Quantum Field Theory
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Module - 4
Quantum Electrodynamics
Lecture - 27
Compton Scattering I

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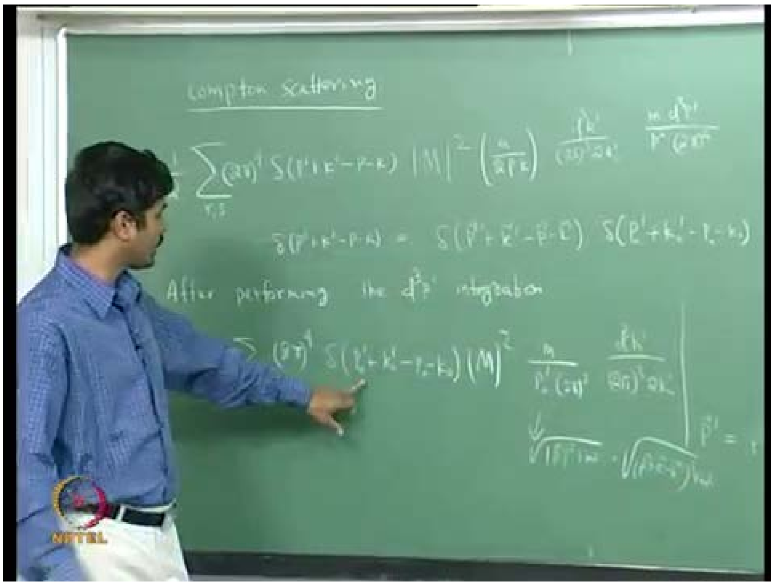

Will continue our discussion on Compton scattering, the differential scattering plus epsilon t sigma is a given by 2 phi the power 4 delta p prime plus k prime minus p minus k mod m squire. Then m divided by 2 p dot k d cube and k prime divided by 2 phi cube 2 k 0 prime and m d cube k prime divided by p 0 prime 2 by cube. Now, what we want to considered is we want to considered the initial electron to be un-polarized and also we do not want to detect the polarization of the outgoing electron, therefore we need to solve some erase and over the spin of the initial electron and sum over all possible values for the spin of the outgoing electron.

When you that, what we neat to do is, we need to consider half the sum over r and s, where s the spin of the initial electron and r is the spin of the final electron. What we will do is that, we will further integrate over the integrate of our p prime remember you have this 4 dimensional delta, p prime plus k prime minus p minus k, which is nothing but the 3 dimensional the direct delta, p prime plus k prime minus p minus k. And then delta p 0 prime plus k 0 prime minus p 0 minus k 0.

When we will carry out the p prime , what will happen is everything will be as it is except that, p prime is evaluated at this value minus of k, prime minus p minus k. So, after performing the d cube p prime integration, what we will get is d sigma will be equal to half sum over r s 2 phi to the power 4, delta p 0 prime plus k 0 prime, minus p 0 minus k 0 and mode m square m divided by p 0 prime 2 phi cube d cube k prime divided by 2 phi cube 2 k 0 prime, where p prime is equal to p plus k minus k prime.

So, you need to replace it everywhere wherever you get p prime you need to replace this for example, in place of p 0 prime which is nothing but square root of p prime square plus m square, you need to write square root of p plus k minus k prime square plus m square. Similarly, where ever you have p 0 prime you need to write this and in the expression for m also whenever you come across with either p prime or p 0 prime you need to put this appropriately this are respectively.

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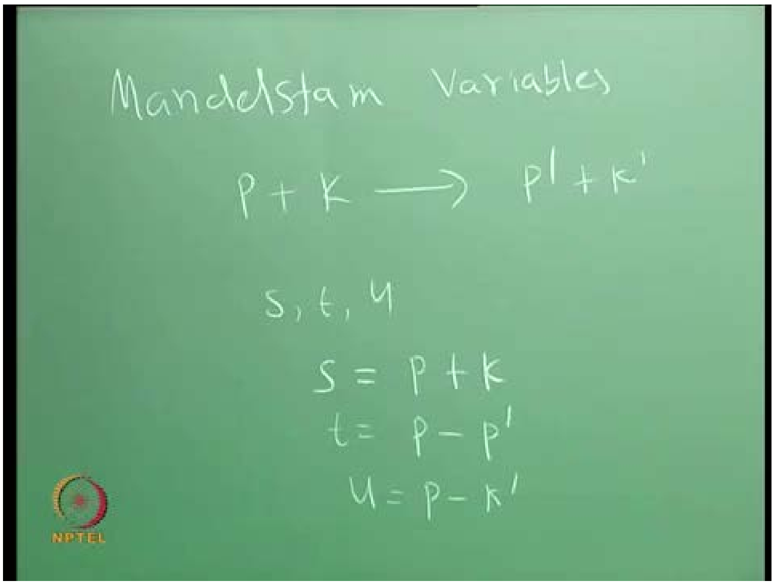

Sure, I would like to introduce something which are known as the Mandelstam variables in process where 2 particles come and then interact and then go. So, for example you have a particle with momentum p and a particle with a momentum k, going to p prime plus k prime, you can introduced the variables s t and u such that, s is equal to p plus k, t is equal to p minus p prime and u is equal to p minus k prime these are known as Mandelstam variables.

