Quantum Field Theory
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Module - 04
Quantum Electrodynamics
Lecture - 30
Moeller Scattering-I

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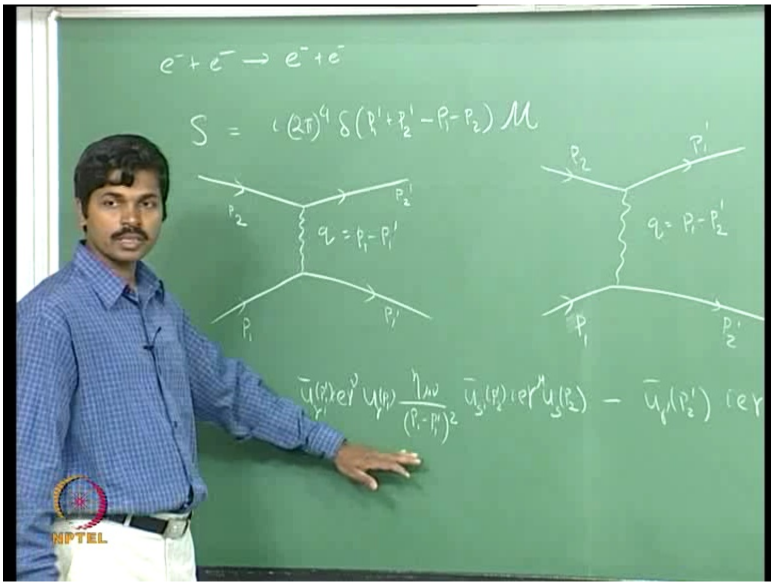

Before we move to a new topic, we will do one more example that of electron scattering. So, e minus plus e minus going to e minus plus e minus and then, we will compute the differential scattering through section. For this process, we already know the S matrix to lowest order which gives a contribution given by i 2 pi to the power 4 delta of P 1 prime plus P 2 prime minus P 1 minus P 2 times amplitude M.

There are two diagrams which contribute to this process at lowest order and these two diagrams are given by two incoming electrons of moments are P 1 and P 2 and then, the outgoing electron of moment P 1 prime P 2 prime. The expense of actual proton momentum is given by P 1 minus P 1 prime. The other diagram is obtained from this one by exchanging these two out going electrons. So, this is the diagram. For the second process here, we have an incoming electron of momentum P 1 P 2, but this one is an actual momentum P 2 prime and this one Lagrangian momentum is P 1 prime. So, the virtual for 10 carries an actual momentum which is given by q equal to P 1 minus P 2 prime.

We can use the fine man rules to write down the amplitudes for these two processes and then, we will write the amplitude keeping in mind that there is a relative minus sign we have to use because of the exchange interaction. So, the amplitude, the fine man amplitude for this process ultimately is given by, here I will start from this one. You have this incoming electron u s of P 2 and then, there is this vertex which is i e gamma mu and then, you have this out going electron which I will write u bar s prime of p to prime. Then, you have to propagate. So, for a proton propagator, I have written it as mu nu divided by mu q square which is P 1 minus P 1 prime square and then, I have an incoming electron for this. I will write u r of P 1 and then, i of vertex for which I will write i e gamma mu and I have on outgoing electron for this. I will write u bar r prime of P 1 prime. The second one is obtained from the first one just by x and the P 1 prime and P 2 prime.

So, the amplitude for the second one with a minus sign and then, I will write u bar r of P 2 prime r prime i e gamma mu u r of P 1 with a mu nu divided by P 1 minus P 2 prime whole square and u bar s prime of P 1 prime i e gamma mu u s, alright. So, what we have to do now is, we have to just consider the differential scattering cross section formula for the differential scattering cross section and then, substitute this for fine man amplitude and then, simplify the formula to get the final answer.

