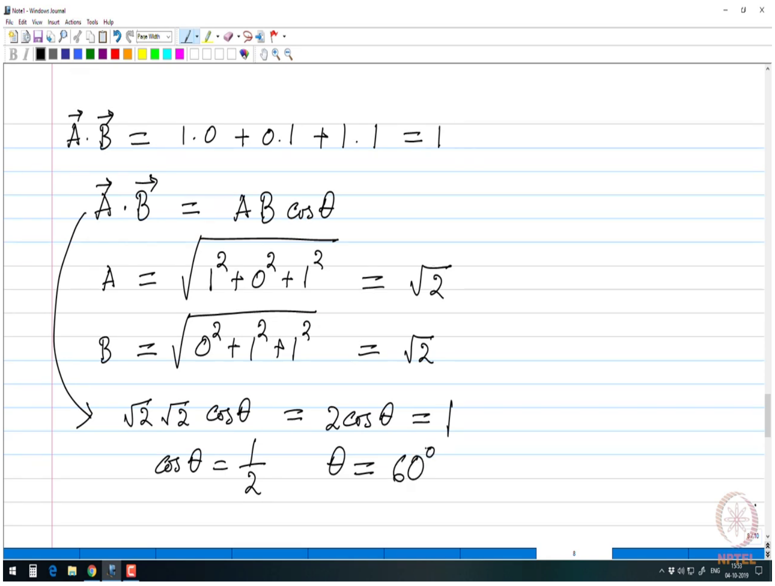

Now, if we perform a dot product of A and B using the component forms that we have just deduced. We have this A dot B equals 1 dot 0 plus 0 dot 1 plus 1 dot 1; that means, its 1 and we know that A dot B equals A B cosine of the angle between them. So, let us find out the magnitude of A vector A that is given as 1 squared plus 0 squared plus 1 squared square root of this entire thing that is square root of 2.

For B vector the magnitude is also the same 0 squared plus 1 squared plus 1 squared square root of this equals square root of 2. And now applying this equation here we find that square root 2 times square root 2 times cosine of the angle, that is 2 cos theta is 1. And if 2 cos theta is 1; that means, cos theta equals half that gives us theta equals 60 degree.