And it is quite convenient to express the cross section in terms of these variables and to do the calculation using the these variables. So, you need to keep in mind that the value of p prime is p prime is no longer than independent variable it is this value. Now, what we will do is that we will work in spherical polar coordinate in the momentum space of the outgoing photon.

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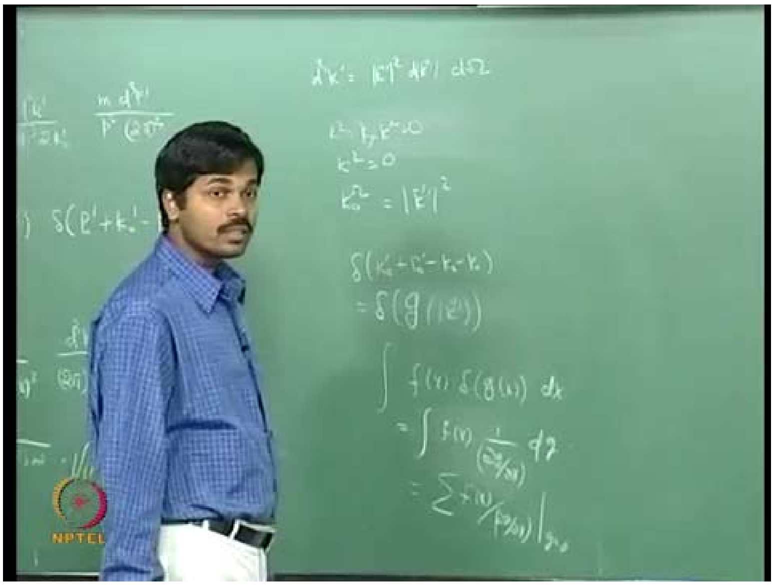

So, the d cube k prime we will write it in spherical polar coordinate, what we will gets, d cube k prime is k prime square d mode k prime d omega. I will call this solid angle to be solid angle in the momentum space for the outgoing photon to be d omega. We will further integrate over this k prime variable and then we will see what we get.

So, you can substitute for k prime, k prime is same as k 0 prime, so this is k 0 prime square d k 0 prime or maybe we will leave it as it is and then we will keep in mind that since photon is a mass less particle. Therefore k square which is k mu k mu equal to 0 and so is k prime square, therefore k 0 square for k 0 prime square for photon is equal to mod k prime square, this you need to keep in mind.

So, whenever you have k 0 prime, you need to replace it by modules of k prime, then you also have to keep in mind that this p 0 prime is I mean in the delta function delta k 0 prime plus k 0 prime minus k 0 minus k 0 is not of this form delta mod k prime minus sum a, this is not of this for because of the k prime dependence in p 0 prime. So, this is of this form delta sum function of I will call it as g, g of mod k prime.

So, when you have such dependence what do you do for example, if you have an integration over f of x delta g of x, d x then this is same as f of 0 divide by modules of del c over del x evaluated at x equal to 0. So, when we need to use this formula in order to carry out the k prime integration. So, what we did is we consider the differential scattering cross section we have assumed that the initial electron is on polarized.

So, erased over the initial electron and also we said we do not care about the spin of the outgoing electron, therefore we sum overall the spin values for outgoing electron, then we carried out the integration of a the momentum of outgoing electron and as well as the we want to carry out the integration over modules of k prime.

So, therefore, so what we need, so what we will get at the end of the day is it is just an integration over modules of k prime, but because of the presence of this delta function we need to be a little bit careful, sum over v 0 of z e of x, absolutely. So, this is actually integration f of x 1 over del z e over del x d g right, so you are right. So, therefore, finally, evaluated at f of x over del z e over del x z e equal to 0 now at x equal to 0.

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What here, what is missing here to here this is their think, this is missing good. So, a m divided by 2 p dot k is already there, So, therefore let us carry out this k prime integration.

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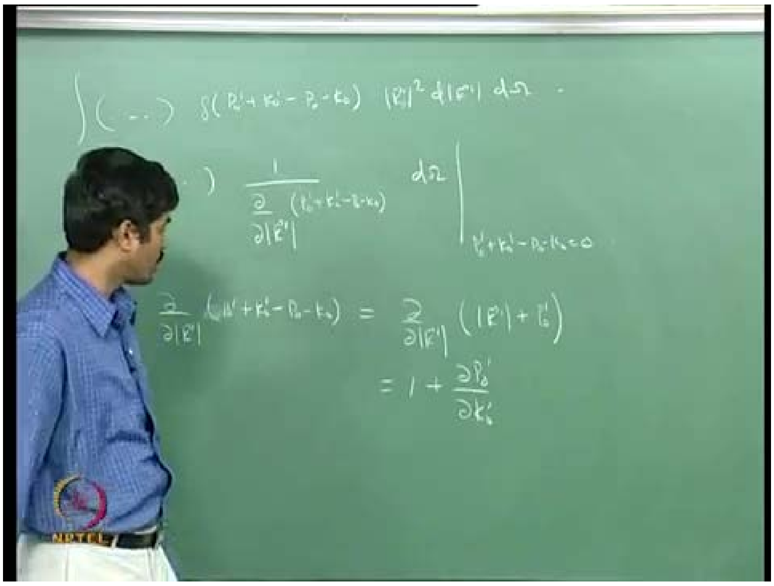

So, you have a hole bunch of things and then you have delta of p 0 prime plus k 0 prime minus p 0 minus k 0 and then mod k 0 mod k prime square, d mod k primed, d omega, this is there. So, I have seen this is just same as ((Refer Time: 13:37)) things times 1 over del over del mod k prime of this quantity, p 0 prime plus k 0 prime minus k 0 minus k 0 d omega, which is evaluated at p 0 prime plus k 0 prime minus k 0 minus k 0 equal to 0 right.