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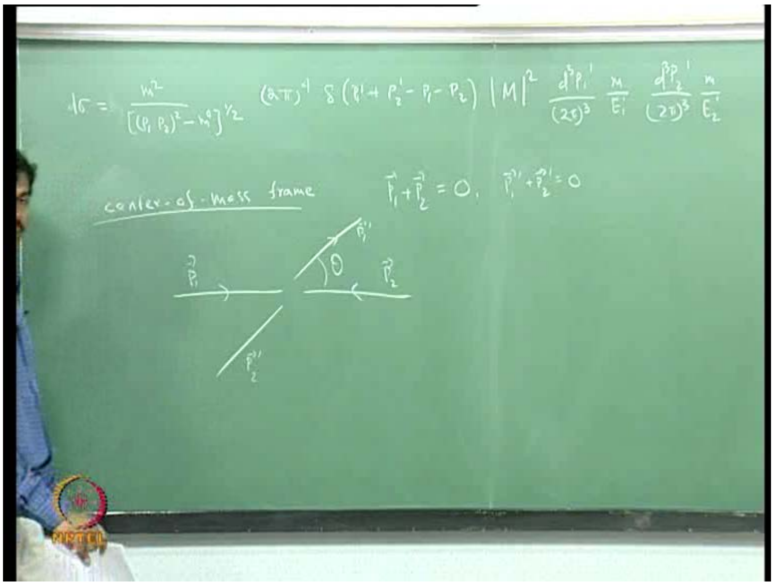

So, what is the differential scattering cross section? When you have two outgoing to incoming fermium and two outgoing fermium, the differential scattering cross section is given by d sigma. You have to remember the phase factor integration method etcetera. We pick it differently for fermium. So, for fermium this is m square divided by P 1 dot P 2 whole square minus m 4 to the power half and then, you have 2 pi to the power 4 delta P 1 prime plus P 2 prime minus P 1 minus P 2 mode m square. Then, I will have d cube P 1 prime over 2 pi cube m over p 1 prime and then, d cube P 2 prime over 2 pi cube m over p 2 prime. This is what is with a differential scattering cross section.

Again, I will just integrate out this delta function and finally, I will get d sigma over d omega. So, all one is to do is to compute the mode m square and then, you already know how to integrate out the delta function. You compute the mode square, you put it, you plug it in this formula and then, you will get the final answer. You have to keep in mind that the energy momentum conservations hold. So, let us do that. What we will do is that we will now work the centre of mass time. So, in the centre of the mass time, the total momentum P 1 plus P 2 is equal to 0, and so is P prime plus P 2 prime. So, you have two electrons. There are two incoming electrons with momentum P 1 and P 2 equal and opposite magnitude and then, there are two outgoing electrons P 1 prime P 2 prime. This scattering angel is given by theta.

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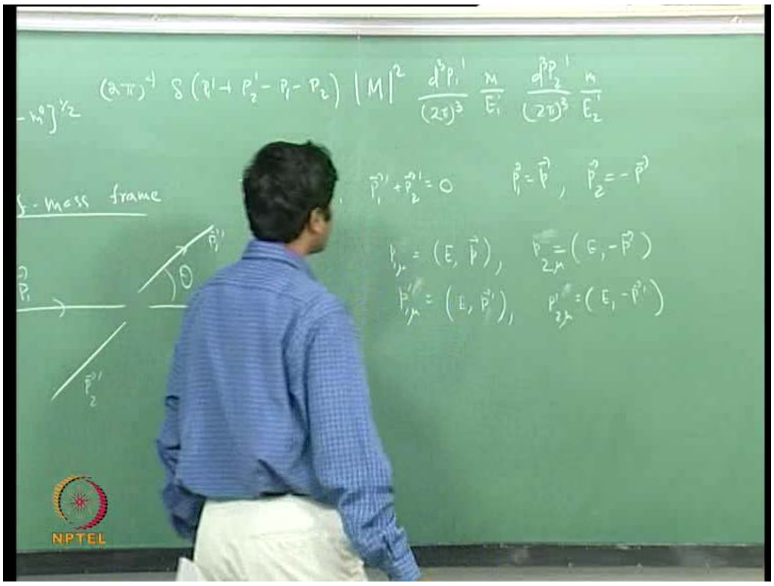

So, therefore, the 4 vector t 1 mu is basically E which is P 1 0 and I will call this P 1 to be, I will denote this to be P. Then, my P 2 will be minus P. So, you have this P 2 is equal to P 2 mu is gain P E minus P. Similarly, you have t 1 prime mu is simply P 1 prime or I will denote this to be P prime and P 2 prime mu equal to P minus P prime. So, I will do the entire calculation in the centre of mass prime where the 4 momentum, this 4 and then, I will evaluate mode square of this matrix element P 1 mu.

