Physics through Computational Thinking
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Non-dimensionalisation and Parametric Plot

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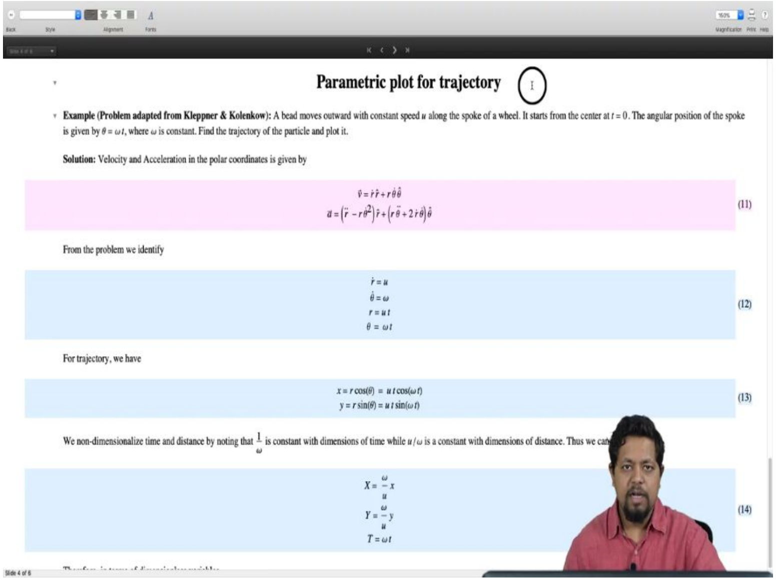

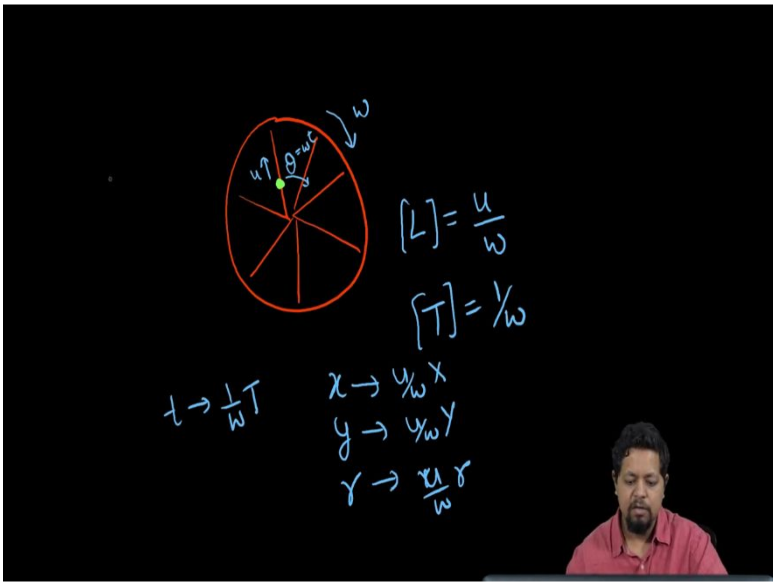

Okay, let us continue our journey on visual thinking. In our last two examples we were working with Cartesian coordinates and this time we will take an example where we will work with polar coordinates. So, in this problem that has been adapted from a problem in Kleppner and Kolenkow.

The problem is a bead moves outwards with a constant speed u along the spoke of a wheel. It starts from the center at $t = 0$. The angular position of the spoke is given by $\theta = \omega t$

where ω is a constant. Find the trajectory of the particle and plot it. So, let me explain to you the problem in terms of a picture.

We consider a wheel, it has got spokes. One of these spokes had a bead. And as this wheel rotates, the angular speed of ω, the angular position of this θ is given by $\theta = \omega\, t$, because it is travelling on the spoke. And on the spoke is travelling with the constant speed u. We have to find out what is the trajectory of the bead. So, let us go ahead and solve this problem.