So, what is this quantity here we have already seen earlier that this k 0 is modules of k prime and there is k 0 dependence in p 0 prime. So, therefore, this quantity in the denominator is nothing but del over del mod a prime is equal to acting on p 0 prime plus k 0 prime minus p 0 minus k 0 is equal to del over del mod k prime; k 0 prime I will right as mod k prime and p 0 prime as k 0 dependents, which I will writing it is p 0 prime this is 1 plus del k 0 prime of a del k 0 prime right; so this is what I need evaluate.

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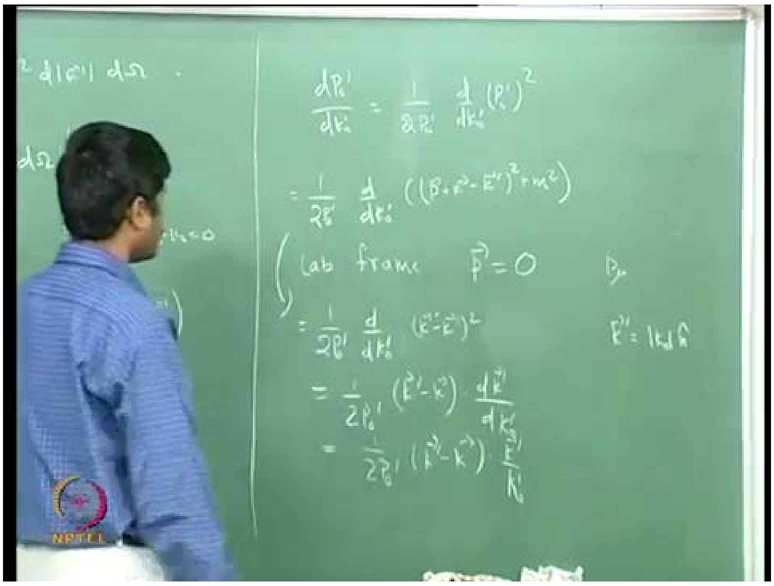

So, let us work out d p 0 prime over d k 0 primed was this d over d, which is 1 over 2 p 0 prime d over d k 0 prime 0 prime square. Now will see this is 1 over 2 p 0 prime, d over d k 0 prime of p 0 prime square is p prime square which is p plus and k minus k prime square plus m square right alright. So, what we will do now is we will work in laboratory frame, where the incoming electron is at rest.

So, we will choose P equal to 0 which saying that p mu is only time direction or you choose the direction to be l on p mu, so we will use this result lateral also. In the last farm this quantity is 1 over 2 k 0 prime d over d k 0 prime k minus k prime square plus m square. m square dependence will any way go away, because we are different setting with respect to k 0 prime.

So, 1 over 2 k 0 prime let us write it as k prime minus k whole square. So, k prime minus k, dot d k prime over d k 0 prime right k prime is nothing but mod k 0 times a unit vector along this direction; therefore, d k prime over d k 0 will be simply equal to k prime over k 0. So, this quantity is 1 over 2 k 0 prime k prime minus k dot k prime over k 0 prime that is all we got for d p 0 prime over d k 0, so I will substitute this here.

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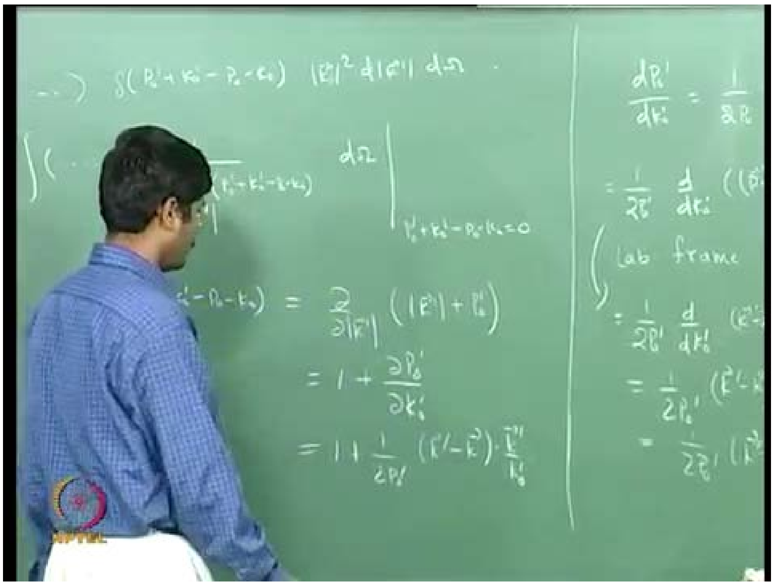

So, this is 1 plus 1 over 2 p 0 prime, then k prime minus k dot k prime over k 0 prime. We can further simplify this result here, so let us do it again by erasing this part some where I missed a factor of 2, when there is a square.

