Electromagnetism

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Lecture - 01
Vector algebra

Hello this is Nirmal Ganguli, we are going to start the course on Electromagnetism. So, before starting the topics of electromagnetism that is electrostatics, magneto statics and electrodynamics some part of electrodynamics we would rather like to brush up our vector algebra and vector calculus maybe many of you did not study vector calculus earlier although you have studied vector algebra. It could be a good idea to go through vector algebra and vector calculus once more at the beginning of this course. If, you are confident about these topics you may skip this part and go to the next part, but I recommend going through this part once again.

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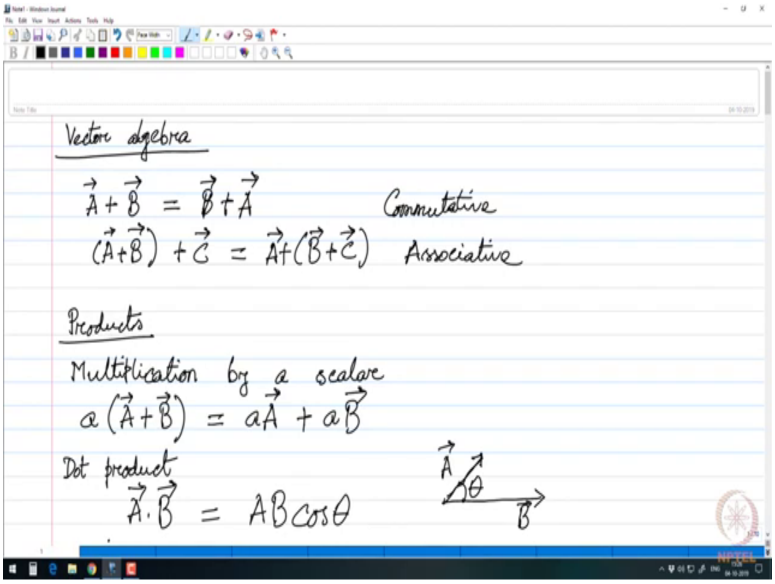

So, let us start with vector algebra. In vector algebra we first briefly mention addition of 2 vectors. So, if we add 2 vectors vector A plus vector B, that is commutative that means, we get vector B plus vector A these 2 are same things, it is also associative meaning A plus B. If, we now add another vector C to it that gives us the same thing as this and, you are familiar with some of the vector products. The first kind of product is multiplication by a scalar. Here small a is a scalar and we are multiplying, the sum of 2 vectors A plus B with this small a, this just gives us small a times A vector plus small a times B vector.

Now, comes product of 2 vectors; one type of product is called dot product. Here A dot B, this quantity is given as A B cosine of the angle between the vectors say this vector say this vector is A, this vector is B, and the angle between them is theta, then the dot product is given as A B times cosine of the angle between these 2 vectors.

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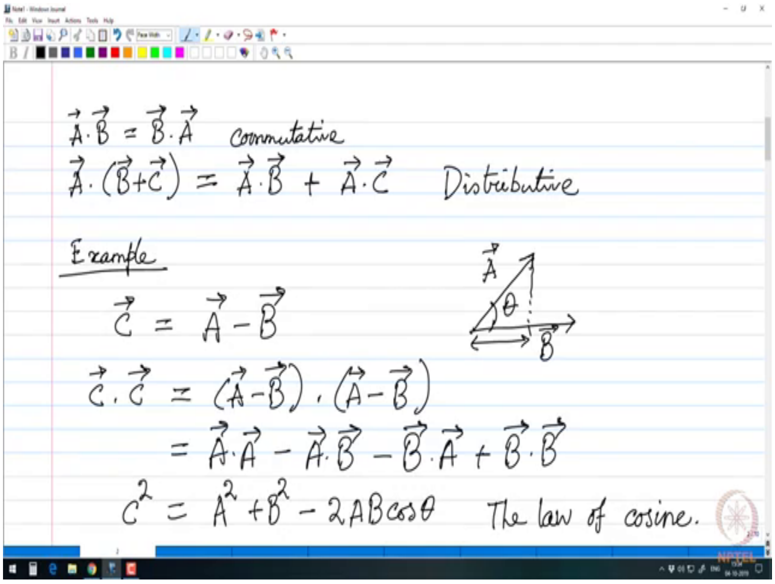

This dot product is also commutative that means, A dot B equals B dot A and, it is distributive that means, A dot B plus C equals A dot B plus A dot C.

Let us see an example. Let us consider a vector C that is given by A minus B and we want to calculate the dot product of C with itself; that means, we are interested in calculating C dot C, how do we do that? Let us draw these vectors A and B, this is vector A, this is vector B. So, C dot C can be written as in the algebraic form A minus B dotted with the same thing that is A minus B.

