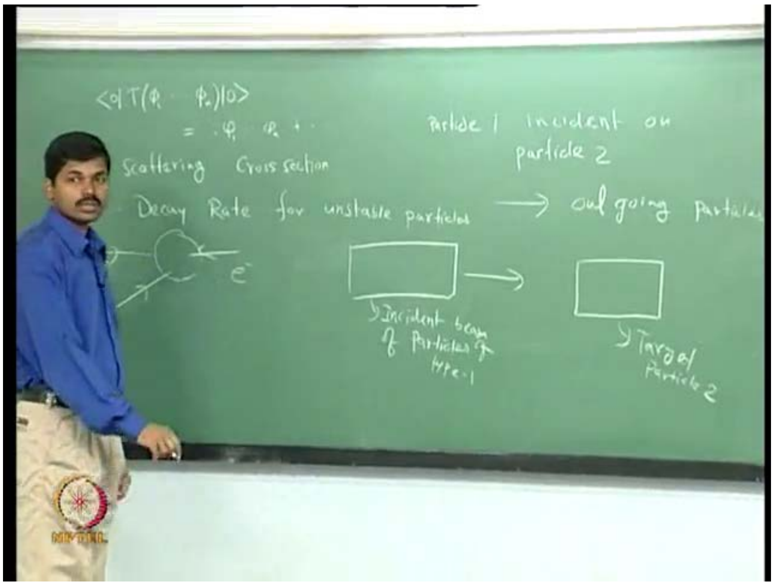
Quantum Field Theory
Prof. Dr. Prasanta Kumar Tripathy
Department of Physics
Indian Institute of Technology, Madras

Module - 2
Interacting Field Theory
Lecture - 11
Interacting Field Theory – IV

(Refer Slide Time: 00:21)



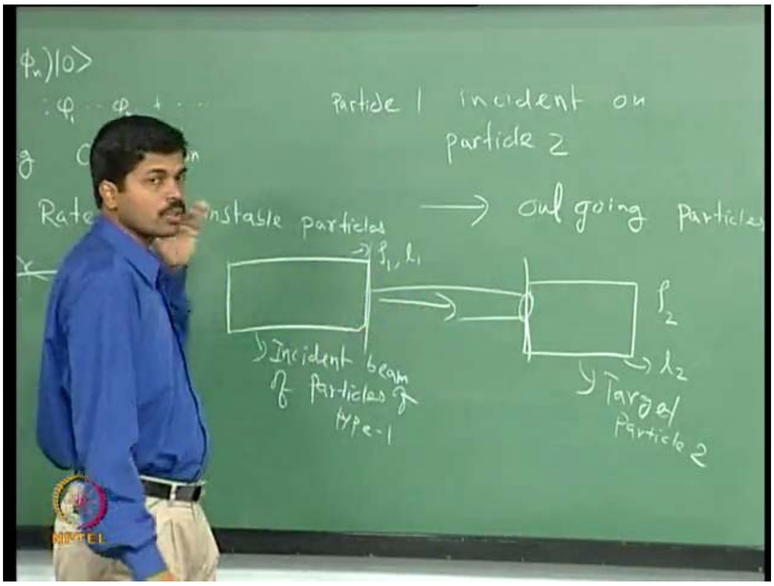
So, in the last lecture we have derived formula for the n point correlation function, which is the vacuum acceptation value time ordered product of n fields. Using the Wik's theorem, this can be expressed in terms of the normal order product of the fields. Then all possible contractions times the normal order products. In today's lecture, we will derived the formula for cross section the k width and so on. Because these are the quantities that are these are the measurable quantities. So, you need to have a formula for these scattering cross section when you consider two or more particles scatter with each of their, if you have unstable particles they can decay by themselves. Then you can determine the rate of the decay width and so on. These are the two basic things that we will be working out. So, we will have a formula for this scattering cross section, as well as formula for the decay rate for unstable particles

So, scattering cross section of course you can have two particles coming and then interacting and then going out. For example, let us say you have e plus e minus or whatever the outgoing particle may or not may be the same set of particles or there may be different number of particles. For example, when you consider e plus e minus scattering you may get e plus e minus mu plus mu minus or mu plus mu minus gamma all kinds of things that you will be getting as outgoing particles. You need to find out the cross section for all these particles. So, let us or you can have also more than two particles, let us say three particles interacting with these. So, there and going out and so on.