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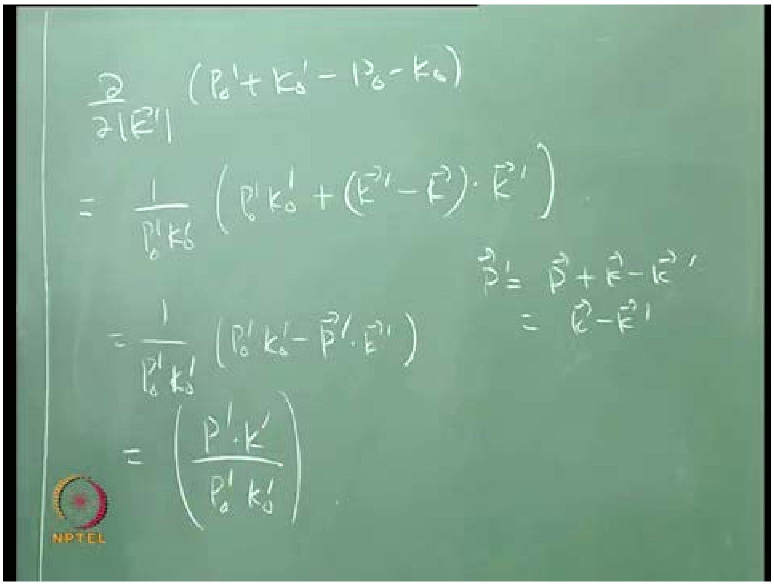

When I consider del over del k 0 prime of k prime minus k square, there is a factor of 2 this is twice k prime minus k dot del k prime over del k 0 prime. So, that to fetor will this 2 here their forte I will left with this alright. this quantity. Now, is equal to del over del k prime of p 0 prime plus k 0 prime minus p 0 minus k 0 is equal to 1 over p 0 prime k 0 prime times, p 0 prime k 0 prime, plus k prime minus k dot k prime.

What is k prime minus k, you remember p prime is equal to p plus k minus k prime, but we are working in the large farm where p is 0 therefore, p prime is k minus k prime. So, I can write k prime minus k s minus p prime, so this is one over k 0 prime k 0 prime p 0 prime k 0 prime minus p prime dot k prime, where for this is equal to p prime dot k prime over p 0 prime k 0 prime. So, what we did is we evaluated this vector and then we found this to be p prime dot k prime over p 0 prime k 0 prime, we will substitute here. So, this comes in the denominator therefore, when we carry out the integration over k prime.

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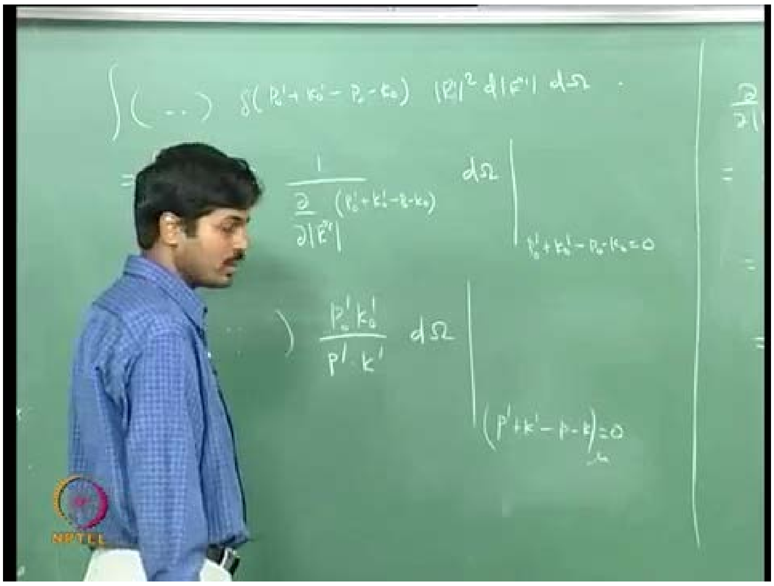

What we will get a result is this whole thing p 0 prime, k 0 prime, over p prime dot k prime d omega we have to remember that p 0 prime plus k 0 prime minus p 0 minus k 0 equal to 0.We have to also remember that we have a integrate intergraded over p prime therefore, p prime is equal to p plus k minus k prime. Or in other words we have to remember that we have carried out the integration over the 4 dimensional delta function. So, this provided p prime plus k prime minus p minus k equal to 0, this mu this 4 vector is 0. So, now we will come back to this formula here this integration over these modules of k prime is carried out already.

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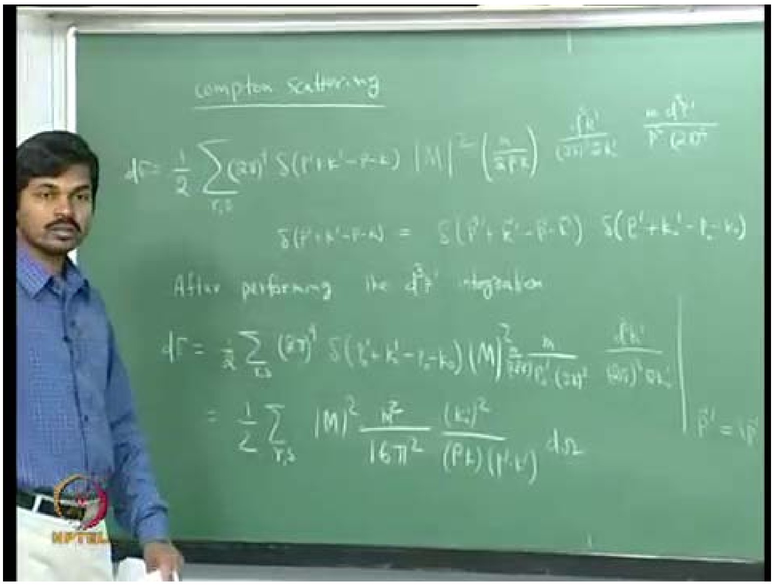

