Quantum Field Theory
Prof. Dr. Prasanta Kumar Tripathy
Department of Physics
Indian Institute of Technology, Madras

Module - 5
Radiative Corrections
Lecture - 34
Vertex Correction III

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We have been computing the one look correction to the vertex operator. And this is the diagram for which we need to evaluate the amplitude, you have an incoming electron p and outgoing electron with momentum p prime. It observes a virtual photon of momentum q, the propagators is momentum k and hence this one is p minus k, this is k prime this is k plus q.

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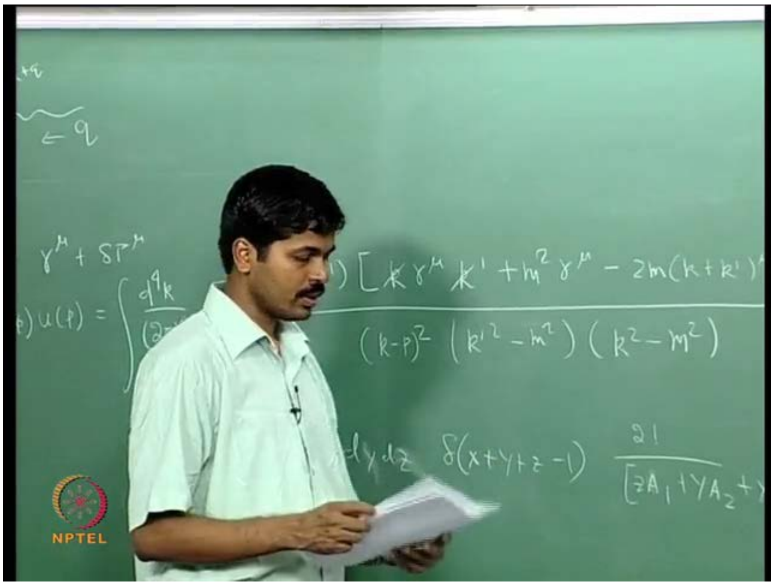

So, this is what we need to evaluate and then the vertex operator gamma mu to first order will be given by gamma mu plus delta gamma mu, where delta gamma mu is such that u bar of p prime delta gamma mu of p prime p u of p it is a given by d 4 k over 2 pi to the power 4th 2 i e square u bar of p prime k plus gamma mu k prime slash plus m square gamma mu minus 2 m k plus k prime mu u of p divided by k minus p wholes square k prime square minus m square k square minus square.

And we need to evaluate this integration to evaluate this integration what we did in the last lecture is we have introduced this identity it is A 1 A 2 A 3, this is equal to integration d x 1 d x 2 d x 3 delta of; or I will use x y z x plus y plus z minus 1 2 factor l divided by z A 3 or z A 1 plus y A 2 plus x A 3 whole cube, this is the identity that we have used to express this denominator in this for.

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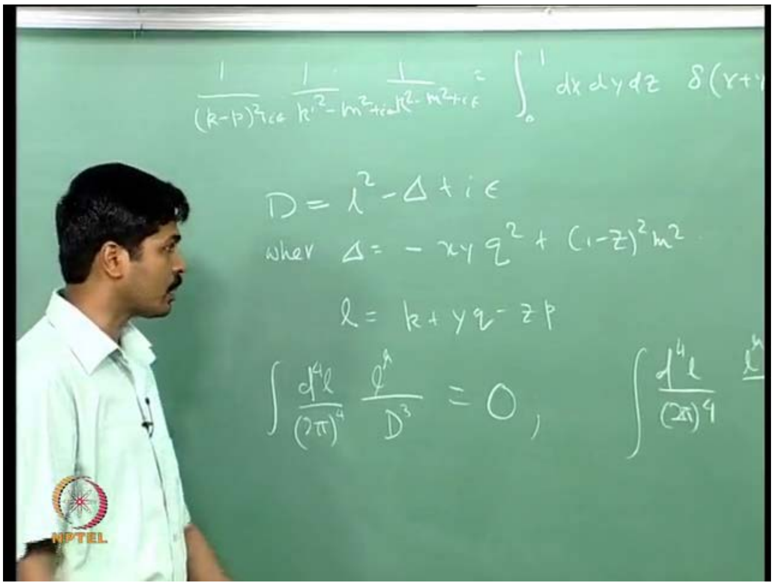

So, when we use the identity this quantity here 1 over k minus p whole square 1 over k prime square minus m square 1 over k square minus square, this turned out to be 0 to 1 d x d y d z delta x plus y plus z minus 1 times 2 over d cube. Where after some simplification this factor in the denominator D turned out to be l square minus delta plus i epsilon because we have to add i epsilons here. Where this delta is given by minus x y q square plus 1 minus z square m square, this is what we have derived in the last lecture.

What we will do, now is we will simplify the numerator and then put all these things here in this integration. Even before simplification we just note is the following think, the denominator here and also finally, what we are doing is we are changing the integration variable from k to l, where l is given by k plus y q minus z p. You see the denominator here is an even concern of l, so any integration this form d 4 l divided by 2 pi to the power 4'th l mu divided by D cube this is going to be 0.

Because, D is an even concern of l, on the other hand this quantity d 4 l divided by 2 pi 4'th l mu over D cube what is this going to be, this is going to be 0 if mu is not equal to mu. And hence it is also symmetric and the exchange of mu and mu, so the symmetry requires that it must be proportional to eta mu nu. Now, if it is proportional to eta mu nu it is very easy to computing the proportional to factor.