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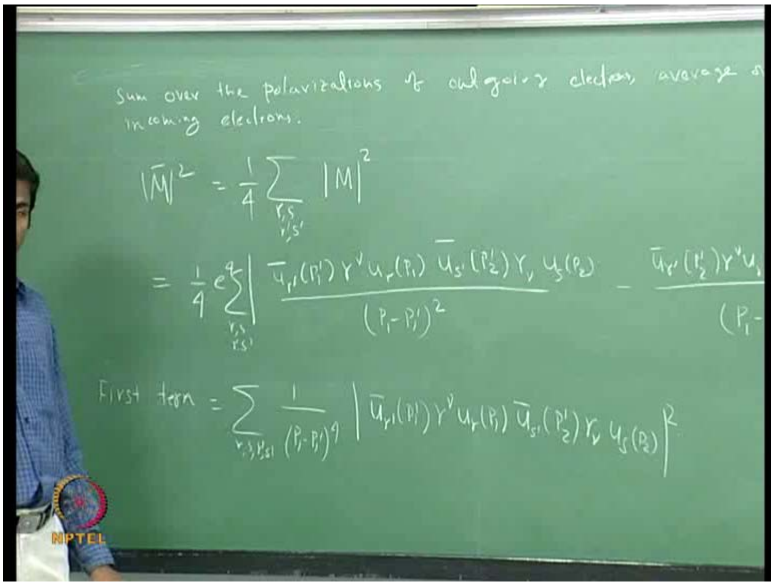

So, again what we will do is, we will sum over the polarization of outgoing electron and average over the polarization of the incoming electrons. So, I have what I will call as m square average is simply sum over r is r prime s prime mode m square. What is the factor? I will put 1 over 4 because there are two incoming electrons this is what I need to do. So, let us do it. So, this is 1 over 4. Now, there is an E one-fourth because there is the e square here, mode square and finally, what I have is u r prime bar P 1 prime gamma mu u r t 1 u s prime bar P 2 prime gamma mu u s P 2. I have used this theta mu nu to contract one of these gamma matrices. This is denoted by P 1 minus P 1 prime square and then, minus the same thing that P 1 prime and P 2 prime are extents.

So, u bar r prime P 2 prime gamma mu u r P 1 u bar s prime P 1 prime gamma mu u s P 2 divided by P 1 minus P 2 prime whole square, this mode square. This is what we need to evaluate. Again, I will get 4 terms, but you will see that we need to evaluate only two of the fourth term. The reaming two we can just obtain by exchanging P 1 prime and P 2 prime. So, the first term in this expression is 1 over 4 P fourth times. So, what I will do is, I will just instead of writing it and then deriving it, what we will do is we will consider mode square of this term and then, see if you can simplify that and finally, we will look at the first term and the reaming crust on and then, mode square of this will be obtained by exchanging 2 m prime and P 2 prime.

So, the first term in this expression is, forget about the 1 over 4 e one-fourth term. We will take care of it later. There is the summation over r s here, but r s r prime s prime. So, first term is some other r s r prime s prime and then, there is 1 over P 1 minus P 1 prime one-fourth and mode square of this u bar r prime P 1 prime gamma mu u r P 1 u bar s prime P 2 prime gamma mu u s P 2 mod square. We need to evaluate this, but you take on mind that this gamma mu here is the domain which is repeated. This is the number and this is also a number. So, this is you cannot simplify. So, the whole mode square is just mode square of this trans mode of this square of this, but all you have to keep in mind is that the first term you are evaluating the mode square, you have to use the different domain for the second one.