To solve this problem we will write down velocity and acceleration in the polar coordinates. Velocity in polar coordinates is given by $\dot{r}\hat{r} + \dot{\theta}\hat{\theta}$. And acceleration is given by $(\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$.

For this problem, we identify $\dot{r}$ is u is always constant; $\dot{\theta}$ is ω is also constant. Therefore, r is $u\,t$ and θ is $\omega\, t$. For the trajectory, the x coordinate is $r\cos\theta$, which is $u\,t\cos(\omega\, t)$. And y coordinate is $r\sin\theta$, which is $u\,t\sin(\omega\, t)$. So, we started out with the polar coordinates because in polar coordinates, it was really straightforward to set up this problem.

We have converted into Cartesian coordinates, which is what we are going to use to plot. For Cartesian coordinates, we have obtained the x and y coordinates but now again, in order to plot, I need to get rid of these equations of the physical dimensions. The physical dimensions both x, y, r, all of these are dimensionful quantities.

So, we had to find out the natural scale for the quantities that are appearing in, over here and use those natural quantities in the problem to non-dimensionalise. In this particular problem, the natural length scale we have we have two constants u and ω and we have to find out the scale for time and scale for length.

Using u and ω, these dimensions of length are given by u/ω, which has dimensions of length and this provides a natural scale for length. For time, we have got $1/\omega$. So, we should go ahead and in order to non-dimensionalise we should replace all quantities of dimensions of length such as x, y, r by these dimensions. So, we can simply go ahead and replace x by $u/\omega\, X$, $u/\omega\, Y$ and $u/\omega\, r$. Similarly, for time we can replace time by $1/\omega\, T$.

And that replacement in the equations will simply give me x coordinate as ω/u times. This is the replacement I am doing or I can also say big X coordinate is $\omega/u\, x$, etc and when I do that for x and y coordinate, I simply get $T\cos T$ and $T\sin T$. x and y are dimensionless coordinates and T is dimensionless time. We will use parametric plot to make a plot for this thing.

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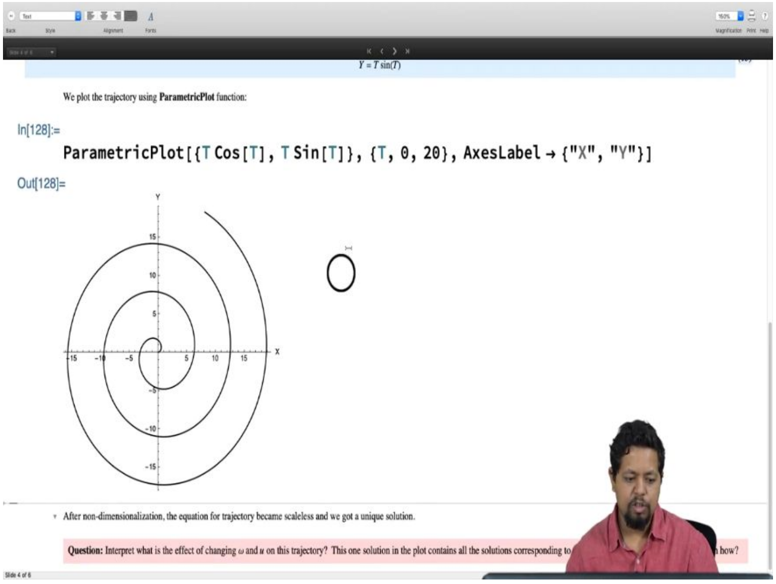