So, what we are left with is the differential scattering cross section is equal to half Sum over r s and then 2 phi to the power 4, there is no delta its mode m square and then this whole thing will be m square over k, so what I will do is that I will cancel this 2 power to the 4 with 2 phi to the power 4 coming from these 2 factors. So, what I will left is here there is a 4 phi square and there is a factor of 2 coming from here and a fetor of 2 coming from here, so that will give me 16 phi square alright.

So, no to 2 phi power 4 mod m square and m square is coming these 2 fetors m in the denominator I have 16 phi square, then k 0 prime square, because of this I will get k 0 prime square and then there is a p 0 prime k 0 prime and this p 0 prime k 0 and the denominator will cancel this p 0 prime k 0 prime here. So, what is will be left with is there is a p dot k and there is p prime dot k prime d omega is there anything else left out.

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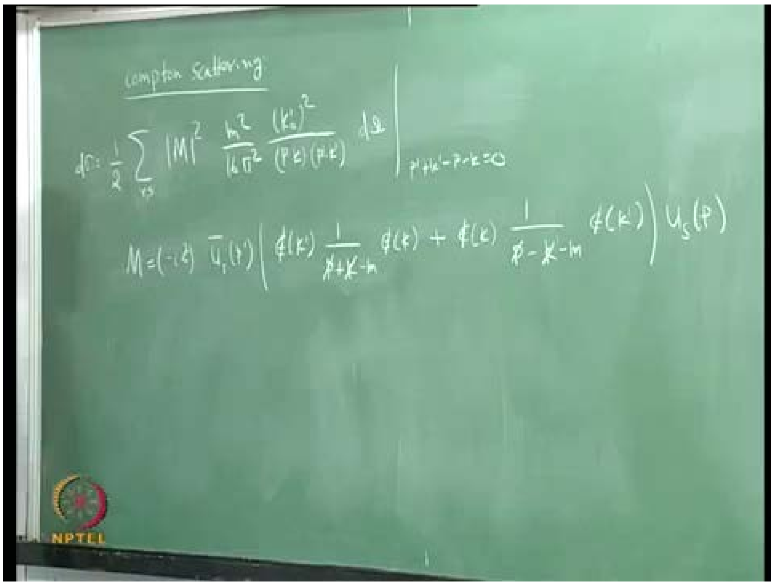

So, to summarize what we got after carrying out the integrations, is this for the differential catering cross section sum over r s mod m square m square over 16 phi square k 0 prime square over p dot k, p prime dot k prime d omega which is evaluated at p prime plus k prime minus p minus k equal to 0.

Now, all we need to know is how to evaluate this mode m square and then we are done. So, let us do that now, so what is m, m if you remember from the last lecture is minus I e square u bar r p prime. I will now put this spin in this is because I need to some over all the spin values and epsilon slash k prime one over p slash plus k slash minus m, epsilon slash of k plus, epsilon slash k 1 over p slash minus k prime slash minus m, epsilon slash of k prime u s p.

Now you remember I need to evaluate modulus of mode m square in the mode m square what will appear is there will be 4 here. So, will I will live this I will erase this minus i, e square from here or in other words what I will evaluate is I will evaluate I will put e 4 here and then it will evaluate mode m divided by minus I e this mode square. So, minus I e square I am interested in this quantity.

So, therefore, what I will what I will be looking is I mean instead of writing this factor of I times e every time I will just concentrate on m divided. By minus that is, which I will call is m till for simplicity and. So, all I need to do is I need to evaluate modulus of m tilde square here. Now in doing this you can see what you get here. So, you just combine e square divided by 4 pi.

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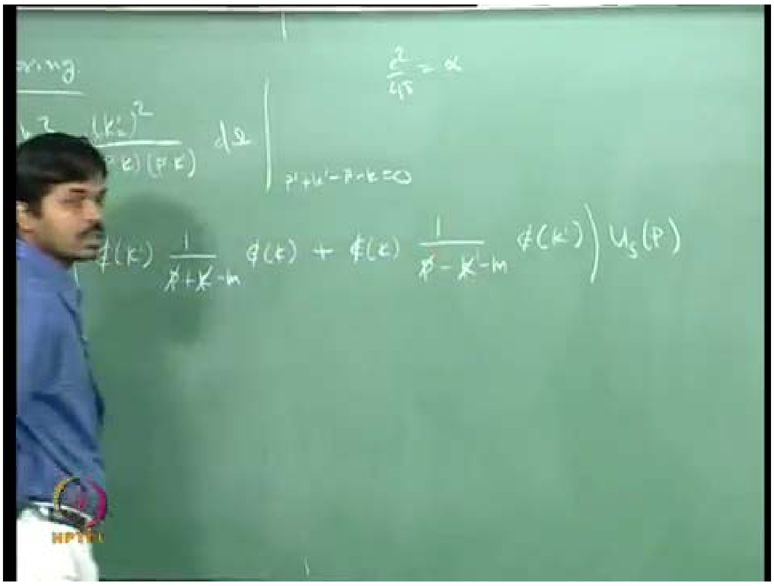

You write it as alpha define structure constant, then what you see is in the in the formula for this catering cross-section what you get here is alpha square divided by 2 instead of all these things you simply get e square of the define structure constant and then here you have m square k prime square anyway. So, that the reason I want to write it in this way. So, that is the define structure constant will appear in the scattering cross-section and also the factor of e will not be there when I evaluate this quantity here.