What we will be discussing in today’s lecture is the case where two particles come. Let us say particle one, which is incident on particle two, which is at rest in the laboratory Then there had a bunch of outgoing particles, we need to find the cross section for such a process.

So, what do what exactly mean by cross section? You consider lets you have a target here, which I will call as the particle two. Then you have a beam of particle one. So, this is the target and this is the incident particle of particle one of type one. So, it falls on this, what you need to know is what are the number of scattering events in this process. Naturally, there will be more events coming out. Let’s say if this target is where it is more number of scattering events will come out. If the incident be, which is also dense, very dense. So, the number of scattering event naturally depends on the density of the target, which I will call as rho 2.

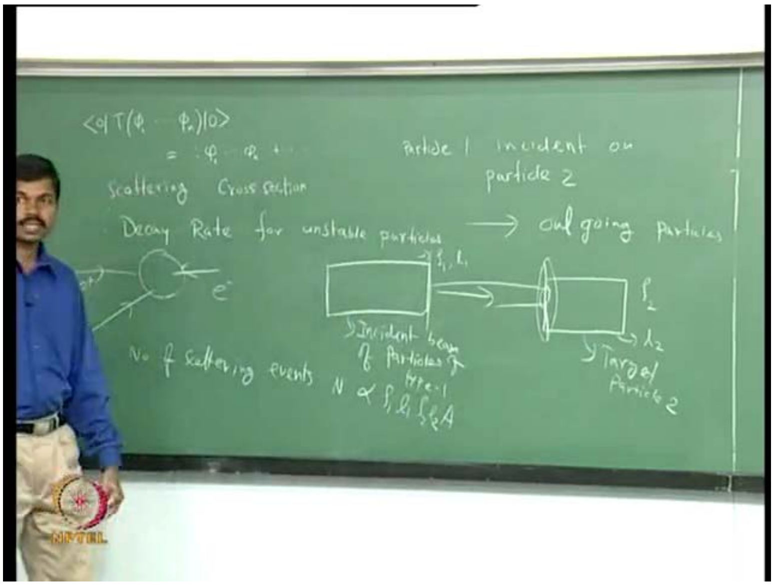
(Refer Slide Time: 05:27)



It also depends on the density of incident beam. If it is not dense at all then the number of events scattering events will be less and so on. This will as depend let us on the thickness of the target here. So, this I will call this as let us say l 2 this length here. Similarly, it will also depend on this l 1, what else it can depend? It can also depend on the cross sectional area here.

If it is the beam incident beam comes only on a very small region here on this target, then the number of events will be less. If it comes on the entire thing it will be more. Let us say both the incident beam as well as the targets here a common cross sectional area. Then the number of scattering events will also depend on the cross sectional area, which is common to both of these thing, so the number of events, which I will call as n.

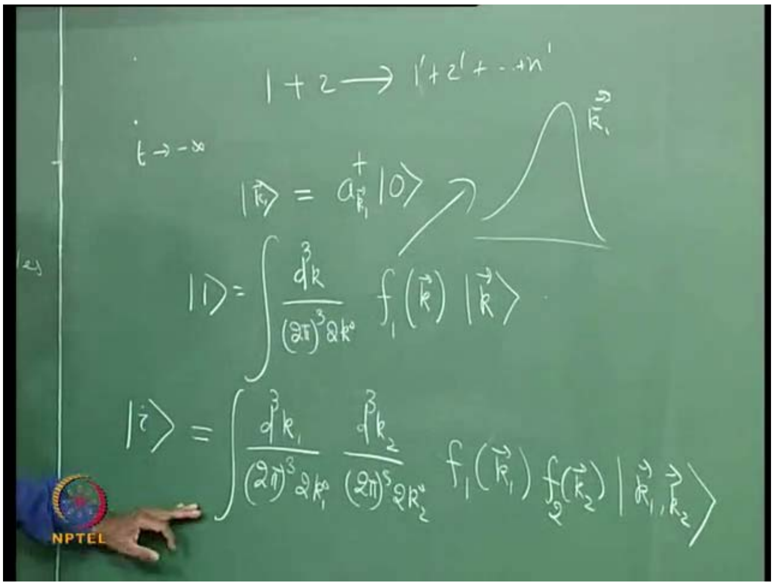
(Refer Slide Time: 06:44)