So, it is just face that this has to be equal to some constant which I will call as a let us call it as rho times d 4 l divided by 2 pi to the power 4'th eta mu nu divided by d cube.

So, what should l be is very easy to compute what l is rho, what is rho it is very easy to compute what row is, you just take the trace here and then you will see that this is nothing but 1 4 th l square. So, these are the two identities that we will be using to simplifying the numerator.

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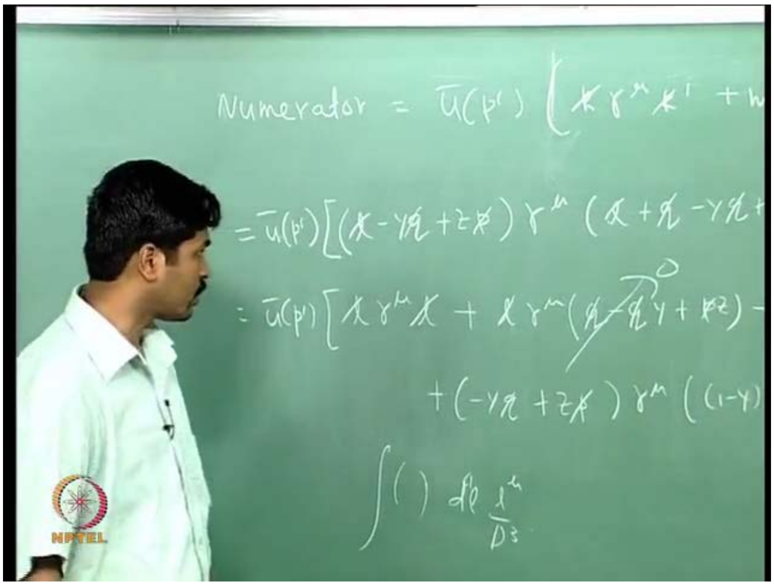

So, let us see what the numerator is now it is a given by u bar of p prime k slash gamma mu k prime slash plus m square gamma mu minus 2 m k plus k prime mu u of p, number of things we need to keep track of k prime is k plus q and l is k plus l q minus z p. So, in place of k at k prime we will use l p and q, so we will eliminate k and k prime from this equation. So, when we do that is you just u bar of p prime k slash is just l slash minus y q slash plus z p slash gamma mu l sash plus q slash k prime is l k plus q and for k we have to put this.

So, minus y q slash plus z p slash and then you have m square gamma mu as it is and then minus 2 m q mu k plus k prime is going to be this minus 4 m l minus y q plus z p mu this time u of p. So, now, what we will do is we will you just see this is u bar p prime the first the term will give me l slash gamma mu l slash, and then you will have a term which is linear in l. So, it will help l slash gamma mu and this quantity here, q slash minus q slash y plus p slash z right, and then again you will have minus y q slash plus z p slash gamma mu l slash.

And, so let us now see this what we will get when they will put this in the inside this integration, this quantity here will contain some operator which is independent of l times d 4 l l mu over D cube right. This is what the second term will give me, none of this thing depend on l, the only thing that depends on l comes as linear. So, when I put it inside the integration it is going to be 0. So, what I will do is that I will drop this term, same thing about this, because again this is linear in l mu. And here this will drop out when I will put this thing inside the integration.

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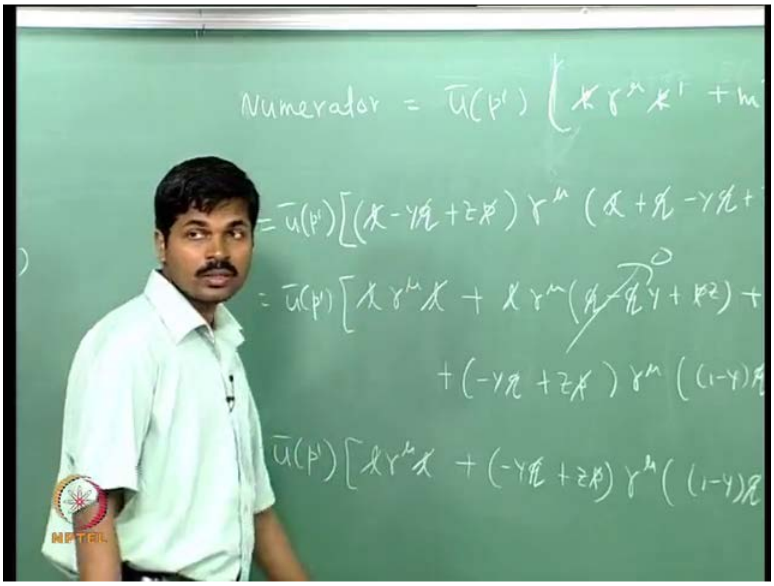

So, what I will do is that I will write this quantity is equal to u bar of p prime l slash gamma mu l slash plus minus y q slash plus z p slash gamma mu 1 minus y q slash plus z p slash plus m square gamma mu minus 2 m q mu minus 2 y q mu plus 2 z p mu u of p, where this quality by this equality what I mean is that this equality hold within this integration.

