Quantum Field Theory

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Module - 1
Free Field Quantization – Scalar Fields
Lecture - 1
Introduction

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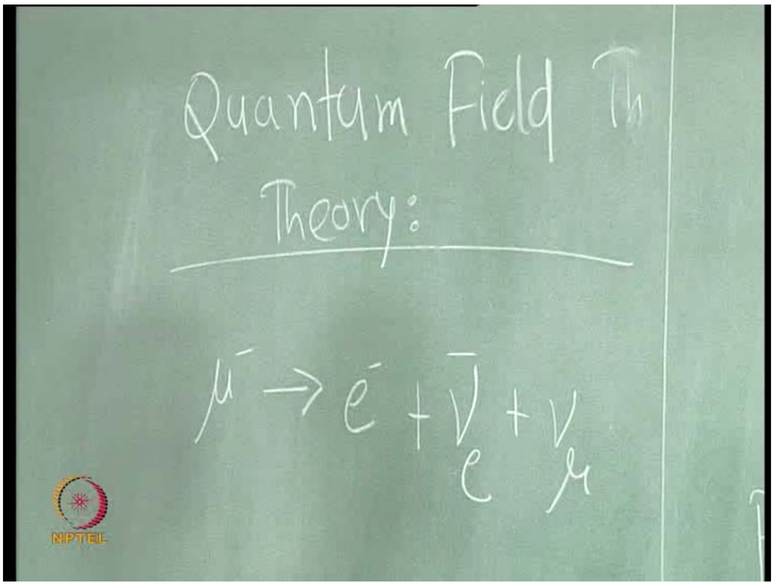

So, in this course we will be studying quantum field theory, you might ask you have already learnt a quantum mechanics in your under graduate in the first year course. So, why do you need to study quantum field theory? The motivation for studying quantum field theory is that suppose you consider a particle the way you do it in your quantum mechanics is you write down Schrodinger equation. And you solve this Schrodinger equation you find the solution to this Schrodinger equation, which is the probability which should. However, there are process which for example, if you consider the decay process; let us assume that you consider the mu and ((Refer Time: 01:15)) mu minus going to e minus plus mu e bar mu nu. Then there is no way you can understand a process like this in your a non relativistic quantum mechanics.

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So, the question that you can ask is suppose you write down the you want to consider this generalization relativistic generalization; relativistic generalization of the Schrodinger equations Klein and Gordon I have already done that. And Dirac also a relativistic generalization; Klein and Gordon generalized it for a scalar field and Dirac generalize it for a spinner; what you find here is that if you want to interprete the Klein Gordon or the Dirac equation the way you interpreted the Schrodinger equation then you run into various inconsistencies.

So, if you want to interpret as a particle mechanics; then you run into various in consistencies. For example, the probability amplitude a does not give the definite positive probability and so on. So, you are force to introduce quantum field theory. In this lecture we will study quantum field theory in much more detail; what I will do that is I will give of the references that will be used in this course. And then we will quickly discuss scalar field theory, classical field theory and then we will study our how to quantize classical field theory?

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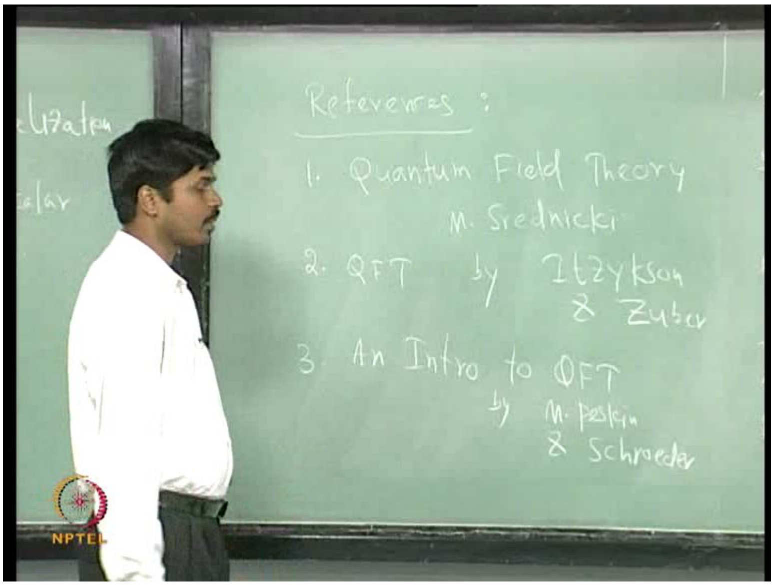

So, the reference are quantum field theory by Srednicki; then quantum field theory by Itzykson and Zuber and an introduction to quantum field theory by Pesktin and Schroeder.

