Electromagnetism
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Lecture – 13
The divergence theorem (Gauss's theorem)

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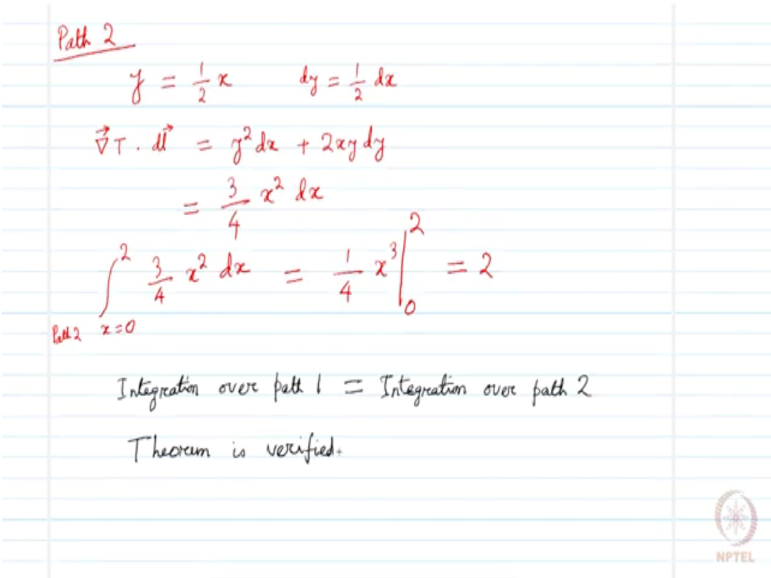

After verifying the fundamental theorem for gradient; let us move want to the fundamental theorem for divergence, also known as the Gauss's law or Gauss divergence theorem.

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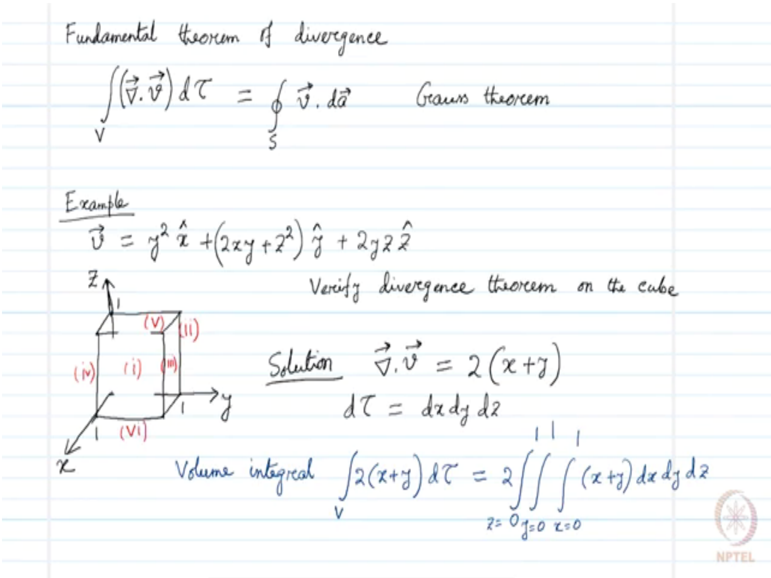

In case of divergence; it is, if we consider the divergence of a vector field v and if we perform a volume integral of it. Here v is for volume and we will write the volume element as d tau as we have done earlier that is nothing, but a closed surface integral of v dot d a. What is, what do we mean by closed surface integral in this context?

So, we are in the left hand side; we are talking about a volume and this is in the right hand side this surface is, is surface that encloses exactly this volume, nothing else. We are talking about that surface in the right hand side. So, this is also known as I mentioned earlier Gauss theorem. Again, we want proof this theorem here, but we will consider an example by which we can verify this theorem.

Let us consider a function v vector field, that is given as y squared x cap plus 2 x y plus z squared y cap plus 2 y z, z cap. And let us consider the cube with one of its corner at the origin. Here is a cube and the axis are relabeled this way; this is x axis this is y axis and this although my drawing is not good this these axis are orthogonal and the last one is z axis. The arm length of this cube is unit 1, 1 and 1.