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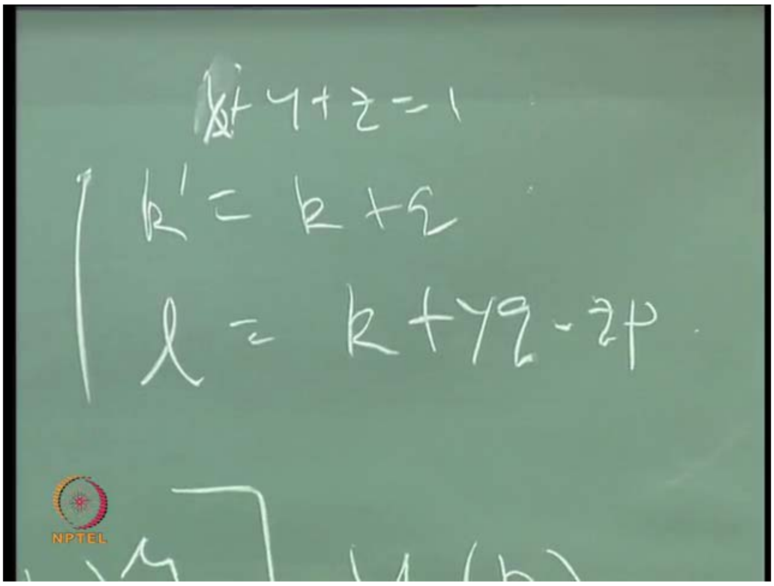

All these equalities here the whole subject to this constraint x plus y plus z equal to 1 and the integration.

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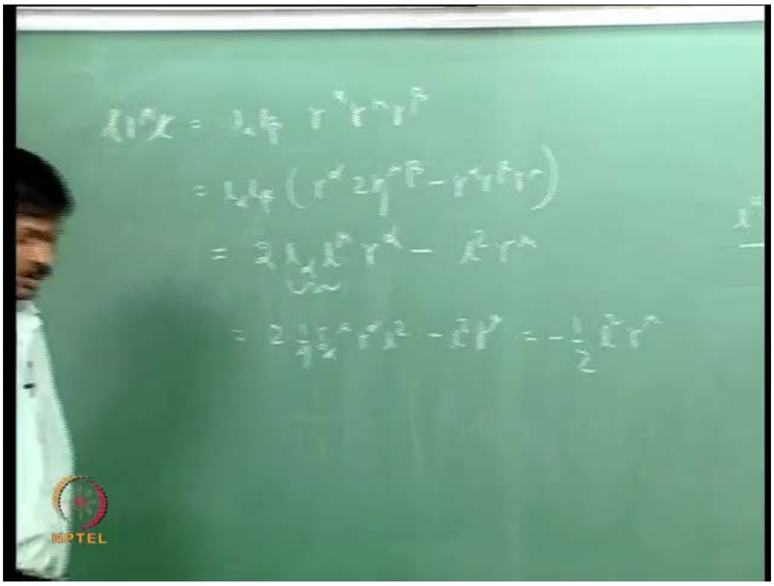

So, we can further simplify this for example, let us look at the first term the first term is l slash gamma mu l slash. So, this is equal to l alpha l beta gamma alpha gamma mu gamma beta I will quick here this alpha and beta, so this is l alpha l beta times gamma alpha 2 eta mu beta minus gamma alpha gamma beta gamma mu, what will I get share I will get a twice l alpha l mu gamma alpha from the first term. And your second term will get me l alpha l beta gamma alpha gamma beta is just l square gamma mu.

Now, again I will use this identity l mu l nu equal to 1 4'th eta mu nu l square, this is not true in general, but this is what is well it inside the integration. So, for that I will just write l alpha l beta is twice 1 4 th delta alpha mu gamma alpha times l square minus l square gamma mu. So, this simply gives me minus half l square gamma mu, similarly is there any will be simplification, so we will put we will substitute this here.

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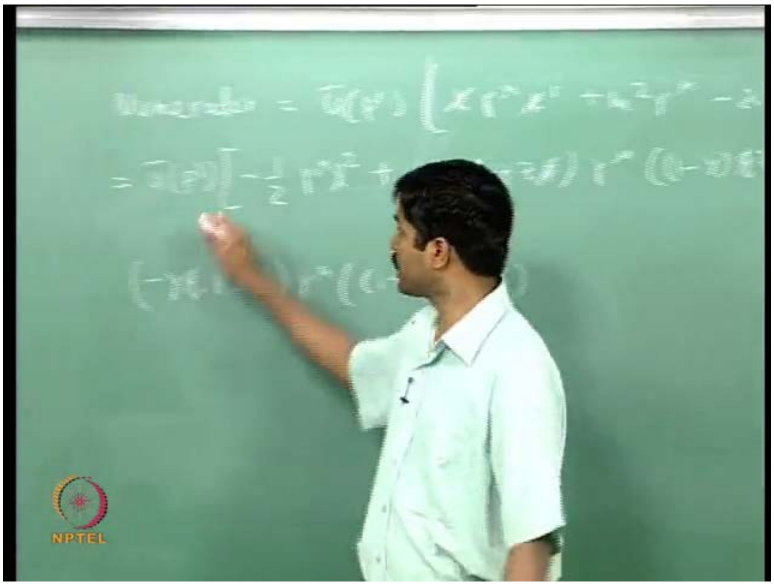

Then what it looks for the numerator looks a look like this is u bar p prime minus half gamma mu l square plus minus y q slash plus z p slash gamma mu 1 minus y q slash plus z p slash plus m square gamma mu minus 2 m 1 minus 2 y q mu plus 2 z p mu u of p, this is what we got for the numerator. So, let us do some more simplification for example, you can concentrate on this term here, this is just minus y q slash plus z p slash gamma mu 1 minus y q slash plus z p slash.

