Electromagnetism
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Lecture – 11
Volume integral

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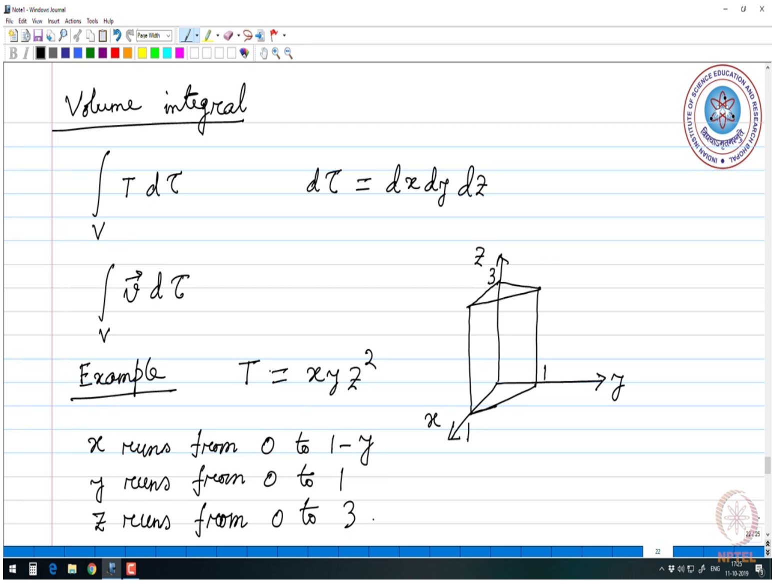

And after knowing the surface integral, we now move on to Volume Integrals. Volume integral is defined as integration over a given volume V any function may be scalar or vector. Here T is a scalar multiplied by the volume element d tau; d tau is an infinitesimal volume element in Cartesian coordinate system it can be given as d x, d y, d z. Also vector volume integrals are possible we can have vector v d tau.

So, the result of this would be a vector, but we encountered the vector volume integrals more rarely than the scalar volume integrals. Let us consider an example here. So, we have a prism and a function T a scalar function of x, y, z is given to us, it is given as x y z square and we will integrate it over the volume of a prism.

So, let us see how the prism is given. This is a Cartesian coordinate system; x, y and z. This is the prism where the coordinate here the intercept is 1; here also the intercept with x axis is 1 and the intercept with z axis is 3; intercept with x and y axis are 1 and the intercept with z axis is 3.

So, we can perform we have three variables x, y and z. And in this case the variable the setting the limits to the variables is very important. If we have this kind of a situation, we can put the limits of the variable in the following way x runs from 0 to 1 minus y; that you can see is the case in on this line this line here.

Then we can set y runs from 0 to 1; because we have already set x in terms of y we can have y independent and then is anyway independent in this volume. So, z runs from 0 to 3. If we have these limits on the variables, we can easily capture the entire volume and we do not really miss anything of this volume. With these limits, let us perform the integral.

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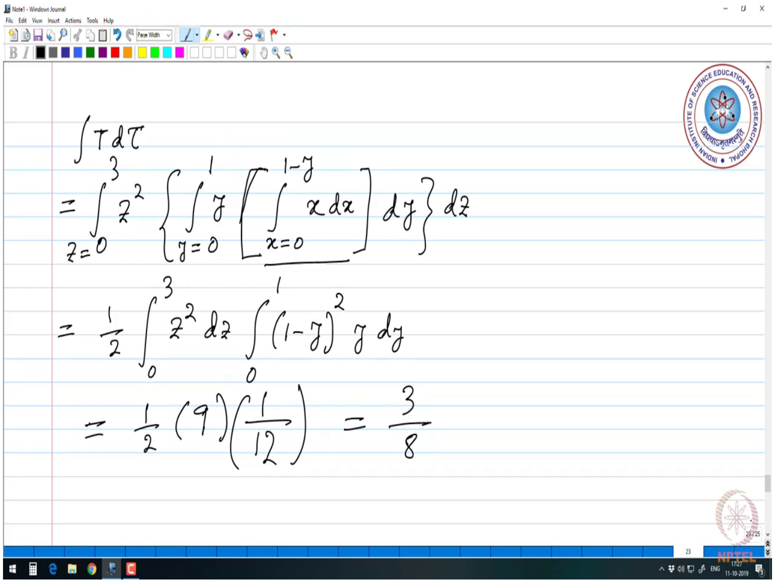

Integral T d tau this will within these limits be given as, the limit over set is 0 to 3 z squared, the limit over y is 0 to 1 y the limit over x is 0 to 1 minus y x d x. First we have to perform this integral then comes the integral over y and then dz the integral over z. We can actually perform the integral over y and z independently because y is not dependent on z or z is also not dependent on y, but we have to perform the integral over x first because if the limit is dependent on y.

So, if we perform it in this order we will have half integration over 0 to 3 set squared d z integration over 0 to 1 1 minus y squared y d y after performing. This x integral we will find this and we have now already made the z and y integrals independent; because that is allowed that will give us half times 9 from the z integral and 1 over 12's from the y integral.

So, we will end up getting 3 over 8; this is how we perform volume integrals in Cartesian coordinate system. So, setting the limits is very important in this occasion.