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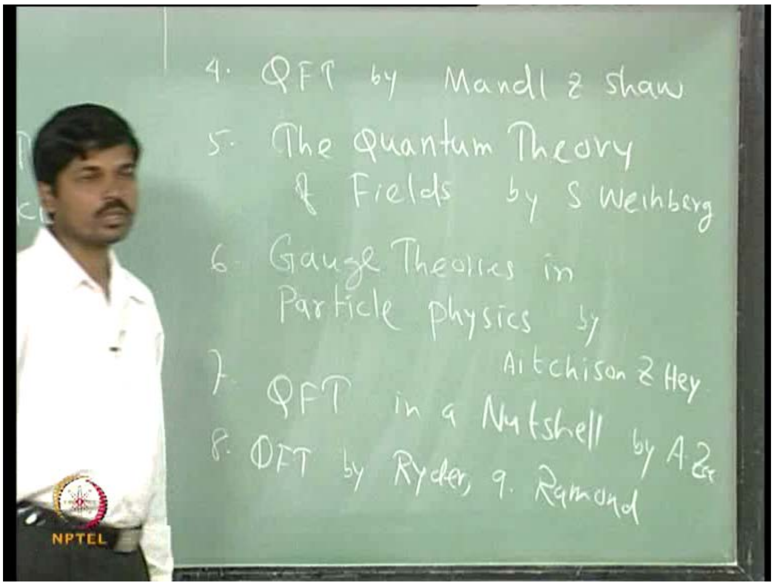

Quantum field theory by Mandal and Shaw then quantum theory of fields by Weinberg gauge theories in particle physics by Aitcheson and Hey. And hey quantum field theory in a nutshell by A Zee; then quantum field theory by Ryder ramanand and so on. So, almost everything that will be discussed in this course will be borrowed from one or two of these books. So, what I will do now is I will briefly review classical field theory. And then I will discuss how to quantize classical field theory and what are their application of the quantum field theory?

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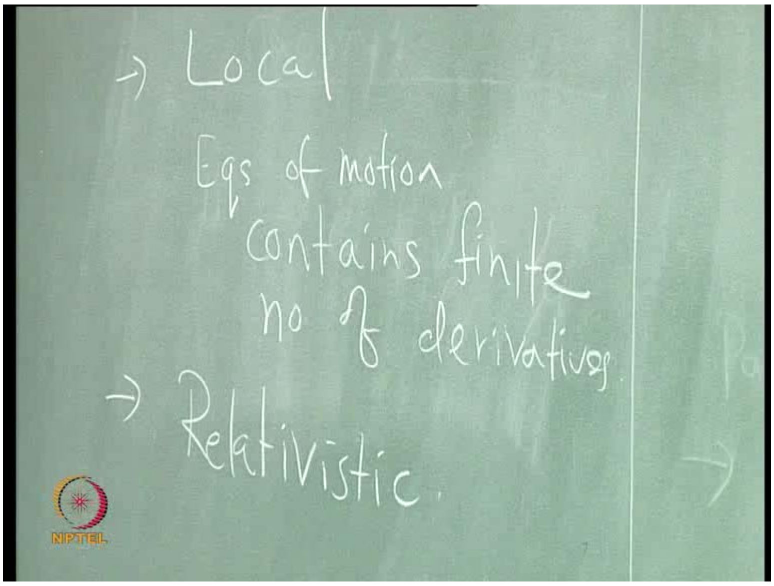

So, let us briefly discuss classical filed theory. So, the classical field theories that we are going to quantize various. So, we are going to discuss the classical field theory feature. For example, local in the sense that the equation of motions content finite numbers of that vectors; equations of motions contains finite number; they will also discuss field theory which are relativistic we will impose Laurence and variance.

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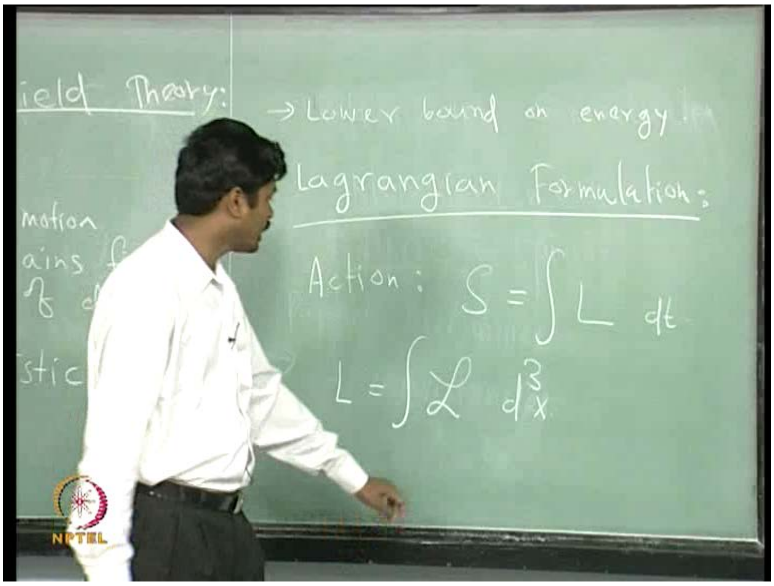