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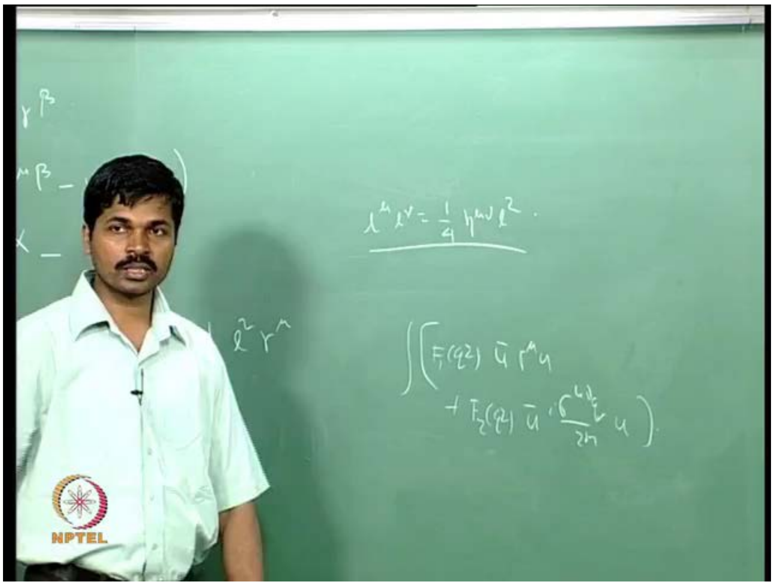

So, the purpose here is the pulling, we know we ultimately this integration the numerator here can be written like this it is the power factor F1 of q square and u bar gamma mu u plus F 2 q u bar sigma mu nu q nu over 2 m or something right i sigma mu nu q nu over 2 m u it should be of this form this is what we had argued in one of this lectures. So, here we can write everything in this form that is what we will try to do what our goal would be is to show this numerator ultimately is a equal to this.

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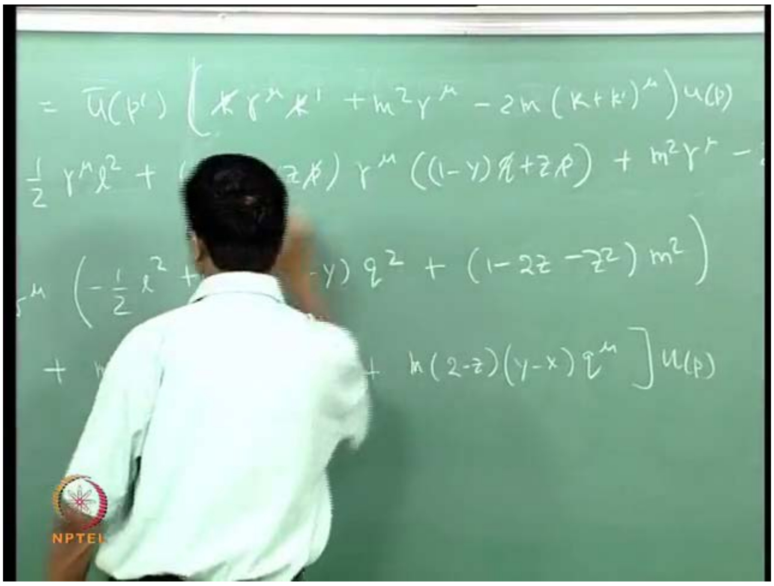

Finally, after simplification what we will show is that this numerator here turns out to be u bar p prime gamma mu times minus half l square plus 1 minus x 1 minus y q square plus 1 minus 2 z minus z square m square plus m z z minus 1 times p prime plus p mu plus m times 2 minus z y minus x q mu this terms u of p.

So, this is what we will show and then we will carry out the integration the l integration, after that we will carry out the x y z integration finally, will get the vertex correction whatever we have we will get. So, let us try to show that this numerator here indeed can be written in this simple form, you how to make some rearrangements, and then you have done.

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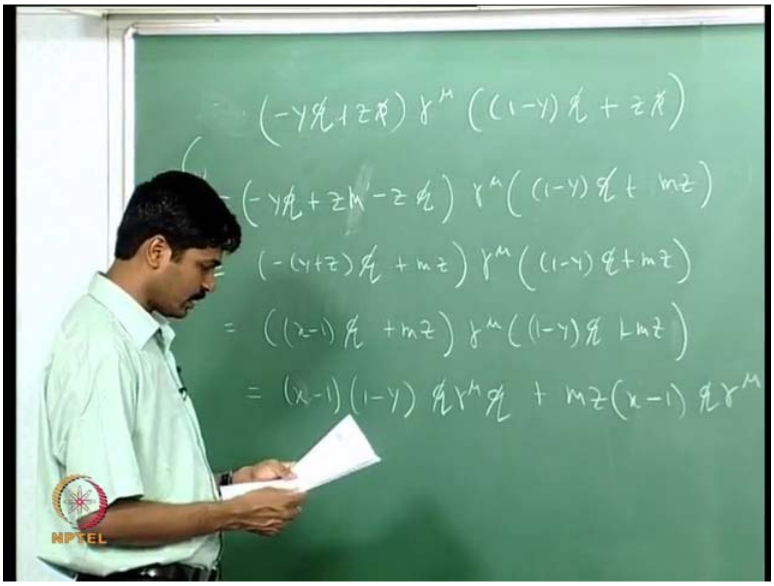

So, let us see how can we show this, so we will first simplify this term here minus y q slash plus z p slash gamma mu 1 minus y q slash plus z p slash. Now, I know this has to be evaluated within u bar p prime and u p and we already know this identities p slash u p is m u p and p slash acting from right will give me u bar p prime m. So, these are the this identity we know, we also know x y and z are constraint by this relation and we know that q is equal to p prime and minus p.