It will be proportional to all these quantities rho one l 1 rho 2 l 2 and A. The proportional to constant is what I will call as the cross section. So, the cross section for this scattering is determine by the dividing this number of events by this quantity here. Similarly, you are when a particle decays decay rate is the formula for the decay rate is even simpler, you consider some substance some unstable particle here. Then there is probability that one of these particle will decay. This decay rate here will depend decide of the system here, the total number of particles. So, the decay rate is basically defined by the number of particles decaying per unit time divided by total number of particles contain in that system. That is what is going to give you the decay rate.

So, in these, so we will be basically discussing scattering cross section and decay width decay rates. We will derive formula for these quantities. Then we will see how can we apply these things. So, in our case we are considering interacting field width. It's basically, let us say you consider two type of particles they were free as at t tends to minus infinity.

(Refer Slide Time: 08:45)



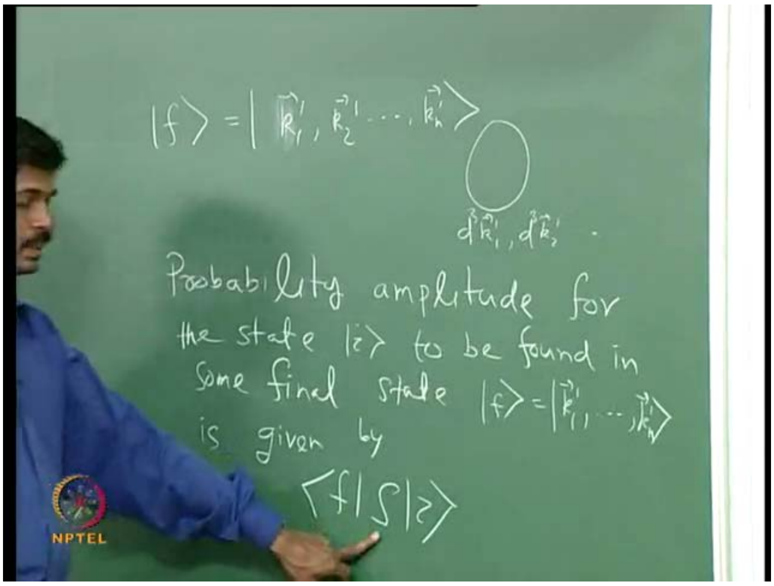
Essentially, they were free particles at some finite time, they come closer they interact they scatter. Then there will be outgoing particle. So, 1 plus 2 goes to a bunch of particles 1 prime, 2 prime, n prime, this is the process that we are going to study. What you want to do is you want to construct a wave packet. Let us say for particle one as well as particle two or a wave packet for the two particles system, let it evolve interact. Then you go what is the probability amplitude for this system to go to final state.

So, essence let us try to construct the wave packet for particle one, which I will denote as 1. Normally, if it is free particle then you normally if the particle is and Eigen state of the momentum, then this is just, let us say a k 1 dagger acting on the vacuum. This is a 1 particle state with a definite momentum k 1. Normally, what we would like to consider is wave packets, which do not have a very well defined momentum, but there is a spread in the momentum. So, there is a momentum distribution with its pit around a ten value let us say, which is k 1 bar.