And, we will also require that the energy have a lower bound. So, we should have we will consider the field theory which have positive definite; let us first discuss the Lagrangian formulation. So, in Lagrangian formulation what you do; you consider the action which is S integration L d t; the Lagrangian itself we can write down as L as an integration of L over the entire space d cube x; over this L here is known as the Lagrangian density.

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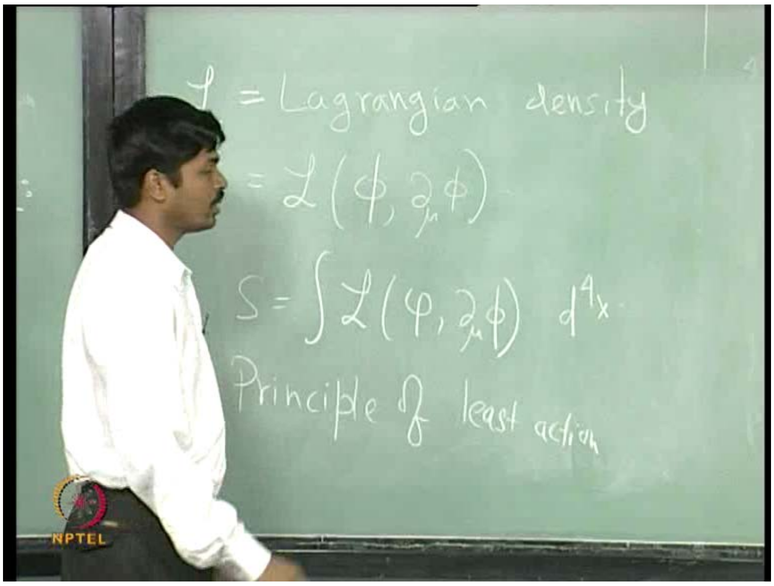

L is the Lagrangian density; in general L is a function of the field which I will denote as pi and its derivatives. So, the action can actually be express is S equal to integration of L pi del u pi d 4 x; the volume element d 4 x is d t times d cube x. We will find the equation of motion from this action by using the principle of least action.

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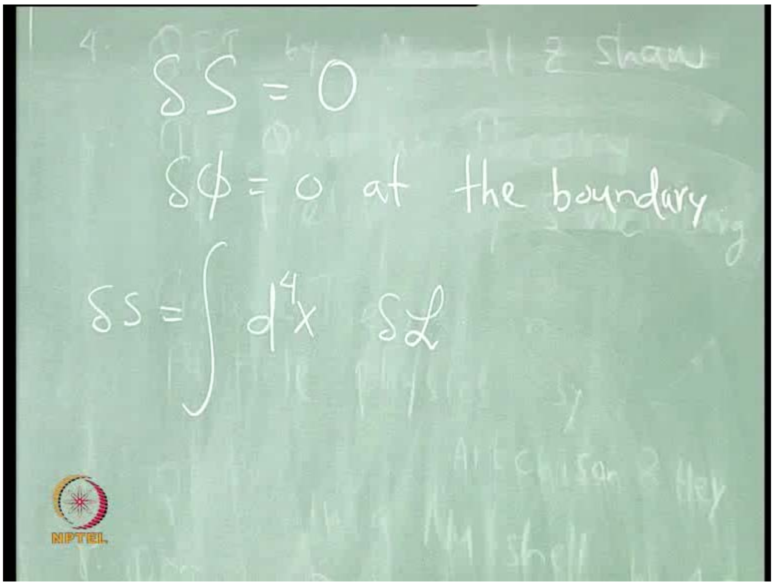

So, what we will do is that we will set the variation of the action to be 0 with the restriction that the variation of the field delta pi equal to 0 at boundary; this condition will give us the equations of motion. So, let us derive the equations of motions from this condition. So, what is delta S? Delta S is integration d 4 x delta L.