So, these are the relations that because you have this is q this is p this is p prime. So, this follows from here, this is the relation we know. So, what I can do is I can straightaway write here, so because this there is a u bar p prime in the right if I eliminate this p in terms of q and p prime, then I can use this identity here p prime slash right. So, let us do that, so this is equal to minus y q slash plus z into p prime minus q slash.

So, z p prime plus minus z q slash gamma mu 1 minus y q slash plus z p slash I will just write is m times z again this we are considering the identities I am in the equalities which are evaluate only within the, so alright let us right this is also it is m times p m times z. So, this now simplifies to minus y plus z q slash plus m z gamma mu 1 minus y q slash plus m z minus y minus z that from here is x minus 1. So, this quantity here is a x minus 1 q slash plus m z gamma mu 1 minus y slash plus m z.

Now, I can multiplied by brute force, so this is equal to x minus 1 1 minus y q slash gamma mu q slash plus m z x minus 1 q slash gamma mu m z 1 minus y gamma mu q slash plus m square z square gamma mu.

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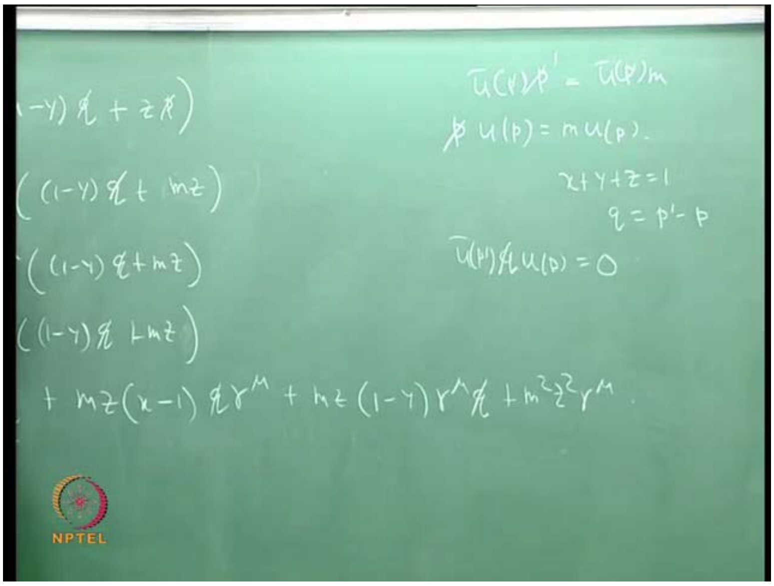

Alright now let us simplify this term here, we will use one more identity which is q slash u bar p prime u of p is equal to 0 why is this, so that because this is simply q slash is p prime slash minus p slash, when p prime adds on this from right it will give you an m when we add on this from the left it will again gives me m. So, that is 0, so therefore, primes if I keep that in mind.

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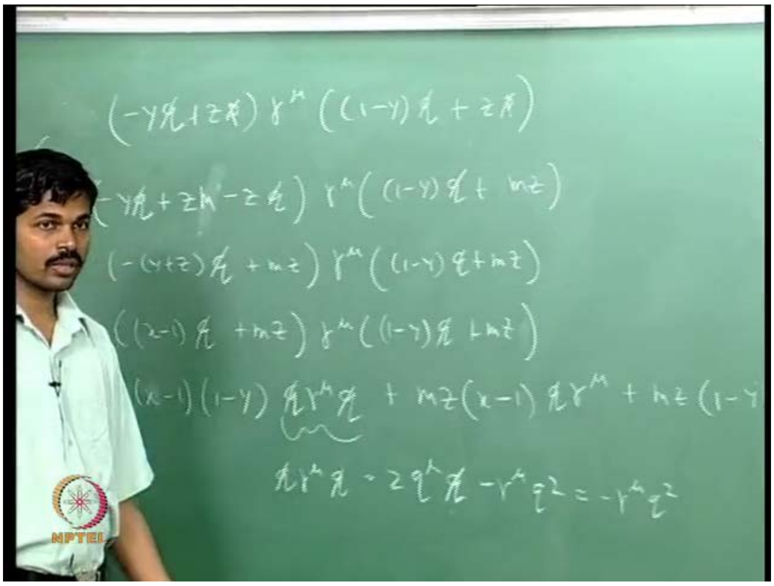

Then this quantity which is q slash gamma mu q slash which is 2 q mu q slash minus gamma mu q square, but the first term will give me 0 because of this identity. So, this can simply be written as minus gamma mu q square.