Now, we have to verify this example; we have to verify the divergence theorem with this example. Let us consider the volume integral first. So, we have to calculate the divergence of v for that. Let us calculate the divergence of v that is; twice of x plus y, you can do it and check for yourself. And the volume element d tau is given as d x d y d z.

So, the volume integral becomes integration over the volume twice of x plus y d tau equals twice of all are variables are independent here x y and z, they do not depend on each other and all three have the range from 0 to 1. So, we can write this integral this way and if we work it out; then it becomes, we can independently work it out because these variables dont depend on each other.

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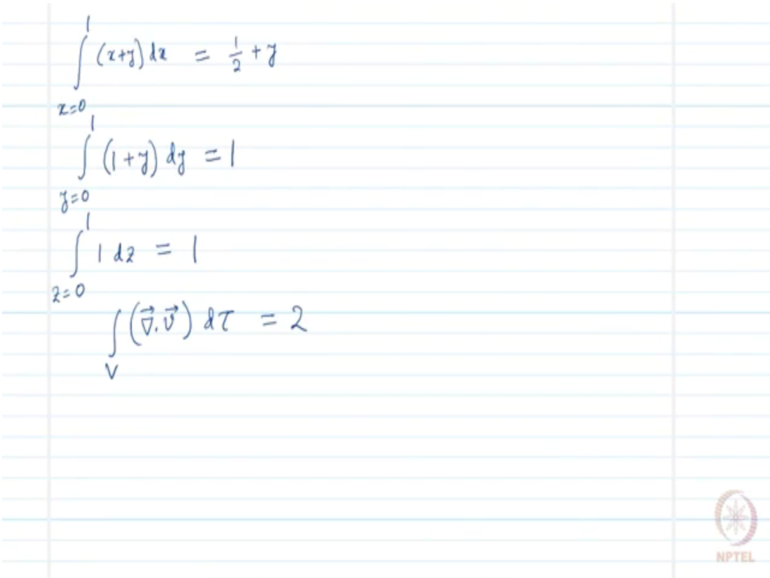
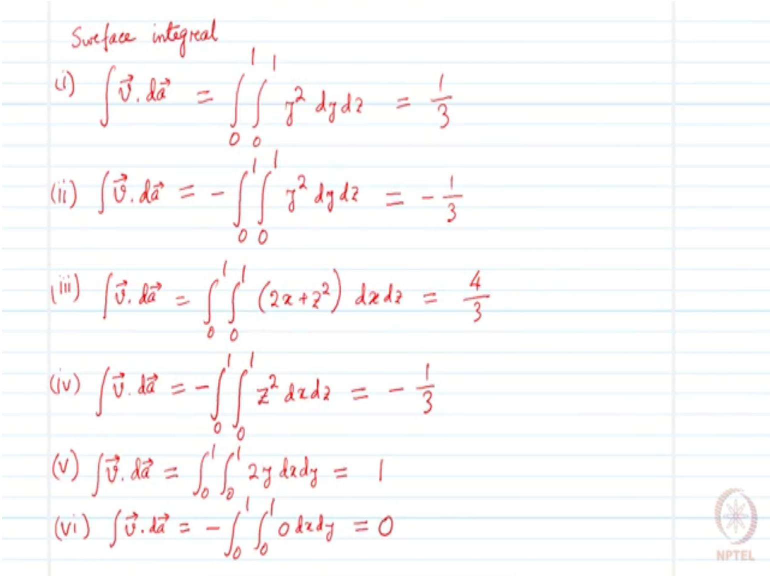


And if you try to independently work it out; we can find that the integral over x equals 0 to 1 x plus y d x is half plus y. Similarly, integral over y equals 0 to 1. We have here, 1 plus y d y equals 1 and the last integral z equals 0 to 1; we will have to integrate over 1 d z that gives us 1.