This is given as A dot A minus A dot B minus B dot A plus B dot B, which means, because of it is commutative property we can write A dot we can have A dot B equals B dot A and that gives us A squared plus B squared minus twice A B cosine of the angle theta between the vectors A and B.

So, this quantity can also be expressed as C squared; this is known as the law of cosine ok.

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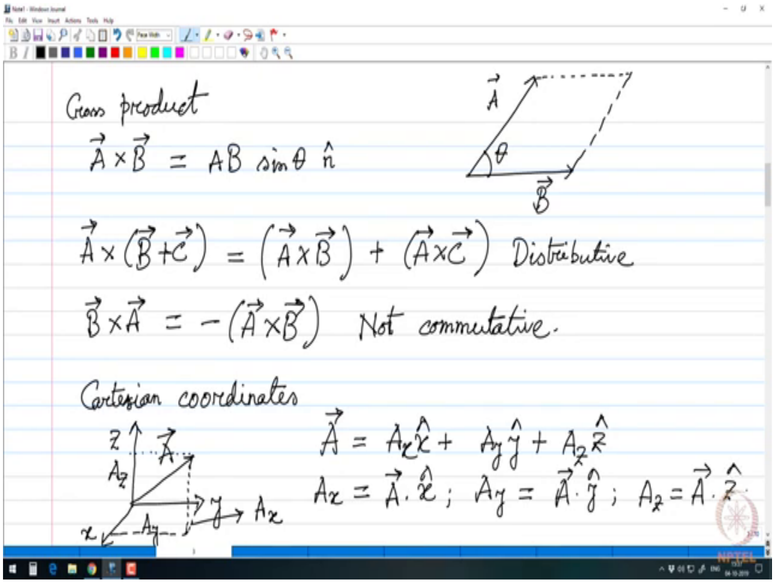

Now, let us move on to another type of product. So, with dot product between 2 vectors we got a scalar here, we will now deal with cross product; cross product for 2 vectors gives us a vector quantity. And, the cross product is defined in the following way: A cross B is given as A B sin of the angle in between and n cap vector which is a unit vector along the perpendicular direction to A and B.

So, if this is A vector and this is B vector, then the angle between them is theta and A cross B would be A B sin theta is the magnitude and the direction would be perpendicular to both A and B that is perpendicular to the plane of this screen, but it is important to know whether it would point outward or inward to this plane. So, the idea is that we will apply the right hand rule. We will align the fingers except the thumb to A vector first and then rotate it to B vector.

In doing so, the direction that the thumb points that is the direction of n cap that is certainly perpendicular to this plane, but in this occasion it shows that the thumb points into the screen. Using this right hand rule we can always find the direction of this cross product. And, then let us see whether what other rules are obeyed with cross product. I am writing A cross B plus C this vector, it can be given as A cross B plus A cross C; that means, distributive property is obeyed here.

Also, B cross A equals minus of A cross B, the sin inverse we can clearly see that from this picture. In this picture that we have drawn here, if we consider A cross B we point the thumb into the plane, if we consider B cross A then we rotate the fingers in this direction and you can clearly see that the thumb points upward; that means, the thumb comes out of the plane.

So, this minus sign is well justified and; that means, cross product is not commutative. And, how can we interpret cross product? The interpretation for cross product would be the area of this parallelogram that would be represented by a cross product; on the contrary a dot product represents the projection of one vector scaled by the other. So, if we had a unit vector one of this say B was A unit vector, then A dot B would represent the projection of A onto B that is this much of length and that is A scalar, while cross product is A vector it represents this area of this parallelogram with a direction that is perpendicular to this parallelogram. And, depending on whether you are taking A cross B or B cross A the direction would change ok.

So, now let us consider Cartesian components in the vector algebra. So, in a Cartesian coordinate system an arbitrary vector can be expanded in terms of basis vectors. So, let us look at it pictorially. Here, I draw Cartesian axis this is x, this is y and this is z. And, this is this is our vector a starting from the origin. Let us project this on to xy plane that looks something like this the projection. And, this part of the length here can be given as A x that is parallel to x axis, this part is A y that is parallel to y axis that is y component of the y component of the vector and the Z component would be it is projection on to Z axis. So, this is A Z.

And, if we have unit vectors along xyz given as x cap, y cap, and z cap, then this vector A can be represented as A x x cap plus A y y cap plus A z z cap. Were mathematically A x can be written as A dot x cap here we see the interpretation the projection interpretation of dot product clearly A y is similarly A dot y cap and A z is A dot Z cap.