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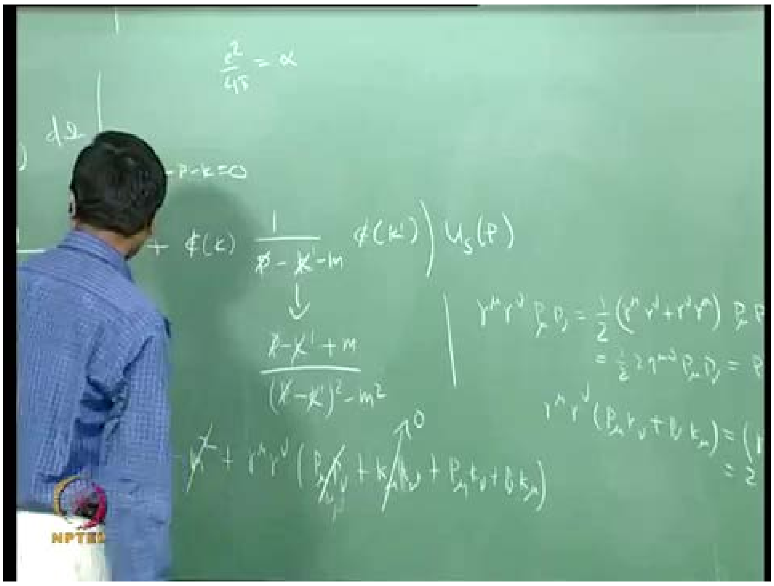

So, Now I can simplify these 2 things by writing this propagator this fermions propagator is p slash plus k slash plus m divided by p slash plus k slash, square minus m square. Similarly, here in place of this I can write it is p slash minus k prime slash plus m divided by p slash minus k prime slash square minus m square. What is the denominator here in this term I can simplify that p slash plus k slash square is nothing, but or let us say this minus m square is equal to minus m square plus gamma mu gamma nu p mu plus k mu p nu plus k nu. So, this quantity again is equal to minus m square plus gamma mu gamma nu times, p mu p nu plus k mu, k nu plus p mu, k nu plus p nu k mu.

What will you get here gamma mu gamma nu this quantity gamma mu gamma nu p mu p nu is nothing but half why is set a half. So, this is half gamma mu gamma nu plus gamma nu gamma mu p mu p nu and this is twice eta mu nu half times to eta mu nu p mu p nu, therefore this is simply p square.

So, this will give me p square this will give me k square. So, this p square is simply m square. So, this turn will cancel this minus m square this will give me a 0. So, all that I am left with is this, so what is this quantity, this is simply gamma mu gamma nu p mu k nu plus p nu k mu this is equal to gamma mu gamma nu plus gamma nu gamma mu p mu k mu.

So, this is twice p dot k, so this denominator here gives me twice p dot k here, because of the minus k prime it will give me minus twice p dot k prime, so let write it

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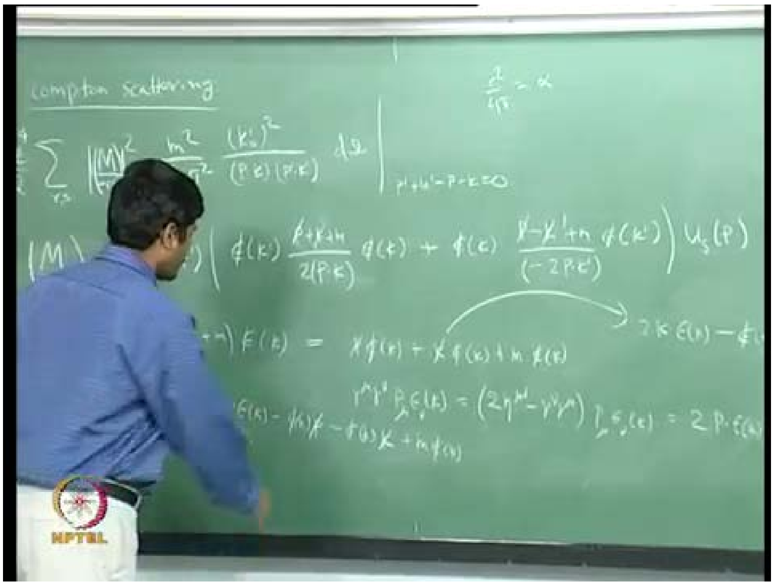

So, p slash plus k slash plus m divided by 2 p dot k, here p slash minus k prime slash plus m divided by minus 2 p dot k prime. This is what I get, I can further simplify these terms let us let see how. So, let us let us look at this term here p slash plus k slash plus m epsilon slash of k, I want to bring the epsilon slash to be left, this will simplify this whole term enormously.