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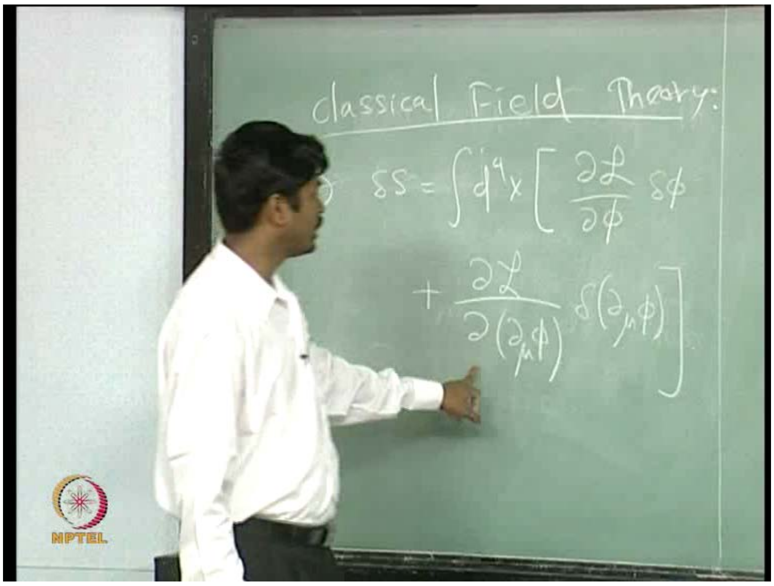

And, this is equal to del L over del pi times delta pi plus del L over del del mu pi delta of del mu pi. Here, I am assuming that the lag range in contains only single derivative of the field pi as well as itself function pi. If there are multiple derivative for example, if the

Lagrangian contains second derivatives of field pi. Then you will have one more term and so on; however we will not consider that case at this moment.

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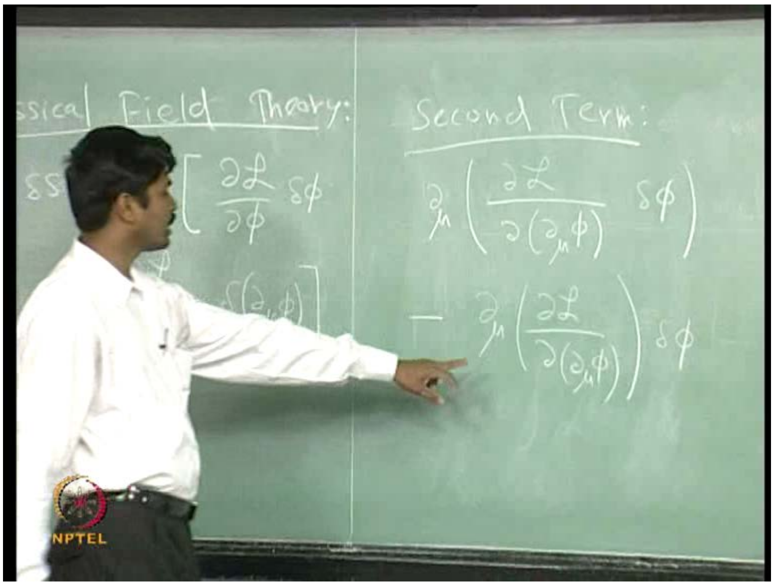

So, let us rewrite the second term; here you can see that the second term can actually be written as del mu of del L over del del mu pi delta pi minus del mu of del L over del mu pi delta pi. Here, I am assuming that this delta actually commutes with del mu; then you can write this term as a total derivative minus this term. So, now what I can do is that I can substitute this here.

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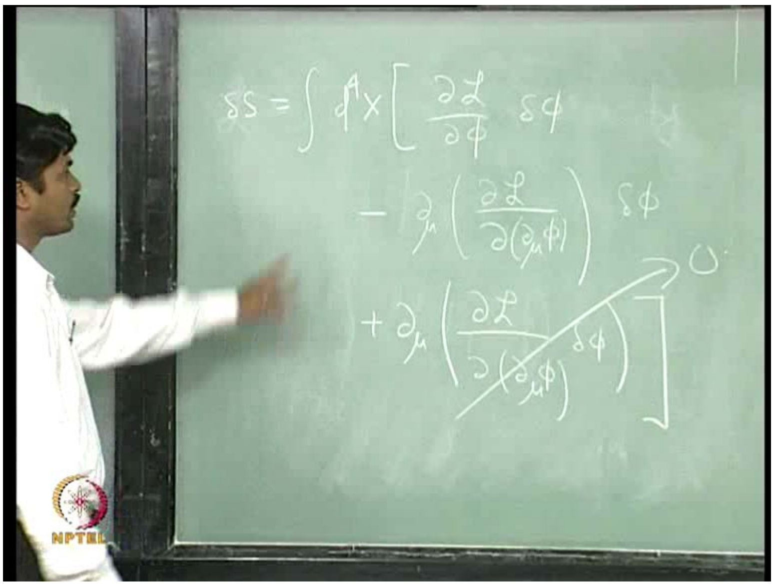

Then, what I get is delta S is equal to integration d 4 x del L over del pi delta pi minus del mu of del over del del mu pi delta pi plus del mu del L over del del mu pi delta pi. Let us now focus at the last term; you can see that this will give you a surface integration. However, here we are imposing the boundary condition that the variation of the field is 0 at the boundary. So, when we use this condition this term will actually vanish. So, the last term in this equation become 0.