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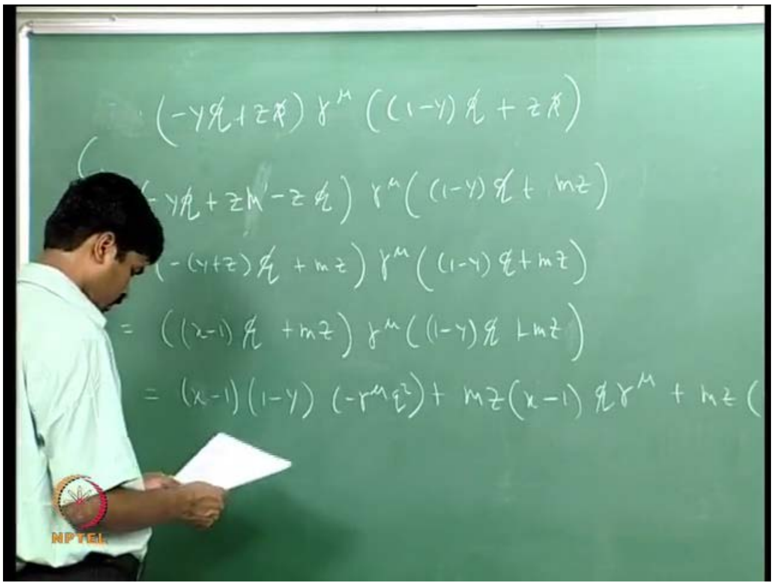

So, let us do that, so here minus gamma mu q square this one maybe I will let us now look at this.

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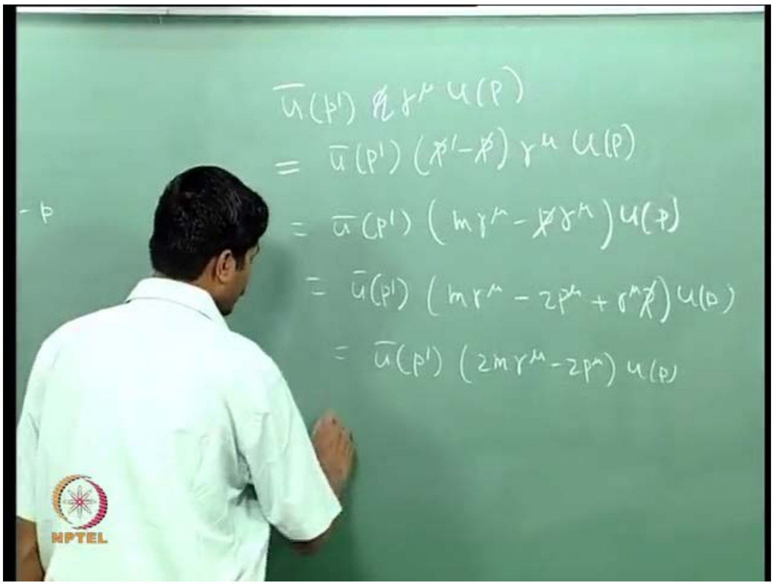

So, what we need to look at is u bar p prime q slash gamma mu u of p, in place of q slash I will use this. So, it is u bar p prime p prime slash minus p slash gamma mu u of p this is simply m here, so u bar p prime m gamma mu minus p slash gamma mu u of p. But, now if I flit this here, then p slash gamma mu is nothing but 2 p mu minus gamma mu p slash. So, when I use this identity then this is u bar p prime m gamma mu minus 2 p mu plus gamma mu p slash u of p. But, now since p slash is on the right it will give me m gamma mu, so what I get is u bar p prime twice m gamma mu minus twice p mu u of p.

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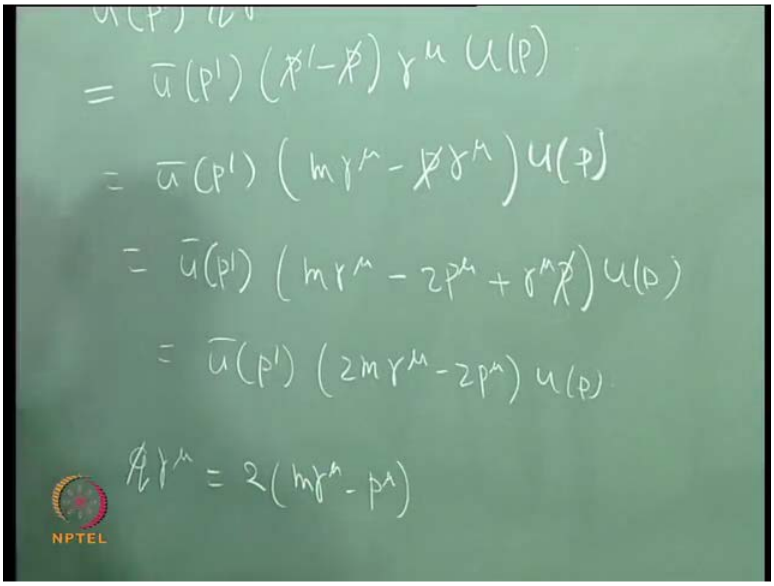

So, this quantity here when I simplify now, what I get s for s this identities are concerned q slash gamma mu is simply equal to twice m gamma mu minus p mu. So, we will substitute that here and then you will do some rearrangements.

