Electromagnetism
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Lecture - 14
The curl theorem (Stokes' theorem)

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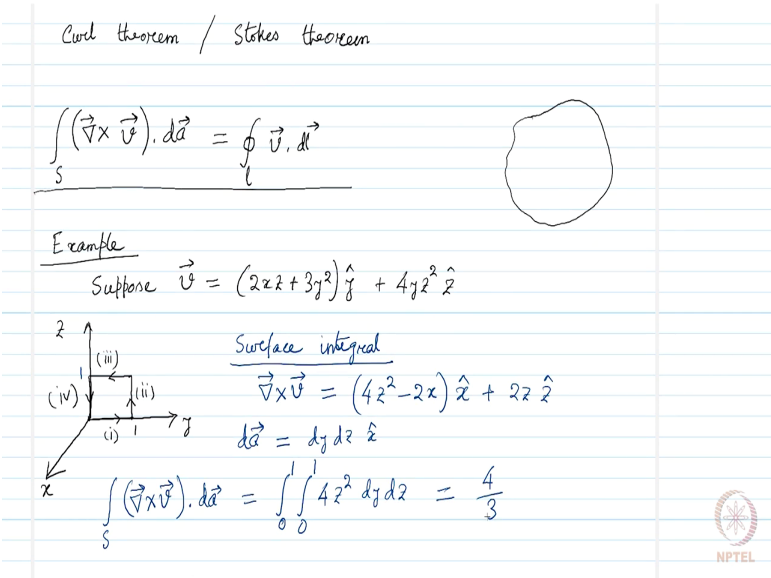

Let us consider another theorem, theorem about curl that is known as Stokes Theorem. Here the statement of this theorem goes like if we have a vector v, if we take the curl of this vector and if we make a surface integral of this curl, then on any surface this would be equal to a closed line integral of this vector dot a line element.

This is the statement of this theorem; there is something interesting in here. If we consider a line a closed line how many surfaces can we make out of that closed line? You can see that, on this plane of on this plane we can consider a line closed line like this and we have enclosed one flat surface in here. But, if we consider this flat surface to be a membrane and if we are allowed to stretch it in different direction, we can generate many surfaces out of this contour.

So, this theorem would be valid on every such surface that we may in principle construct it is a very powerful statement. Once again we will not go deep into the proof of this rather we will see an example. We suppose a vector field v given by 2 x z plus 3 y squared y cap plus 4 y z squared z cap and we check the Stokes theorem for this vector field over a closed contour. This is x direction, y direction, z direction and we consider a line from here to here that is line segment i and the length here is 1. Then a line along the z direction that is line segment ii, then a line along minus y direction that is line segment iii and closing it in the minus z direction so that is line segment iv.

So, we have this line the closed line and one flat surface. So, we will verify it on the flat surface enclosed by this line. Here if we are supposed to verify the surface integral, if you are supposed to calculate the surface integral here, we will first have to calculate the curl of v and curl of v is given as 4 z squared minus 2 x x cap plus 2 z z cap and d a. As you can see this plain surface is on the y z plane and its direction is along x cap.

So, it can be given as d y d z x cap. Then the integral of curl of v, the surface integral dot d a; this can be evaluated as integration from 0 to 1 because each arms length is 1 and other integral from 0 to 1 4 z squared d y d z. We did not consider this 2 x, because on this plane x equals 0 and this if you evaluate, you will find this value is 4 over 3.

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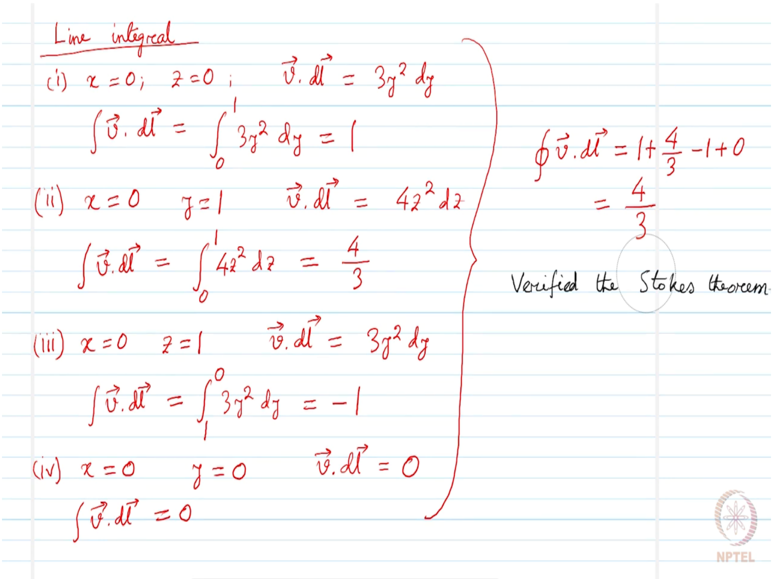

Now, comes the line integral. Line integral over the first segment, on the first segment we have x equals 0, z equals 0 and v dot d l is given as 3 y squared d y. With this v dot d l on this line segment will be integration over the range 0 to 1 3 y squared d y equals 1. On the second line segment we have x equals 0 y equals 1 and v dot d l is given as 4 z squared d z. With this we can write the integral as v dot d l equals integration over the range 0 to 1 4 z squared d z is nothing but 4 over 3.

Third segment, there we have x equals 0 z equals 1 and v dot d l is given as 3 y squared d y. So, we integrate over this v dot d l that becomes the range be careful this here the range of y is from 1 to 0. And, once we perform this integral of 3 y squared d y over this range we get minus 1. Then comes the last that is fourth segment, fourth line segment we have x equals 0, y equals 0 and v dot dl that is also 0.

Therefore, integral of v dot d l over this segment is certainly 0. Now what do we have? After adding all these up we will find that the cyclic integral, the closed integral of v dot d l that becomes 1 plus 4 over 3 minus 1 plus 0; that is 4 over 3. What was the value earlier? Earlier we have found that curl of v integrated over the surface area was 4 over 3. Now, we have found that using the line integral we also found the value to be 4 over 3.

So, we have verified the Stokes theorem.