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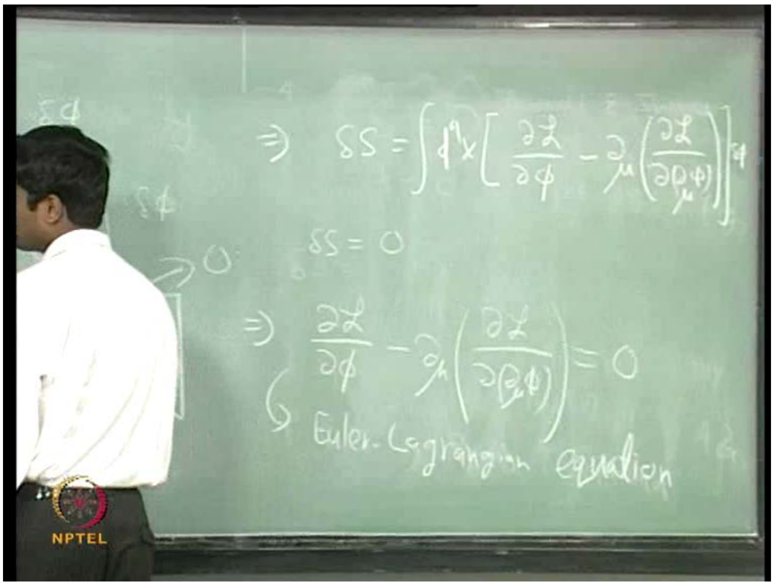

Therefore, delta S is actually equal to delta S is equal to d 4 x times del L of our del pi minus del mu of del L over del del mu pi times delta pi; however delta pi is arbitrary. Therefore, the integrant must be equal to 0 if delta S is equal to 0 then the integrant must be 0. So, this implies del L over del pi minus del mu of del L over del mu pi is equal to 0; this is our earlier langrage equation of motion. So, let us now find the Hamiltonian for the system; again you do it just the way you find the Hamiltonian in mechanics; in mechanics what you do?

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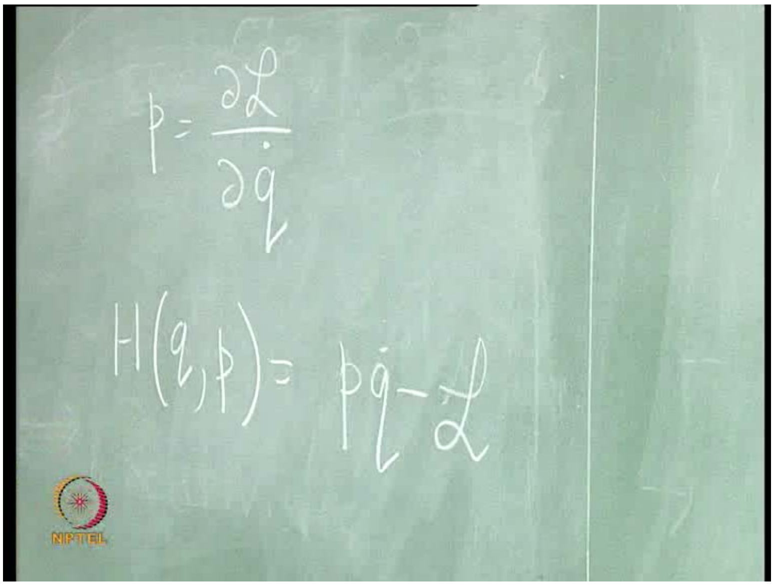

You have you introduce the conjugate momentum P which is del L over del q dot. And then you define the Hamiltonian which is a functional of q and P to be P q dot minus L.

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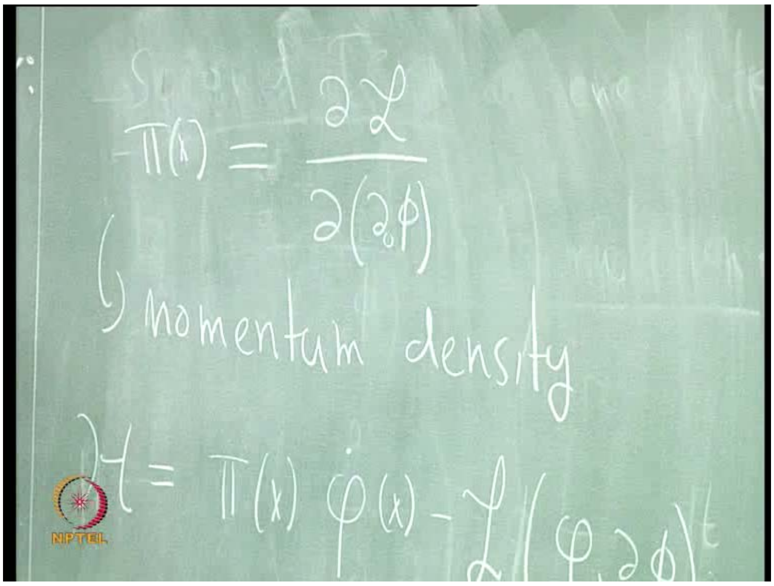