So, what did we do? We have first perform the integral over d x; we, the range was from 0 to 1 and after performing that integral, we are left we got half plus y and then we perform the integral over y. So, there once we perform the integral we got the result as 1. And then after performing this we are left with the integral over z and we perform the integral to get again only 1.

So; that means, we have evaluated the volume integral of the divergence of this vector field v and that becomes 2, because we had a 2 factor in front. So, this quantity becomes 2. In order to perform the surface integral; we have to mark the surfaces. So, let us go back to this picture let us consider the front surface to be surface 1, front is surface 1, back is surface 2, surface 3 is this one, this side is surface 4, 5 is above and 6 is bellow. We consider these to be the surfaces.(Refer Slide Time: 09:59)



Now, let us try performing the surface integral on surface 1; we will have integral v dot d a, this is given as. So, on surface 1 d x is not a variable x is constant. So, d y and d z would be the variables its integral over 0 to 1, integral over 0 to 1, y square d y d z because the direction is along x.

So, we will only have the x component of the vector surviving here, the other components would not come as we will take a dot product with the area element. And this will become 1 over 3; you can find it on surface 2, we will have integration over v dot d a and that is minus integral 0 to 1; integral 0 to 1, y square d y d z. Just because, we have the direction of this surface element in the opposite sense we always consider the direction of a surface outside the box and because these surfaces are opposite; we have different direction and given the value of y, we will find that this is nothing, but minus 1 over 3.

Now, let us come to surface 3; here, integral over v dot d a, this becomes integral from 0 to 1 integral from 0 to 1 2 x plus z squared d x d z. Here, the surface was along y direction, so y component of the vector comes in and that becomes 4 over 3. Surface 4 let us consider integral v dot d a equals minus 0 to 1, 0 to 1 minus sign once again for the opposite since of the direction of this surface and here its z square d x d z, that becomes minus 1 over 3. On surface 4 as we can see that x equals 0. So, we instead of 2 x plus z square we only have z square and this is the value.

On surface 5 we have v dot d a; that gives us integration over 0 to 1, integration over 0 to 1, 2 y d x d y and that becomes 1 on surface 6; we have integration over v dot d a equals again a minus sign 0 to 1, 0 to 1 to integrals. And because on surface 6; we can see y equals 0 here, so we will have 0 d x d y. So, the contribution from here will be 0. Now, if we add of the contribution from all the surfaces, we can write down.

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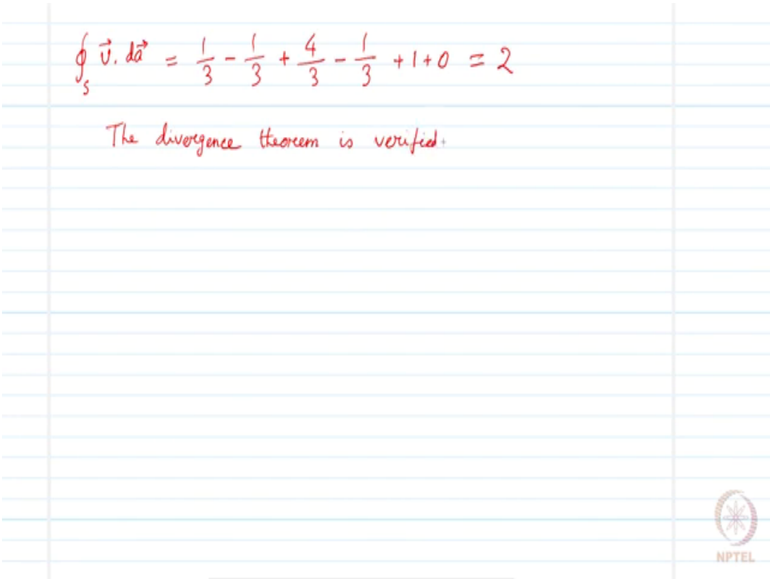

Closed integral over the cubic surface v dot d a is 1 over 3 minus 1 over 3 plus 4 over 3 minus 1 over 3 plus 1 plus 0 equals 2. And the value from the volume integral was exactly 2. So, that means; the theorems stands verified. The divergence theorem is verified.