So, let us do that systematically this is p slash epsilon slash k plus k slash epsilon slash k and m epsilon slash of k, first term is nothing, but gamma mu gamma nu p mu epsilon the mu of k, this gamma mu gamma nu I can write it as twice eta mu nu minus gamma nu gamma mu.

I will use the enti-commutation relation of gamma mu gamma nu to write gamma mu gamma nu to be this. So, this times p mu epsilon nu of k, so as a result what I get here is twice p dot epsilon of k from the first term and from the second term, I will get minus epsilon slash of k, p slash. Similarly if I evaluate the second term here I will get, twice k dot epsilon of k minus epsilon slash k, k slash.

So, let us put these things here together when I put these things together what I get is twice p plus epsilon sorry p plus k dot epsilon of k from this and this then I have minus epsilon slash p slash minus epsilon slash k slash, plus m epsilon slash of k I will use that we will use that in the moment.

So, two thinks we can use, first of all, we know that this u s satisfies the direct equation and the second thing is that if this is a if epsilon represents the transfer polarization for the incoming photon, then it needs to be orthogonal to k. And also we have chosen the direction of time along p, therefore epsilon k will again do orthogonal to p. So, that will simplify enormously because the direct equation tells that there is a p slash minus m here because I have taken it to that side.

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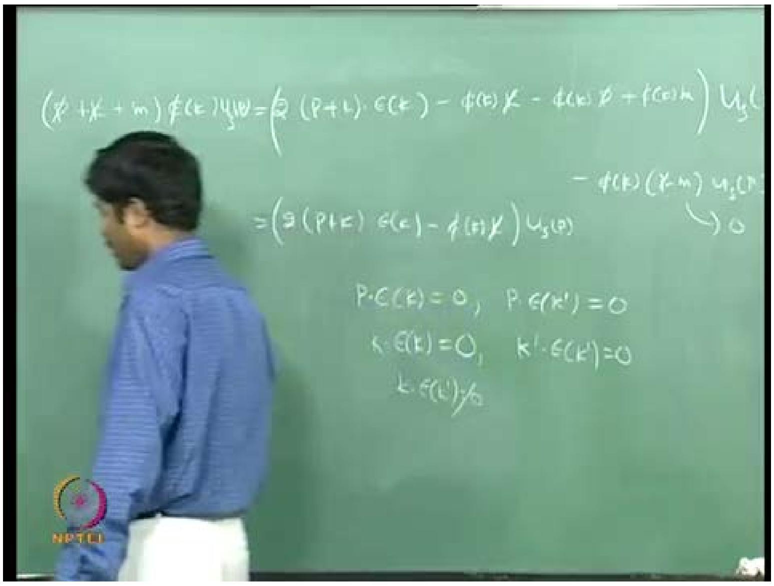

Anyway, let us let us look at this, so this term actually it did not simplify it is gave me a complicated expression p slash plus k slash plus m epsilon slash of k is equal to twice p dot p plus k dot epsilon k minus epsilon slash k slash minus epsilon slash p slash plus epsilon slash m, you have to remember that this whole thing actually acts on u s p.

So, these acting on the u s p will give me this whole thing acting on u s p, but the last term you can see is nothing, but minus epsilon slash k p slash minus m u s p, but u s p satisfy the direct equation p slash minus m u s p equal to 0. Therefore, this term is equal to 0. So, when it action u s p, I get this simpler expression twice p plus k dot epsilon of k minus epsilon slash k slash acting on u s p.

We will further assume that this epsilon k is transfers polarization therefore, it will orthogonal to k and also we will choose we have already chosen the time direction along p, Therefore p will again the orthogonal to epsilon. So, therefore, p that epsilon k equal to 0 p dot epsilon k prime equal to 0.

Because p is along the time direction on the other hand, since epsilon represents the transfer polarization at will be orthogonal to the corresponding momentum 4 momentum of the photon or for k dot epsilon k equal to 0 where is k prime dot epsilon k prime equal to 0, remember transfers does not mean that k dot epsilon k prime equal to 0 it is not true. So, do not make a mistake of setting this quantity to be 0 these are the things 0. So, when I set that this whole thing actually simplifies to minus epsilon slash k, k slash u s p. So, that is what I get from the numerator of this past term I can similarly simplify the second term.

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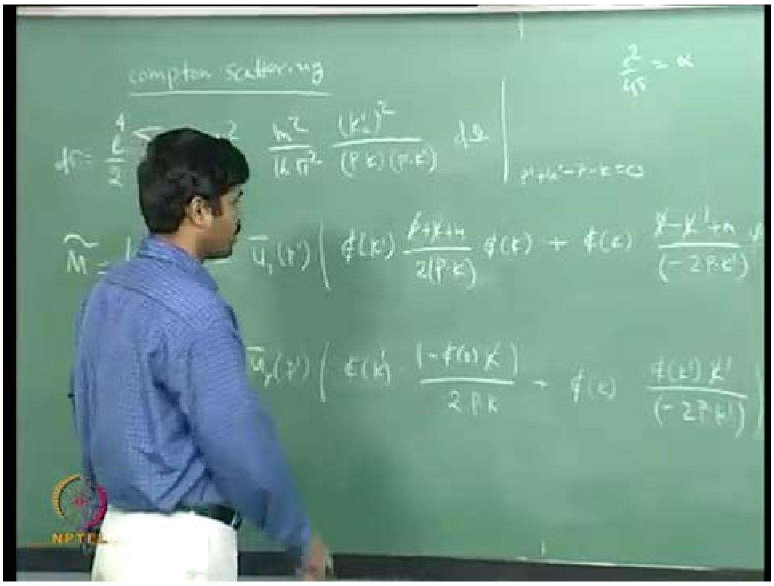

So, let us simply substitute that here, this is u, r bar of k prime then epsilon slash of k the whole thing here simply become minus epsilon slash minus epsilon slash k, k slash divided by 2 p dot k, here what I claim is if you do a similar analysis because of the minus here it will give me a plus epsilon slash k prime, k prime slash. So, this is epsilon slash k then a epsilon slash k prime, k prime slash divided by minus twice p dot k prime this acting on u s p.