So, the particle one is a wave packet, which is basically integrated over d cube k divided by 2 pi q 2 k 0 time some distribution, which I will call as f 1 of k and then k here. So, this if this f 1 is a delta function if f 1 k is delta k minus k bar, then you get the particle one to be a Eigen state of the momentum. It has a very well define momentum otherwise it's you can consider some momentum distribution here. Then you can construct a wave packet this way. This f 1 is some distribution you do not need to know the exact for of this f 1. Except, that it has a fairly well define momentum with it is the average momentum is peak around at ten value, which I will call as k 1 bar. So, you have a distribution with a peak around k 1 bar.

This is what as the initial state is not a 1 particle state it is a 2 particle state, you we are considering a process, where particle of type one and type two interact with each. So, there and go to some final state. So, the initial state is basically d cube k 1 divided by 2 pi cube two k 1 0 d cube k 2 divided by 2 pi cube 2 to k 2 0. Then f 1 of k 1 some distribution f two k 2, so this state evolve. The two both the particles interact at some finite time. Then what you would like to know is the probability amplitude, for this state to evolve to some final state, which I will as f with its some momentum k 1 or k 1 prime k 2 prime up to k n prime.

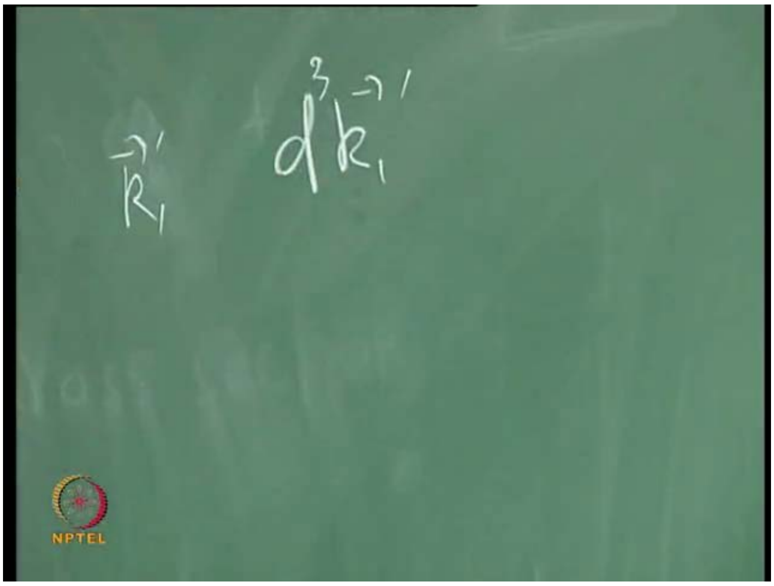
(Refer Slide Time: 13:35)



Normally, the outgoing particles can have fairly well defined momentum. You can detect these outgoing particles by your by some detector, which have very good resolution. So, that you can consider these outgoing particles in some momentum range d cube k 1 prime d cube k 2 prime and so on. We already know what is the probability amplitude for such a process, the probability amplitude will be given by. So, if this the fine final state then the final state for the state i to the found in is given by. Where, s is the s matrix for which we have derived the expression in one of the earlier lectures.

We do not need to assume this the outgoing state to be a momentum Eigen state. So, what you will do is when calculate the cross section. We would like to find the differential cross section, where the outgoing particles will have momentum with lies in some range d cube k 1 prime.

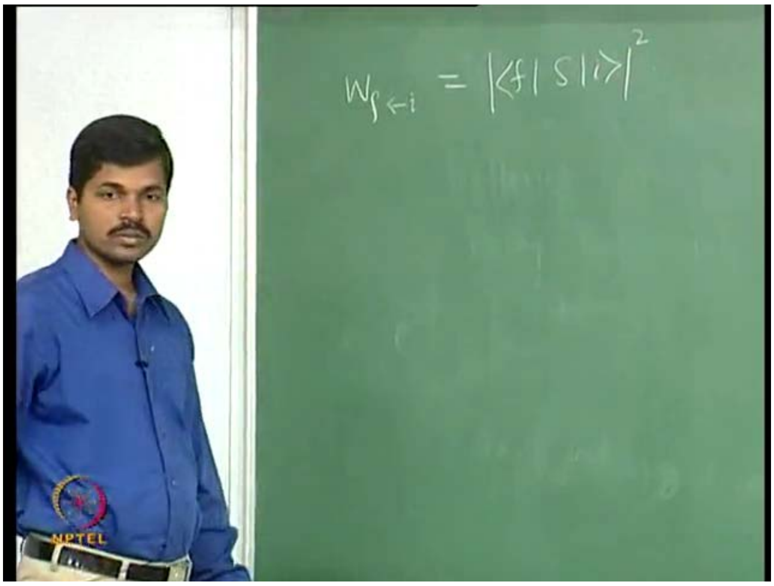
(Refer Slide Time: 16:39)