Here, you do exactly the same thing what you do is you introduce the conjugate momentum density which I will denote as pi of x; and the momentum density is defined to be delta L over del del 0 pi. And the Hamiltonian density so this is the momentum density then the Hamiltonian density H pi of x pi dot of x minus L of pi del pi. Let us consider a very simple example; the example of a real scalar field and then let us derive the equation of motion as well as Hamiltonian.

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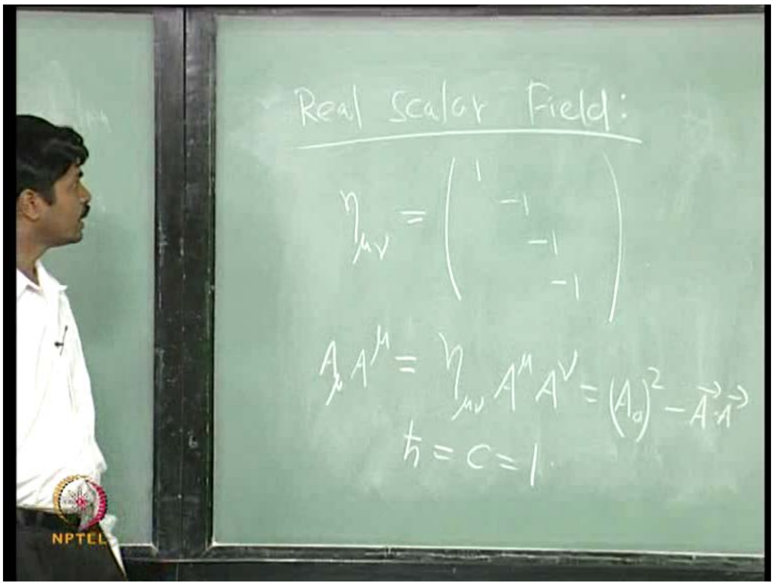

So, you will consider the example of a real scalar field. And then we will think what the equation of motion is and what is that Hamiltonian and so on? Before that let me explain the notation that we will be use it throughout the course; I will use the space favor matrix. So, eta mu nu it is 1 minus 1 minus 1 minus 1 or in other words the invariant length of a vector A mu A mu which is equal to also eta mu nu A mu A nu mu; this is a 0 square minus A dot A; throughout the lecture we will be using this method. And also we will use natural units. So, we will set h bar equal to C equal to 1throughout the course. Now, let us consider a real scalar field.

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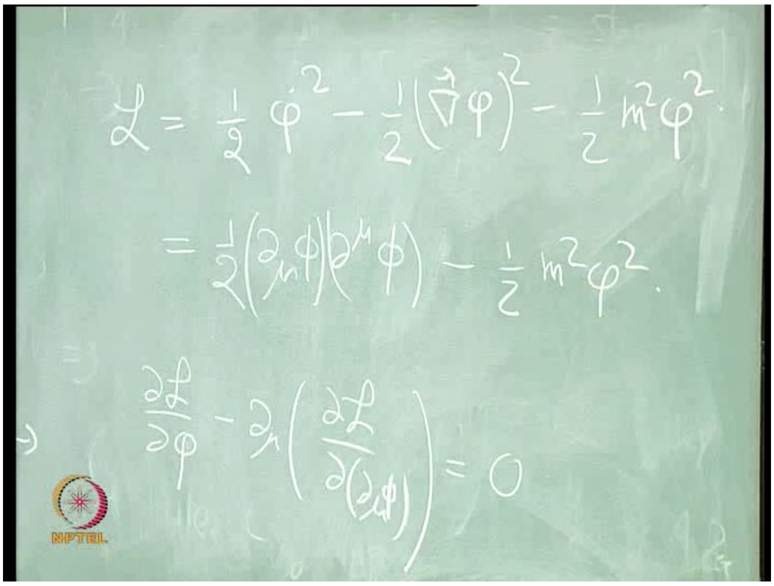

The Lagrangian density for real scalar field is half pi dot square minus half del pi square minus half m square pi square; you can rewrite this is half del mu pi del mu pi minus half m square pi square all right. So, what is the equation of motion for the system? The equation of motion is given by del L over del pi minus del mu del L over del of del mu pi s equal to 0. So, let us derive each of the term.