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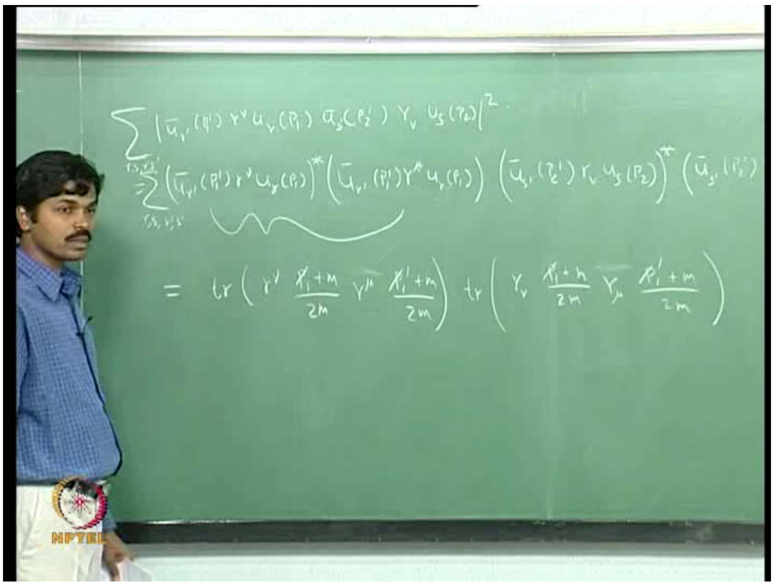

What I am saying is if you look at u bar r prime u n prime gamma nu u r P 1 u bar s prime P 2 prime gamma nu u s P 2 mode square by this, what I mean is u bar r prime P 1 prime gamma mu u r P 1 conjugate because this is the number and then, u bar r prime P 1 prime gamma mu instead of gamma nu u r P 1. Similarly, the second term you will give u bar s prime P 2 prime gamma nu u s P 2 conjugate of this and u bar s prime P 2 prime gamma mu u s P 2, right. This is what you are doing. So, I am using gamma mu, the index and leveling the index new for the complex conjugate of this and using the level mu for the term itself and then, I will just rearrange the term.

Now, what I need to do is, I need to consider this and sum over r s r prime s prime. So, here again I have to sum over the screen indices r s r prime s prime. You look at this only involves r, and r prime. The first two terms and the second two terms on the s and s prime and then, we know what we have already done this when we were scattering. So, what you get when you carry out this in sum in this fermium? Simple. So, if you just consider the first two terms, this will give you place of gamma mu and T 1 plus m divided by 2 m. Then, gamma mu bar P 1 prime plus plus m divided by 2 m. This is what you will get from the first two terms.

Similarly, you will get exactly the same expression from the second two terms. So, this whole thing is going to the product of two traces. One is this and then, the one is again case of the same thing, but mu and nu indices are called at it properly. So, this is gamma mu P 1 slash plus m divided by 2 m gamma mu bar P 1 prime slash plus m divided by 2 m, but gamma mu bar according our definition o bar is just gamma 0 over dagger gamma 0. Therefore, gamma mu bar is gamma 0 gamma mu dagger gamma 0 which is nothing, but if you use this expression for gamma mu dagger, it is gamma mu gamma 0 square. So, this is simply gamma mu. Gamma mu bar is gamma mu. Therefore, I will just remove the y here as same as from here. So, this is what we will get when we sum over this spin indices.

Now, all we need to do is, we need to evaluate this and then, we need to forget to get the first term. So, this is of course evaluating this is very easy because these symbols can be four gamma matrices. So, let us do that. So, what I will do is, I will just look at the first term here and then, the second term will have the similarity.

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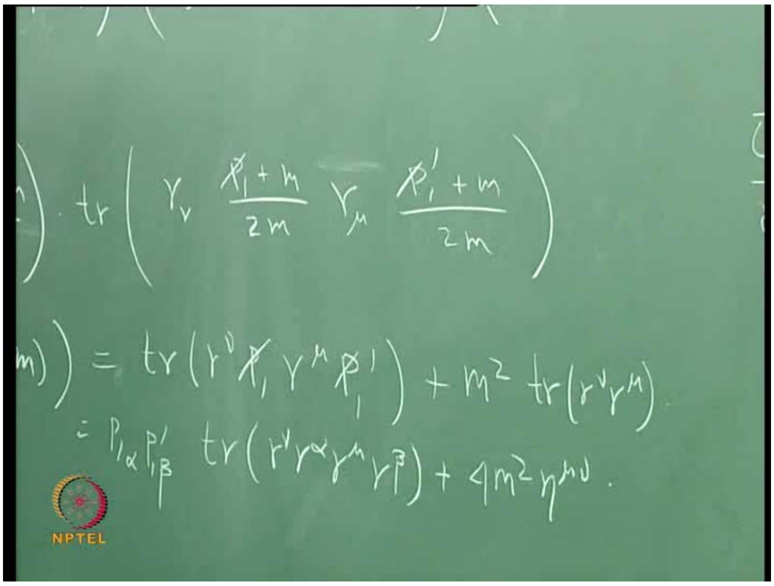