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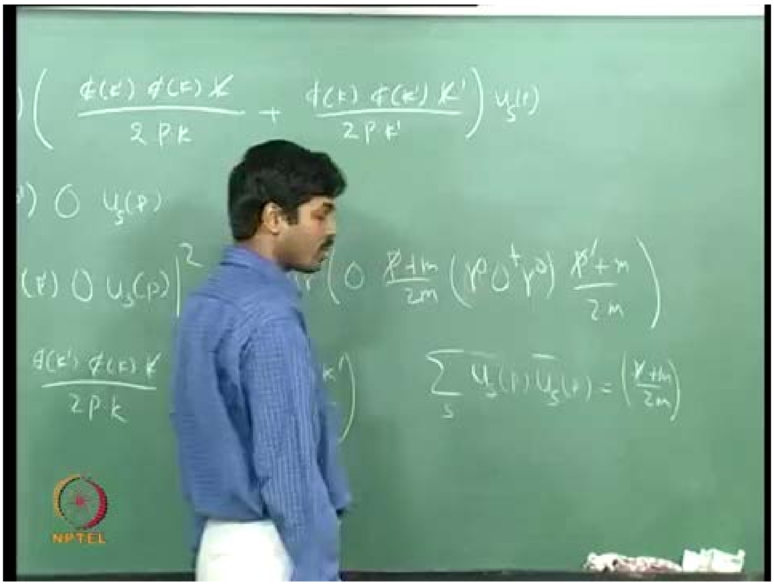

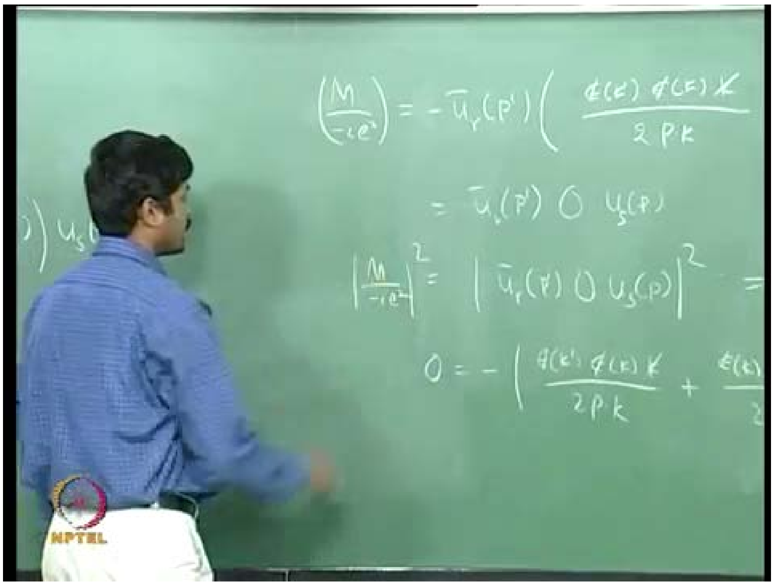

So, let us rewrite the scattering cross-section. D sigma or we are evaluating mode m square mode m square m over minus i e this is the quantity that we are took in at it is u r bar k prime, then there is a numeral minus sign coming from both the terms. Epsilon slash k prime epsilon slash k, k slash over p dot k now epsilon slash k epsilon slash k prime, k prime slash divided by p dot k prime acting an u s p.

So, what we need to do is we need to compute the mode square of these things, this is of this form some u bar r p prime some operator o u s p. So, this is the this form all we need to find it m over minus I mode square.

So, therefore, we need to consider this quantity u r bar p prime u s p mode square where o is the operator, minus epsilon slash k prime epsilon slash k, k slash divided by twice p dot k plus epsilon slash k epsilon slash k prime slash over twice p dot k prime, so I e square.

So, what we will do is that we will evaluate these and then we will show that this quantity is actually equal to trace of o p slash plus m over to m gamma 0 over dagger gamma 0 p prime slash plus m over to m. This what, we will be showing in the next lecture and then we will evaluate this trace here, so you can see that something similar.

So, it come here because you already know there is a mod square at the end of the day, you will get some expressions which involve u s p, u bar s, u s p bar s p sum over s. And we know this quantity is nothing but p slash plus m over 2 m. So, we will use similar relation twice and then will right the things properly at the end of the day we will show that this is equal to this.

So, ultimately what we see is that evaluating mod m square actually turns out to be evaluating the literate of bunks of gamma matrixes. So, finally, we will evaluate this and then at the end we will get this hole square this modules of this quantity square to be sum function of p and p prime, which we will get a nice expression for p and p prime which we will plug in the scattering cross section formula and then will see what we get?