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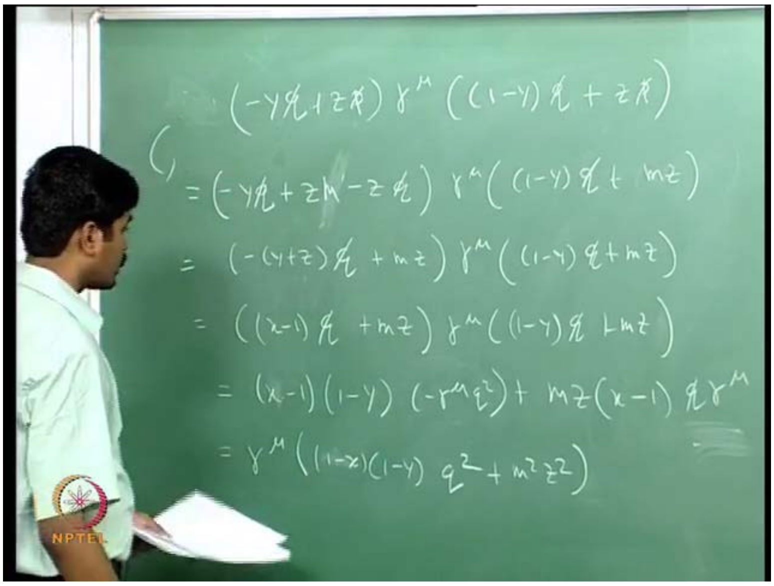

So, let us write it as gamma mu times 1 minus x 1 minus y q square that contribution for this term and from here I get m square z square plus m square z square.

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And then there is a term it is linear in q mu, so this gamma mu q you write it as, again if you flit this what you get is 2 q mu minus q slash gamma mu. So, that I will combine here to write plus 2 m z 1 minus y q mu from here, then what I will do is that I will combine this term and determine coming from here that will give me plus m z 1 minus y x plus y minus 2 q slash gamma mu.

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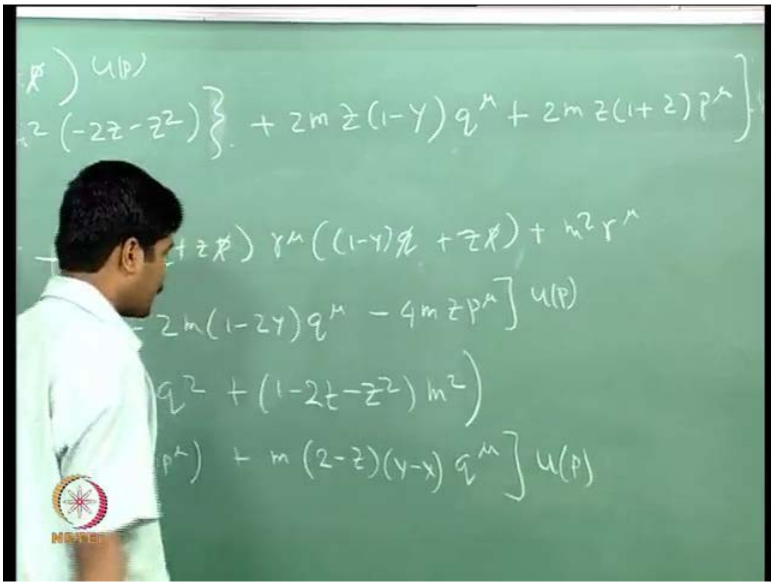

Now, what I will do is, I will in place of x plus y minus 2 what I will do is I use this identity here x plus y is 1 minus z x plus y minus 2 is minus 1 plus z. So, and finally, I will use this identity here, which is q slash gamma mu is equal to twice m gamma mu minus p mu. So, when you put all these what I will do is that I will not do is, but it is a very straightforward algebra, you have to do is two three more steps.

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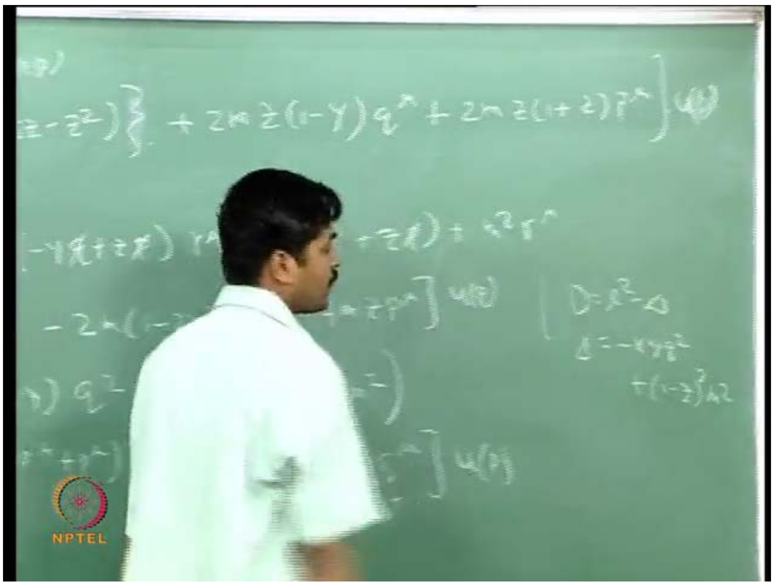

What you find is minus y q slash plus z p slash gamma mu 1 minus y q slash plus z p slash that is given by gamma mu 1 minus x 1 minus y q square plus m square minus 2 z minus z square plus twice m z 1 minus y q mu twice m z 1 plus z p mu, this is what you will get it is a very straightforward thing. This quantity here again what I can do is that I can write it as, p prime plus p mu time some quantity and q mu times some quantity right, what you do is the following.

So, what I do is I mean you can do it a whatever way you like, so this is already there you I mean it is already simplified, you need not simplify the first term the last two terms here. This is only one of the terms in the numerators, the numerator is whether terms also over let me remind you u bar p prime minus half gamma mu l square plus this quantity here minus y q slash plus z p slash gamma mu 1 minus y q slash plus z p slash plus m square gamma mu minus twice m 1 minus 2 y q mu minus 4 m z p mu u of p this is our numerator.