So, let us say the outgoing particle with momentum k 1 prime. That is the one prime is the particle its momentum lies in certain range d cube k prime. Because, when you have the detector the detector not only can measure what is the particle type. It can also measure the momentum of this particle in fairly, accurately. So, if we do not need the momentum of the outgoing particle, what we can do is. We can consider the differential scattering cross section. We can integrate over all possible value of the momentum that will give as, what is the total cross section irrespective of, what is the momentum of the outgoing particle.

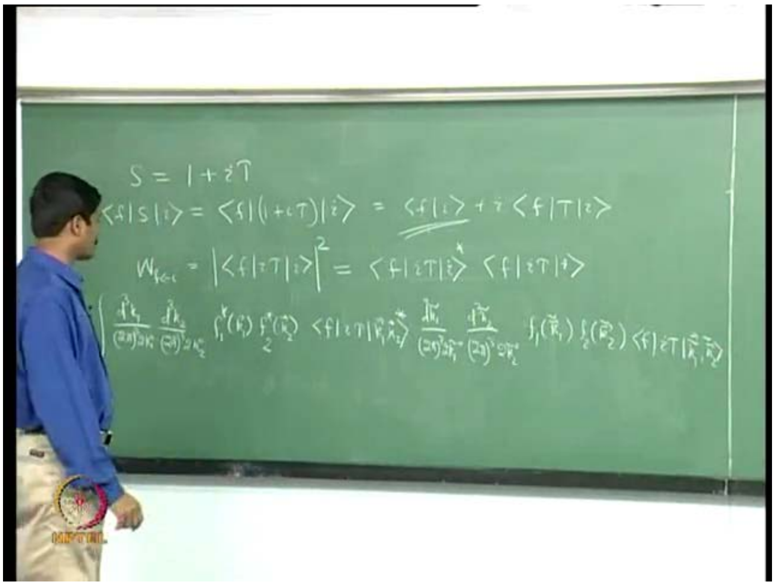
So, at this moment we will not assume anything about the final state. We will just assume this initial state to be two different wave packets coming out close to each. So, there and then interacting and then there is some outgoing set off particles, that is all we are assuming. We will also not assume any particular form for these function f 1 k and f two k. Except, that they there is some momentum distribution with its peak with is a peak around certain value.

(Refer Slide Time: 18:26)



So, let us compute the probability of going some initial for the particle from some initial state to some final state W f i. That will be given by the modes square of this quantity. We will be computing this quantity in a moment. You know this S matrix has an exponential form.

(Refer Slide Time: 19:07)