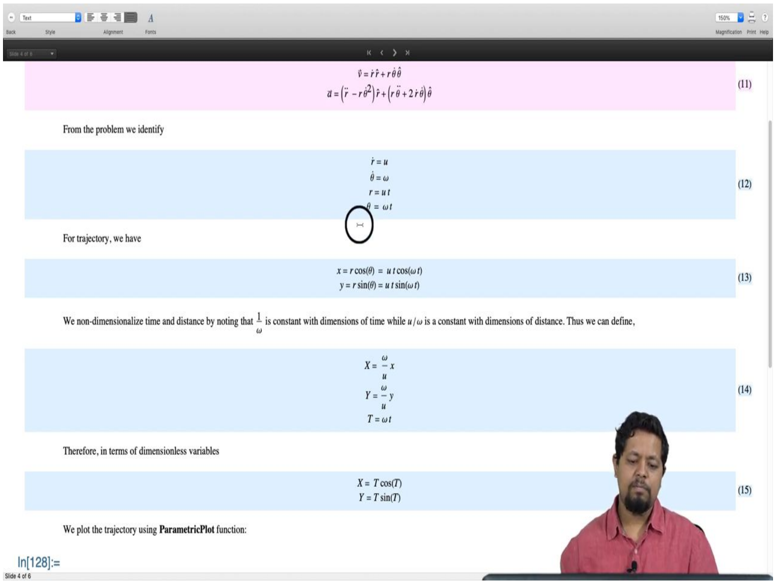

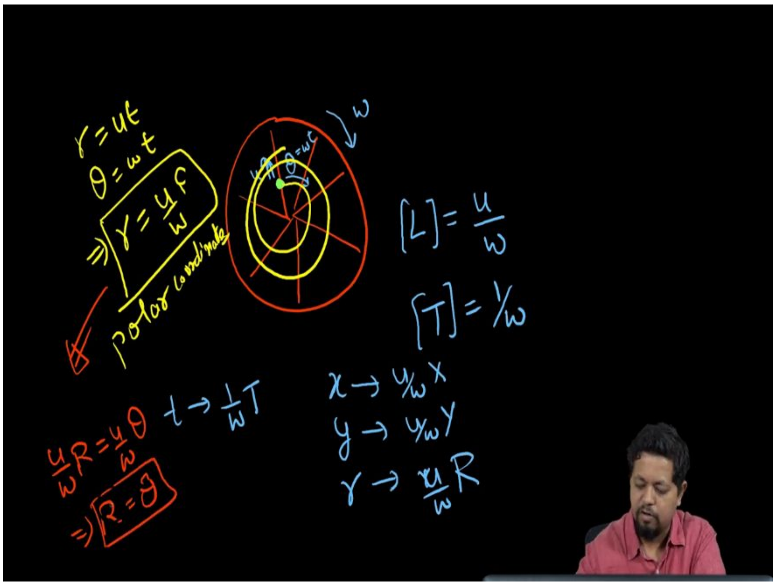

For parametric plot, we will use a function called parametric plot. This is a new function, you are seeing it for the first time. So, parametric plot works in the following way. Inside curly brackets, you get the x coordinate and the y coordinate. The x coordinate, in this case are $T \cos T$ and y coordinates are $T \sin T$.

I am going to make a plot for T from 0 to 20 and I will label the axes as the x and y. When I execute that, I got the out-spiral. This is exactly what we expect the trajectory of the bead to be on the wheel as the bead bead moves outwards and the wheel rotates with an angular velocity of ω. So, the bead, bead is going to have a trajectory that is spiralling out like this.

And that is what this plot shows. In order to make this plot we have to use the parametric plot function. Alternatively, we could also have used a polar plot function by using the r and θ coordinates.

So, for polar plot function, we need to know the equation of this in terms of polar coordinates and in terms of polar coordinates, we simply get $r = u\,t$ and substituting time in terms of θ that is eliminating time out of this equation, we get $r/\theta = u/\omega$ or in other words, $r = u\,t$, $\theta = \omega\,t$ eliminating time we get $r = (u/\omega)\,\theta$. So, that becomes my equation in polar coordinates.

For this equation in polar coordinates, I can make a plot directly in terms of polar coordinates, I can go ahead and do a non-dimensionalisation by making this translation r goes to u/ω and let me call it big R. So, for that particular purpose, this equation becomes

$(u/\omega)R = (u/\omega)\theta$, which gives me $R = \theta$ and that is nothing but an equation of an out-spiral. I can also make this by a polar plot.