So, the first part involves evaluating p s of gamma mu P 1 slash plus m gamma mu P 1 prime slash plus m. So, this will help four terms, but two of them will have 3 gamma matrices will vanish identically. So, the reaming two terms are v s of gamma mu P 1 slash gamma mu P 1 prime slash plus m square phrase of gamma mu. So, this is what we have in the first part. This one is simply P 1 alpha P 1 prime beta times phase of gamma mu gamma alpha gamma mu gamma beta, where here this is plus 4 m square beta mu nu because it has gamma mu nu. It is of gamma mu nu is 4 times theta mu nu.

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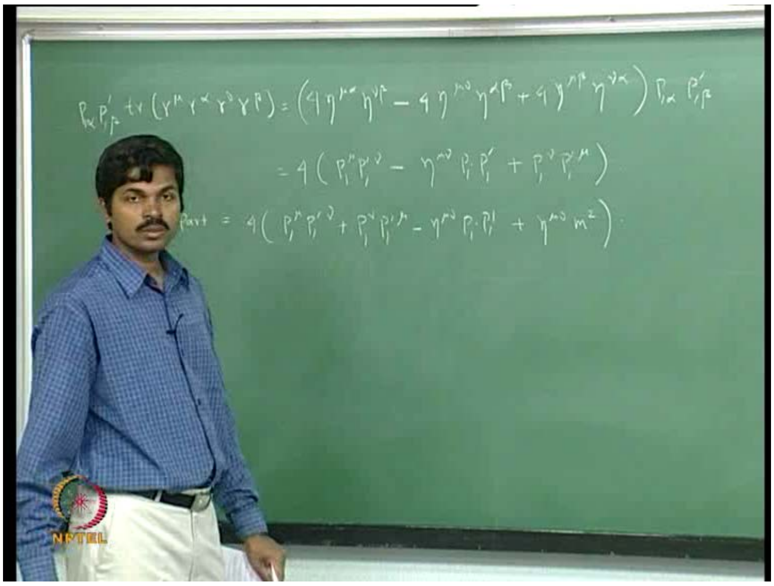

Now, we have evaluated this phase in the last lecture. This is simply given by phase of gamma mu gamma alpha gamma mu gamma beta is simply 4 beta mu alpha beta mu beta minus 4 beta mu nu beta alpha beta plus 4 mu beta mu alpha. This is the phase.

Now, what I have to do is, I have to consider this and multiply it with P 1 alpha P 1 prime beta. So, P 1 alpha P 1 prime will give me 4. Over all 4 of 4. I will just check it out. First term will be P 1 mu P 1 prime mu minus beta mu nu times P 1 dot P 1 prime, and this term plus P 1 mu P 1 prime mu. So, this is what I get for this term here. Now, I have to write 4 m square eta mu nu here to get the first part. So, when I do that I get the first part to be equal to 4 P 1 mu t 1 prime mu minus. I will write it first P 1 mu P 1 prime mu minus theta mu nu P 1 dot P 1 prime and then, finally plus eta mu nu times m square. So, the first part gives us this, the second part gives us the same term. Expect that mu and mu are the indices, mu and mu are contracted there propagate. Therefore, this quantity here will be there are 2 m to the power fourth one o over 2 m to the power forth.