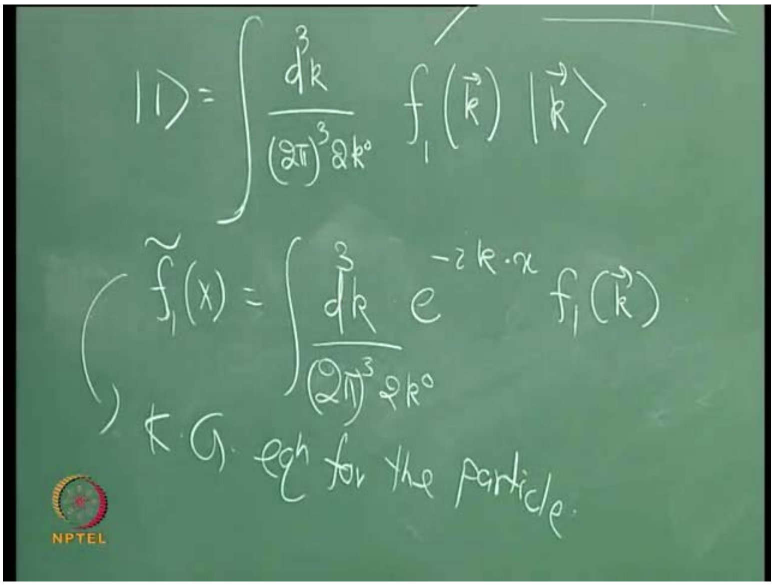


Hence, I can basically write this as the identity operator plus the rest of the term, which I will denote as i times T. Then the probability amplitude will basically be this one. What will the first term give? So, this term basically says that there is a finite probability. That is why you consider particle one incident on particle two on some target. There is a finite probability that these particle do not interact at all there are just go without interacting. This is the term even if there is an interaction in the system, this particles can simply pass without interacting each. So, there and this term represents that, so this is the forward scattering part we will we will not be interested in such a term.

So, will just ignore the first term and this expression we will be evaluating the second term. Then when the interaction takes place what is the probability of this system of two particles? Some, which are in this state i going to some final state. So, we will be evaluating the mode square of this quantity, which is a W i i will define this to be mode square of f i T. Lets substitute this expression for the incoming state in this formula. Then let us try to simplify the expression. So, let us say what we get when we substitute that this quantity here? I will call this is again f i T i star times f, it’s absolutely trivial.

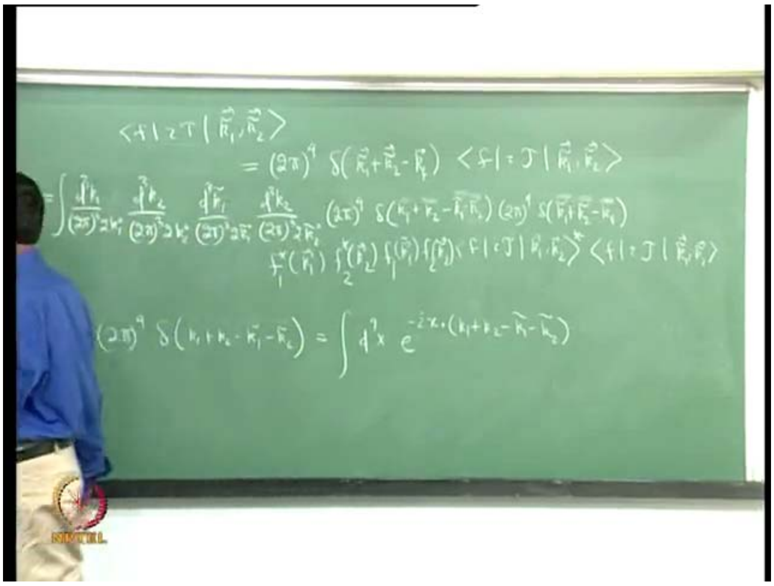
Now, I will put this formula there. So, what I will get is for the first term d cube k 1 divided by 2 pi cube to k 1 0 d cube k 2 over 2 pi cube two k 2 0. Then f 1 star of k 1 f two star of k 2 then f i T k 1 k 2 star. Then I will have integration over d cube k 1 tilde over 2 pi cube 2 k 1 tilde 0 d cube k 2 tilde over 2 pi cube 2 k 2 tilde 0. Then f 1 of k 1 tilde f 2 of k 2 tilde final state inner product with i T times k 1 tilde k 2 tilde. So, this is what we have to simplify. Then I have to express in. To do that, let’s see this expression, here I can take a Fourier transform of this quantity.

(Refer Slide Time: 25:06)



So, I will define as f i will define some quantity, which I will call as f tilde 1 of X, which will be d cube k times a to the power minus i k dot x f 1 k. You define f 1 tilde of x to be the Fourier transform of this f 1 k. This distribution function you can show that the this complex killer field, in fact satisfies the Klein Gordon equation. There is this integration measure 2 pi cube 2 k 0. This is first of all a complex killer field, this field will satisfy the Klein Gordon equation for the particle. What we will do is, we will consider this term here we will insert one identity. So, you consider one of these amplitudes here. This lets say this one f i t k 1 tilde k 2 tilde.