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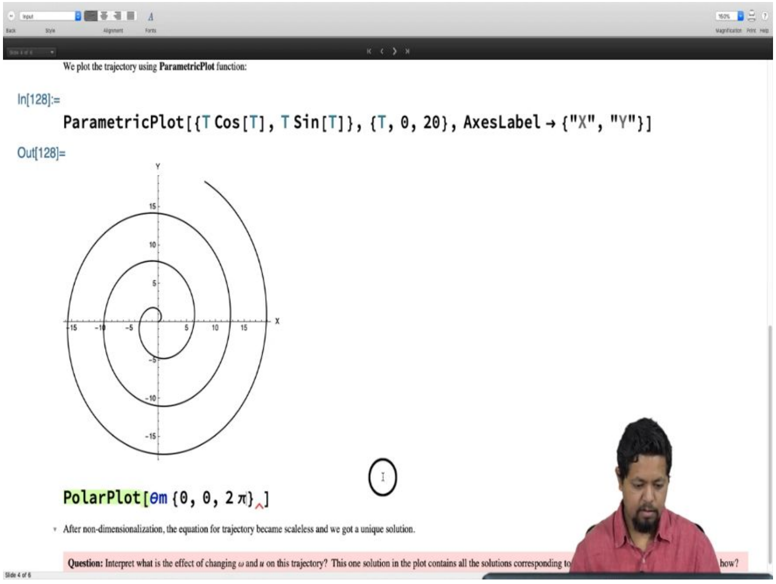

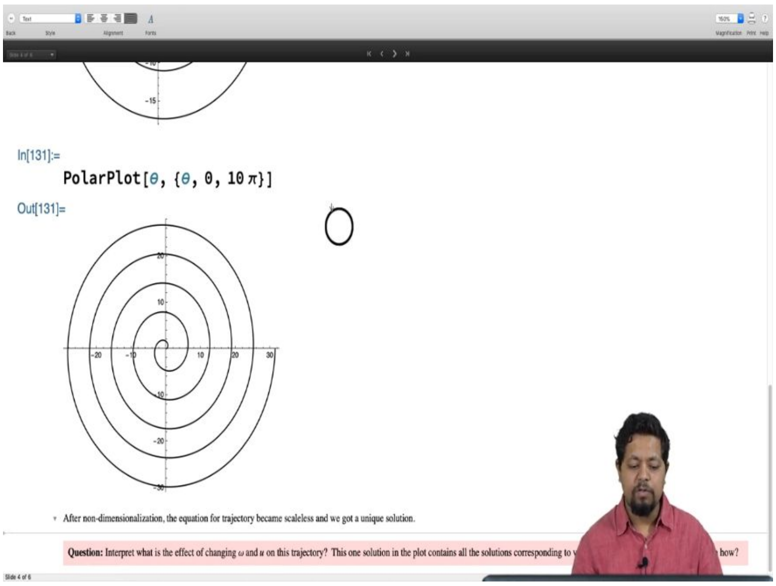

For polar plot, I give equation of R in terms of θ, in which case this is just θ and I say θ goes from 0 to 2π. This should be θ. I got an out-spiral just like over there, but since I want to cover a few circles I should make this maybe 10π and there we go. We get the same plot as over there.

In this particular case, we use the parametric plot by conversion, converting the coordinates to x and y coordinates and making a parametric plot with respect to dimensionless time for x and y coordinates. Over here we use directly the polar plot function to make a plot using polar coordinates where you have to specify the first argument as R as a function of θ and provide a range for θ.

So again, there are multiple ways of making the plots and you have to find the most reasonable way of making a plot given the problem, given the coordinate system and whichever is most suitable and and easier to use. So, finally, let us interpret from this solution. So the question that I have for you is what is the effect of changing ω and u on this trajectory, if I change ω and u, how is this trajectory going to change?

And as you already know that changing ω and u is not going to change this trajectory. That is this one solution in the plot contains all the solutions corresponding to various values in u and ω. Can you explain how that is possible that a single solution covers all the values of u, ω, or what is the effect of u and ω on this plot? Give it a moment of thought, and I will explain it to you in a moment.