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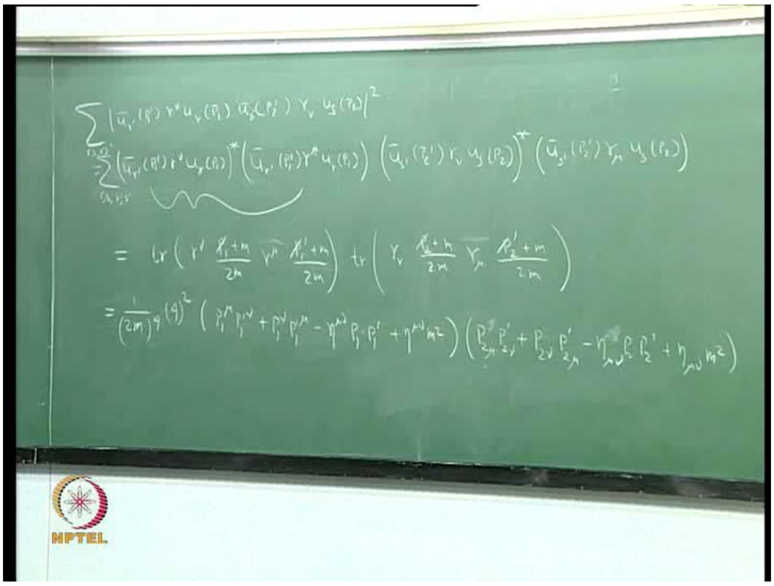

So, let us do that when I substitute that for the first part. What I get here is 1 over 2 m to the power 4 and then, there are 4 square two factors of 4. This quantity is P 1 mu P 1 prime mu plus P 1 mu P 1 prime mu minus theta mu nu P 1 dot P 1 prime plus beta mu nu m square, that is from this part and then, from this part again I will get an identical term. Second place is P 2, ok. Good. Thank you. So, this is P 2 and P 2 prime. So, all I will get is P 2 mu. So, the mu and mu are now co-variant indices and P. Instead of P 1 prime, I will get P 2 prime and P 2 prime mu plus P 2 mu P 2 prime mu minus theta mu nu P 2 dot P 2 prime plus theta mu nu m square, alright. So, good. So, now what I have to do? I have to multiply this term and then, I have to simplify them.

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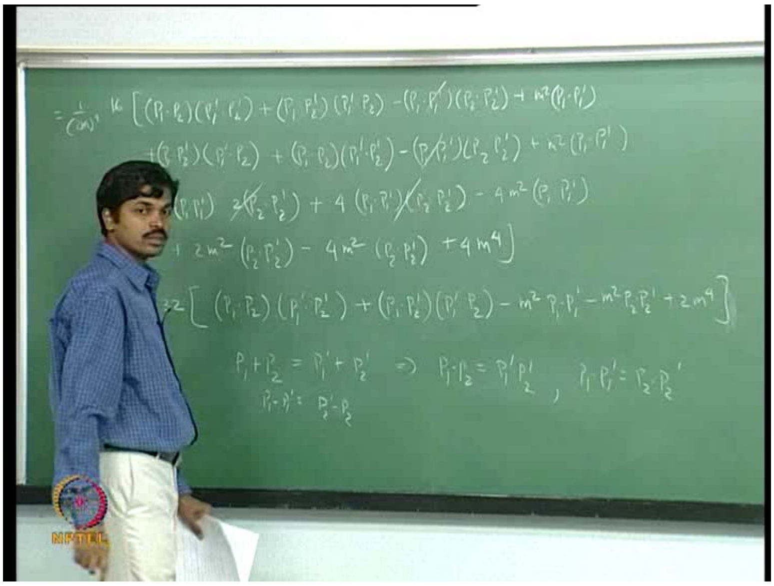

So, let us do that. So, this is equal to 1 over 2 m to the power 4, and this is 16. The first term will give me P 1 dot P 2, P 1 prime P 2 prime. So, P 1 dot P 2 P 1 prime and then, I will get P 1 dot P 2 prime and P 1 prime dot P 2. Then, I have to multiply this minus P 1 dot P 1 prime P 2 dot P 2 prime plus m square P 1 dot, that is for the first term and then, there are three more. So, the second term will give me P 1 dot P 2 prime. Yes sir. P 1 dot P 2 prime times and P 1 dot P 2 times P 1 dot P 1 dot P 2 prime. Now, that will multiply to the third. This will become P 1 dot P 1 prime P 2 dot P dot prime and finally, I will again that plus m square P 1 dot P 1 prime.