And we have evaluated this term here, actually we have evaluated u bar p prime u p this you have shown is u bar p prime u p. This is what we have evaluated, so we will substitute this here, and then what you do is that you write in terms of the coefficients of p mu and q mu, what you will get when you put this result here is u bar p prime gamma mu minus half l square plus 1 minus x 1 minus y q square plus 1 minus 2 z minus z square m square.

And then twice m 1 minus y times z minus 1 minus 2 y q mu plus twice m z square minus z p mu u of p this is what you will get, these two terms here you can rewrite them in the following manner you can, so that these two terms can actually be combined. And then I can write this as m z into z minus 1 p prime mu plus p mu plus m 2 minus z y minus x q mu u of p right. This is what we got, after doing this radius competition.

But, now you see, if you remember this the contribution from the term which is linear in q mu actually what 0, when we use to word identity we have argued that word identity implies this term here is 0. But, now you get something which is none 0, we will show that this term actually when says when we put it inside the integration, the argument is the following they here this function is odd in x and y.

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But if you look at here denominator the denominator contains D equal to l square minus delta, and this delta here is minus x y q square plus 1 minus z whole square m square. So, this is given under the exchange of x and y whereas, this is odd under the exchange of x and y, so if you carry out this d x d y integration x minus y times some even function of x and y, then this is going to be 0.

Because, you can change the variable sectioned y, and then you will get a minus sign, so this quantity will be equal to itself minus time itself. So, this is 0 therefore, this the contribution from the last term is 0, so ultimately what we get when we use that is that the numerator will simply be equal to this times u of p alright.

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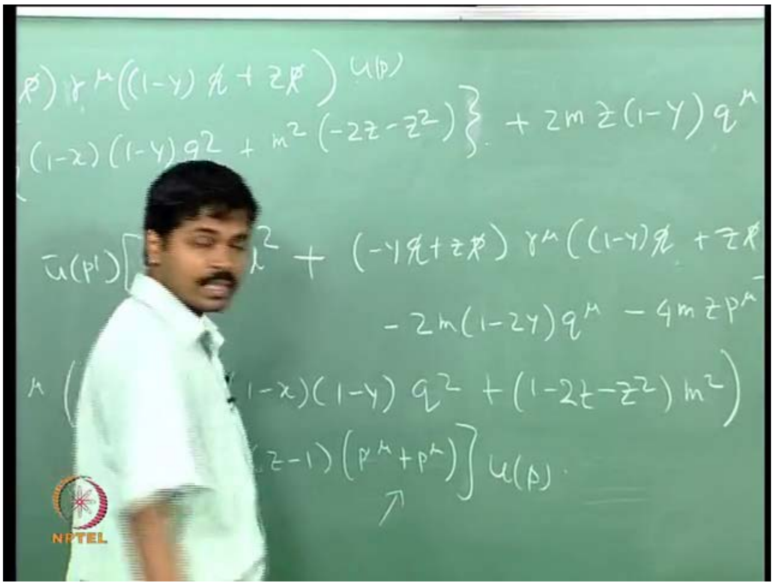

Now, what we will do is the again we are not done completely done here, we want to write this the integration in terms of the form factors. So, you want to derived correction to the form factor, so far that we will use the golden identity which we have proved earlier.

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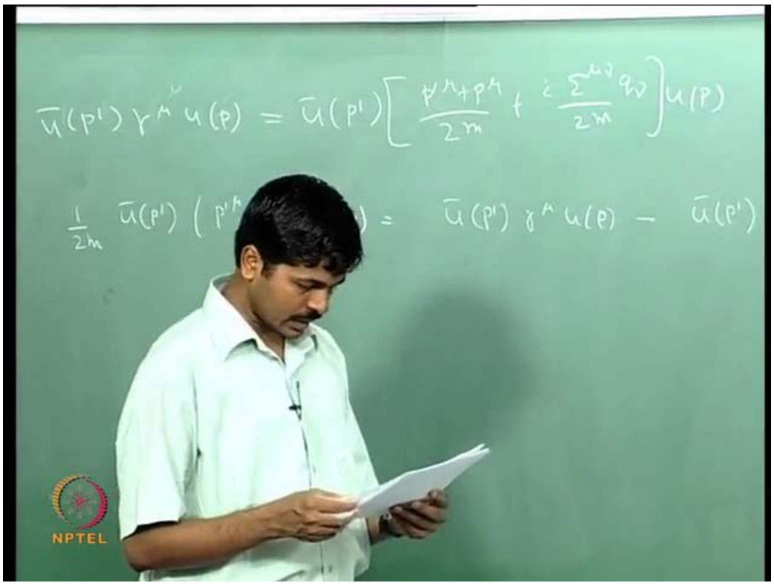

So, if you remember u bar p prime gamma mu u of p we have shown is equal to u bar p prime p prime mu plus p mu divided by 2 m plus i sigma mu nu q mu divided by 2 m u of p. So, we will consider this term here 1 over 2 m u bar p prime p prime mu plus p mu u of p, we will write this to be equal to u bar p prime gamma mu u of p minus u bar p prime i sigma mu nu q nu over twice m u of p.

We will use this, we will replace this with this in this numerator here, and then we will carry out the integration. And finally, we will see that we get a first order correction to the anomalous magnetic moment for electron, but that we will do in the next lecture.