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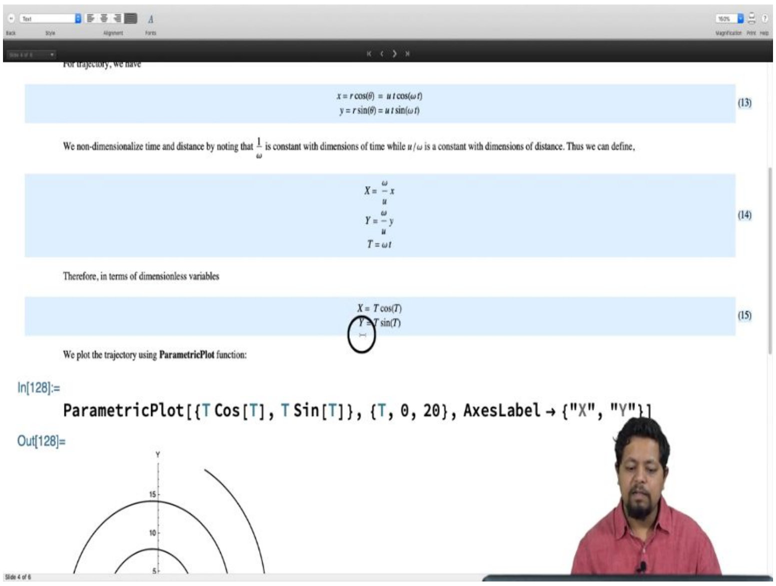

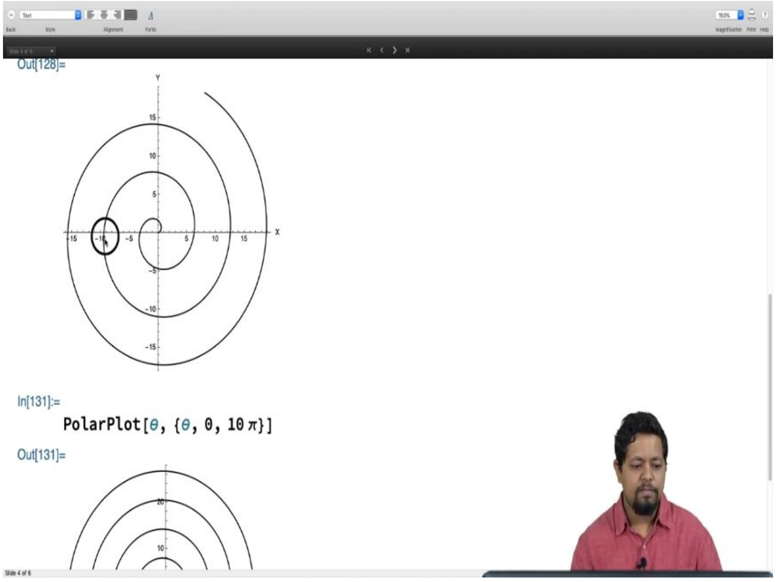

The equations that we have obtained over here do not contain any u and ω, they are independent of u and ω that is why our solution or this plot is independent of u and ω. So, what does u and ω actually change? u and ω in this case change the length scale, the units in which we measure X, Y and T.

As we change u and ω, we change the scale which measure u, Y and T. On this particular plot the meaning of one unit of X or one unit of Y will change if I change u and ω. Similarly, if I change ω, the unit, the time is moving along this trajectory whether this is a fast out-spiral or a slow out-spiral that will depend on what is the frequency omega.

If the omega frequency is very high, then the time scales are small and the consequence is that wheel spinning faster than and it is moving slowly outwards and the consequence of that will be, this will be a slow out-spiral. If ω is small then this is going to be a fast out-spiral that means the bead will move very fast on this out-spiral. If the ω is small the bead will be moving very slowly on this out-spiral.

So, by changing u and ω, we change the length in the time scale. In this particular problem, our solution remains, the non-dimensionalised version of the solution remains independent of the parameters of the problem that is u and ω.