So, in other words, you could have simply concluded it that this term multiplied by this identical to this term multiply by this because this is symmetry conduct exchange of mu and mu and hence, they will just head up anyways. So, then I have to multiply eta mu nu with this last term. So, this will give me minus P 1 dot P 1 prime time, beta mu nu will give me P 2 dot P 2 prime with the factor of plus 2 P 2. Then, minus minus plus beta mu nu beta mu nu. What is this? It is 4. 4 P 1 dot P 1 prime P 2 dot P 2 prime and then, minus 4 m square P 1 dot P 1 prime. Finally, the last term will multiply with the second part which is plus twice m square P 2 dot P 2 prime minus 4 m square P 2 dot P 2 prime plus 4 alpha. That is all you have.

So, now, you see that sound will cancel here and some of them will head up. For example, P 1 dot P 1 prime this and this will head up and none of these terms will cancel to m square. This and this, two will have to cancel and then, these two will be cancelled and then, you will get factor of 4. So, what I will do is, I will just write down the answer for you is that m to the power 4 terms that will P 1 dot P 2 P 1 prime dot P 2 prime plus P 1 dot P 2 prime P 1 prime dot P 2 minus m square P 1 dot P 1 prime minus m square P 2 dot P 2 prime plus 2 m fourth, and I have taken the effectors of 2 over all factor of 2. So, this gives me 2 m 4. This term and this term will give me this. Adding these three terms will give me this. Similarly, you have this that will give me this one and adding this P 1 dot P 2 prime will give me this one. Finally, what is left is this. They will cancel, alright. So, there is sector of two here and there are 4 minus and these are cancelled.

So, these two head will give this and then, you have these two, these three, alright. This is what you get for the trace. Now, what I will do is, I am using defect that the energy moment is conserved. So, P 1 plus P 2 is just equal to P 1 prime plus P 2 prime. This implies the number of thing. First of all, this implies P 1 dot P 2 is equal to P 1 prime dot P 2 prime. This also implies P 1 dot P 1 prime is equal to P 2 dot P 2 prime. So, I will use these two relations here. You just square it and then, you remember that P 1 square P 2, both are equal to m. That will give you this relation. If you write this identity P 1 minus P 1 prime equal to P 2 prime minus P 2, then if you square now, then you will get this identity. Now, I will substitute it here.

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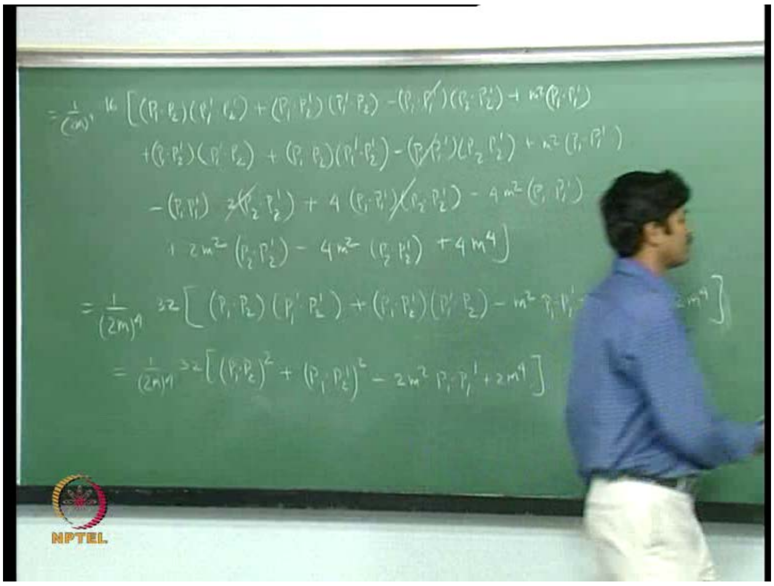

So, when I substitute this here, what I get is the following. This simply becomes 1 over 2 m to the power forth 32 times P 1 dot whole square. Similarly, these two are equal of, what I will get is P 1 dot P 2 prime whole square and these two will head up minus twice m square P 1 dot P 1 prime plus twice and forth. Now, what I claim is the following. I will just combine these two terms and then, write it in a simpler question.