(Refer Slide Time: 27:15)



Then outgoing particles have momenta k 1 prime up to k n prime. They need to conserve the total momentum, the momentum of incident particle are k 1. This state has well defined momentum, this is a two particle state with momenta k 1 tilde and k 2 tilde. So, the momentum conservation tells you that this amplitude here must be proportional to the delta function. It will be zero, unless k 1 tilde plus k 2 tilde is equal to the sum of all these outgoing momenta, of all these particles.

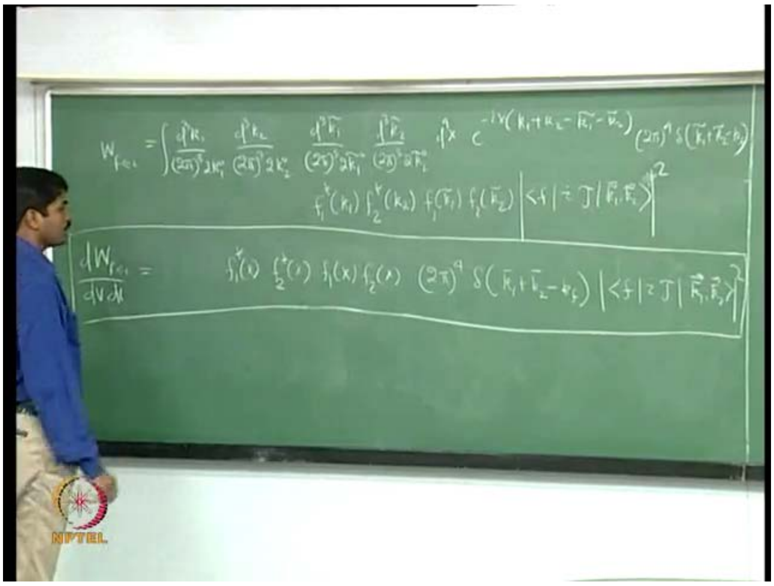
So, this quantity here will be for convenience I will put some factor of 2 pi to the power fourth. Here, it will have a delta function delta of k 1 tilde plus k 2 tilde minus k f, where k f is the sum of momenta of all the outgoing particles. So, it will have such a term here, then the remaining part of the amplitude I will denote this as f i this curly T k 1 tilde k 2 tilde.

So, similarly you have the second term here you can, so let us consider this expression with this w f i will become d cube k 1 over 2 pi cube 2 k 1 0 d cube k 2 over t pi cube 2 k 2 0, then d cube k 1 tilde. Then there are factors 2 pi fourth delta of k 1 plus k 2 minus k f. Then 2 pi to the power fourth delta of k 1 tilde plus k 2 tilde minus k f, then these two matrix elements. So, which are f i T k 1 k 2 star times f i T k 1 tilde k 2 tilde. Then also I have f 1 star k 1 f 2 star k 2 f 1 k 1 tilde f 2 k 2 tilde, this is what I get. Now, notice this there are these two delta functions here I can combine these two delta fruitions. Then I can write this as delta of k 1 plus k 2 minus k f delta of k 1 tilde plus k 2 tilde minus k l a is equal to delta of k 1 plus k 2 minus k 1 tilde minus k 2 tilde times delta of k 1 tilde plus k 2 tilde minus k l.

So, we will use this form here. Therefore, instead of this I can simply write here this minus k 1 tilde minus k 2 tilde. Then the remaining part will go as it is this is just delta of x minus a f of x is basically delta of x minus a f of x. This is what is the formula that I have used here. Now, I can consider this term and I can write delta of k 1 plus k 2 minus k 1 tilde minus k 2 tilde as if you consider the four dimensional delta function.