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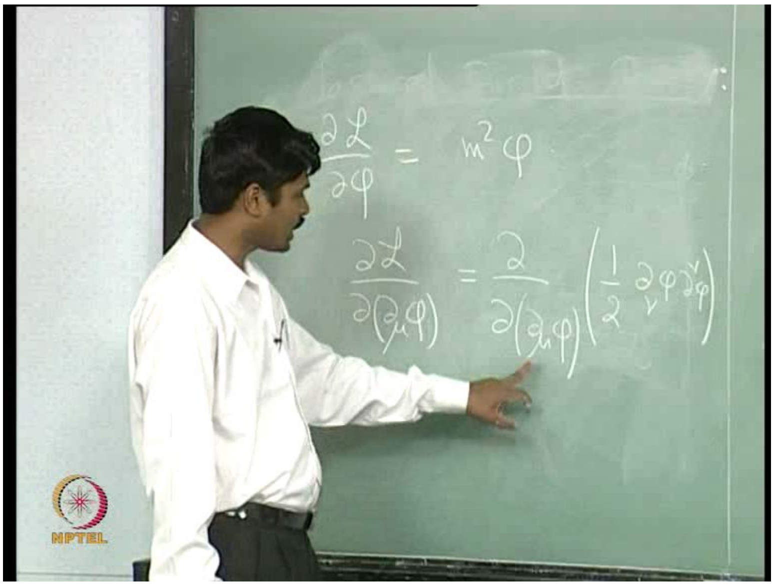

Del L over del pi is equal to m square pi and del L over del del mu pi. So, how will you derive del L over del L del mu pi? This is in the Lagrangian only this first term will contribute. So, as you can see these terms only depends on del mu pi del is independent of del mu. So, we will consider this ((Refer Time: 24:41)). So, this is equal to del of del mu pi times half; this acting on half del mu pi del mu pi. If you note this I have use this symbol nu here instead of mu; that is because I have the free index mu here; I must say that I am using Einstein summation convention. So, whenever a symbol an index is repeated it is summed over the value it takes.

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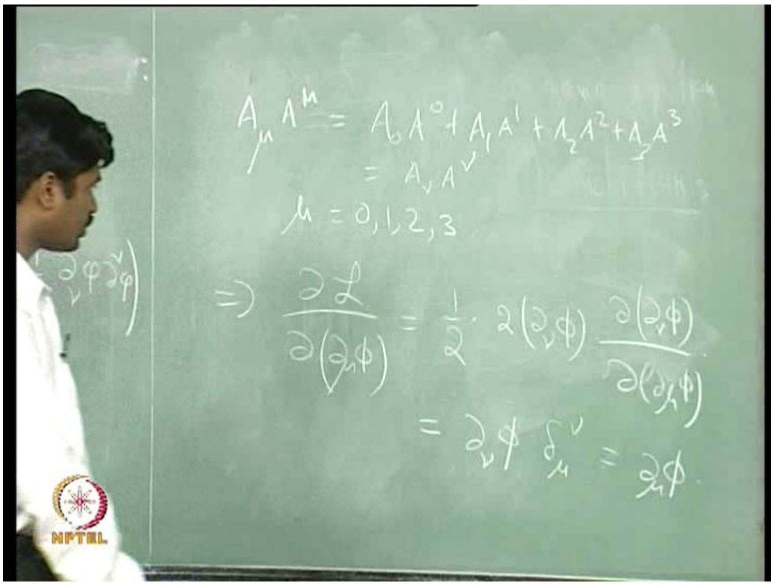

So, for if I consider for A mu A mu the index mu here runs from 0 over 1, 2, 3. So, A mu A mu is that equal to A 0 A 0 plus A 1 A 1 plus A 2 A 2 plus A 3 A 3. And hence A mu A mu is also equal to A nu A nu; this mu here and this expression is a dummy index. So, we can put any level we want for this here; that is what I have done in this expression instead of mu I have used nu here. So, this is now equal to so this implies this del L over del del mu pi is just equal to half times twice del mu pi times del of del mu pi divided by del of del mu pi. Now, what is this quantity here? This is just delta mu you no. So, this is nothing but del mu pi delta mu nu. So, this is just del mu pi.