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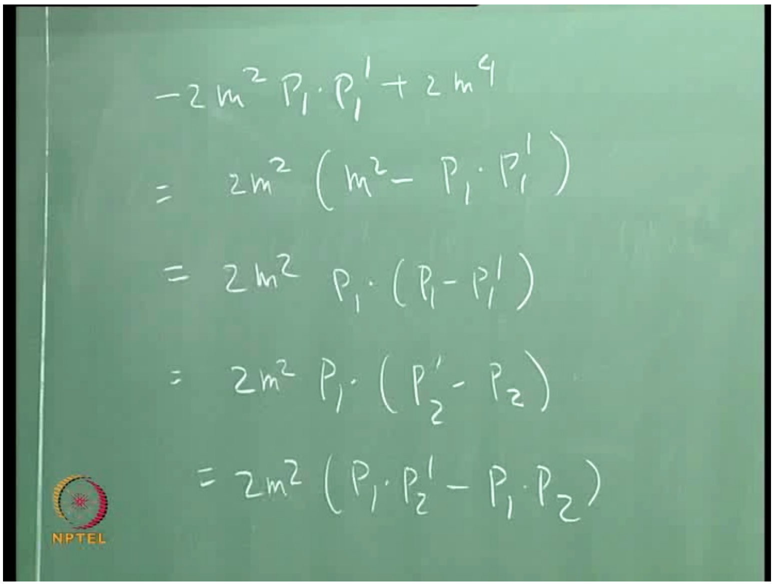

So, minus twice m square P 1 dot P 1 prime plus twice m 4 is simply equal to twice into m square minus P 1 dot P 1 prime which is nothing, but m square. I will write it as P 1 square. This is square m square P 1 dot P 1 minus P 1 prime, alright. Then, what I will use? I will use the energy momentum conservation rules to write it as twice m square P 1 dot P 1 prime is equal to P 2 prime minus P 2. So, this is just twice m square P 1 dot P 2 prime minus P 1 prime minus P 1 dot P 2. I will substitute this here.

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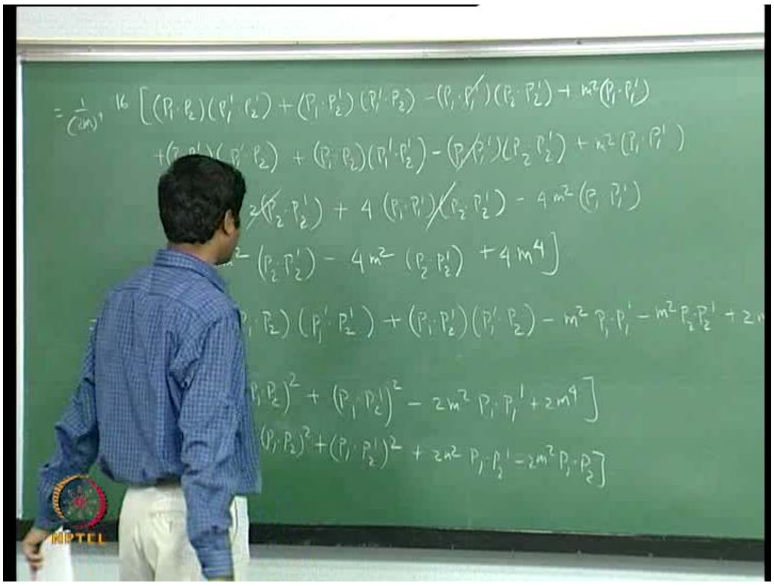

Then, what I get is this is equal to 1 over 2 m 4, 32 times P 1 dot P 2 square plus P 1 dot P 2 prime square minus twice m square P 1 dot P 1 prime plus twice m square P 1 dot P 2 prime minus twice m square P 1 dot P 2, alright.

Now, what you see here, it is from the last three terms. You will just write. You can write them and then, here I wrote it twice, ok. That is the problem. Thank you. So, plus twice m square P 1 dot P 2 prime minus twice m square P 1 dot P 2 square. That is all I get for the first term in the term. So, now, what I have to do is, I have to evaluate one of the cross term and then, remaining two terms in the mode square of the matrix element, I will just obtain by exchanging P 1 prime and P 2 prime.

So, then finally, we will put everything in the formula for the differential scattering cross section and then, obtain the scattering cross section for electron scattering, alright. So, this we will do in the next lecture.