So, not only the energy, but the momenta are also not only the momenta, but also energy also needs to be conserved. So, you have to have four dimensional functions here instead of 3 d delta. I will use this formula for the delta function there is a 2 pi to the power 4. So, 2 pi to the power fourth times this is given by this I will substitute here and when I substitute what I will get is the following, d cube k 1 over 2 pi cube 2 k 1 0.

(Refer Slide Time: 36:04)



Then d cube k 2 over 2 pi cube two k 2 0, then d cube k 1 tilde over 2 pi cube 2 k 1 tilde 0 d cube k cube tilde over 2 pi cube k 2 tilde zero. Then d 4 X times e to the minus i x dot k 1 plus k 2 minus k 1 tilde minus k 2 tilde. Then 2 pi to the power 4 delta of k 1 tilde plus k 2 tilde minus k f i have f 1 star of k 1 f 2 star of k 2 f 1 of k 1 tilde f 2 of k 2 tilde f i star tilde.

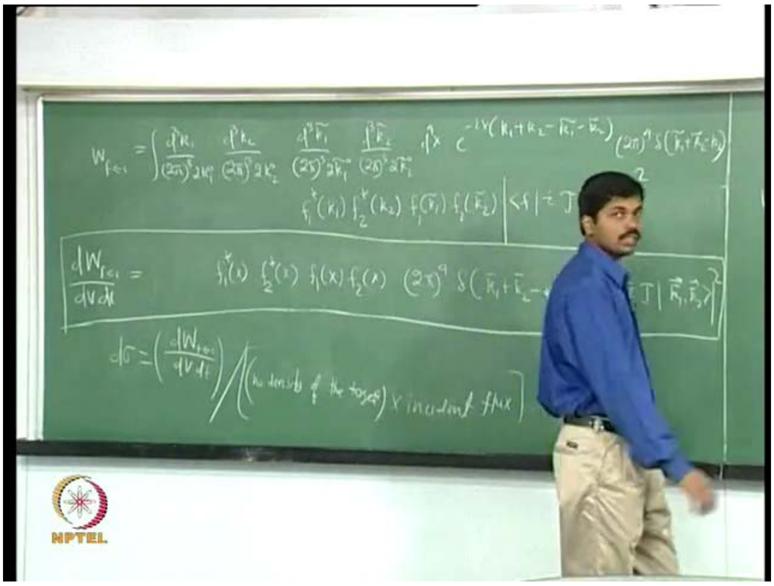
So, it is a fairly straight forward job. Now, what I will do is that I will combine this k integration will just carry out the k integrations by making the following assumption, what is the assumption is that. Because, we have assumed this function f 1 k etcetera at some distribution, which is a peaked at some value k bar. Let us say f 1 k is peaked at k 1 bar and so on. Therefore, the amplitudes here I will assume that these amplitudes are basically equal to k 1 k 2. I will say that this is nearly equal to f i T k 1 bar k 2 bar and also this is fairly close to i k 1 tilde bar k 2 tilde bar.

Then what I can do is that for this I will just substitute modes square of this. So, therefore this, the assumption here is that these matrix elements are slowly varying. Then they are close to this they are nearly equal to this quantity here. Therefore, this probability density here is, nearly equal to this modes square of this quantity with this k 1 and k 2 are replace by k 1 bar and k 2 bar

So, I am not assuming this k dependence here then of course, it is fairly simple. Because, what you get here is basically the Fourier transform of these four S. So, they are nothing but integration d 4 X. This one will give you f 1 of X this one, one of these thing. This factor will combine this to give you f 2 star of X. Similarly, here you will get f 1 of X f two of X. Therefore, the transition probability per unit volume per unit time, which I will denote as d w f i over the d v d T will be given by simply this expression without this integration over d 4 X.

So, what you have derived here is the transition probability per unit volume per unit time. We need to find the scattering cross section, which is basically this quantity divided by the incident plugs, when you have a single scattering centered. So, what you need to do is, you need to the differential scattering cross section is…

(Refer Slide Time: 43:59)



Basically, the transition probability per unit volume per unit time divided by the density of the number density of the target times the incident plugs. So, let us derived the formula for the incident plugs and the number density for the target. Then plug it here we will get the formula for the differential scattering cross section.