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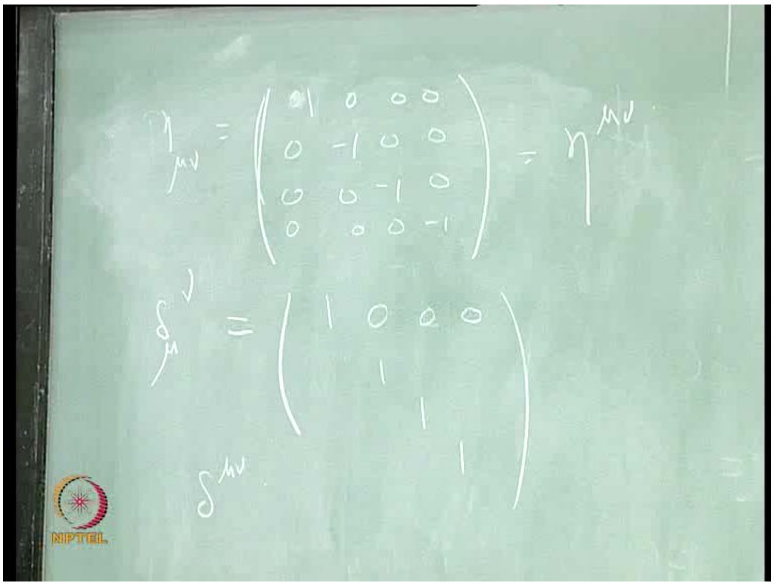

So, what we saw here is that

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No. So, let me explain this symbol again eta mu nu is equal to 1 0 0 0 0 minus 1 0 0 0 minus1 0 0 0 0 minus 1. And now delta mu is the identity matrix instead of 1 minus 1 this is just 1 0 0 0 and so on; all the diagonal elements are 1 and all of diagonal elements are 0. Now, when you look at this; if mu is not equal to nu this will just gives you a 0. If mu is equal to nu this is 1.

So, however eta mu nu as well as it inverse eta mu nu which is numerically equal to the same thing eta 0 0 is 1 whereas eta 1 1 eta 2 2 and eta 3 3 are minus 1. However, here you can see if no matter whether mu and nu whether mu is 0 or 1 or 2 or 3 it always gives you 1; if identity if mu is equal to nu otherwise it gives you 0. Therefore, this quantity here is has to be equal to delta mu nu not eta mu nu; we are not using something like this delta mu nu we are using and also the tensional property here is such that it is calorie mixed tensor. If you look at the Laurence transference property of this quantity this does not transfer like contra variant tensor of rate 2; it transfer likes a mixed tensor of rank 2. So, therefore the index structure has to match. And also the notation that we have we are using is such that this is equal to delta mu nu right.

Student: ((Refer Time: 29:58)).

Thank you; this mu here is actually a contra variant index not a co variant index.

Student: ((Refer Time: 30:12)).

No, I see ok thank you. So, this is actually equal to eta mu nu. So, this is del mu pi clear let us so summaries what you have seen here.

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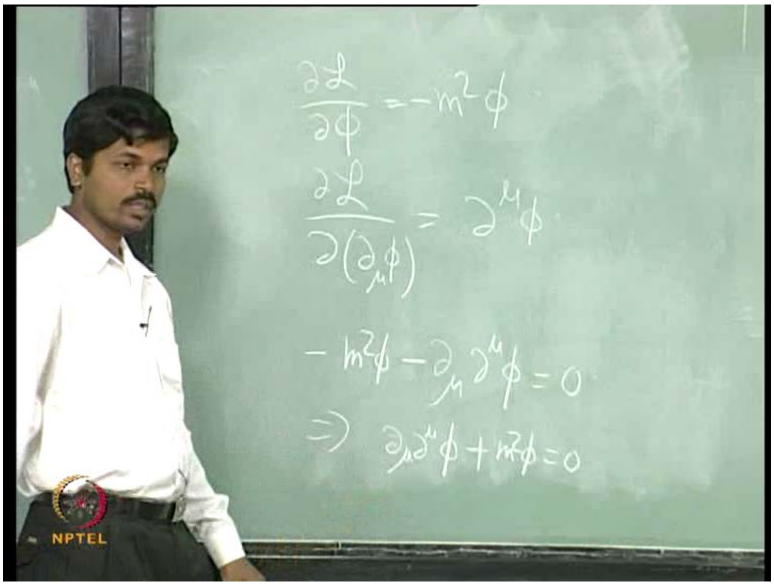

What he say is I define del L over del pi is equal to m square pi and

Student: ((Refer Time: 30:58)).

Minus m square pi thanks again minus m square pi and del l of del mu pi is equal to del mu pi. So, now we can substitute this in to equation of motion. And what is see is minus m square pi minus del mu del mu pi is equal to 0 or in other words this is just del mu del mu pi plus m square pi s equal to 0; what you define is the Klein Gordon equation. So, this field here is this actually a Klein Gordon field. Now, what we will do is we will find the Hamiltonian density for this system and you will see what you get?

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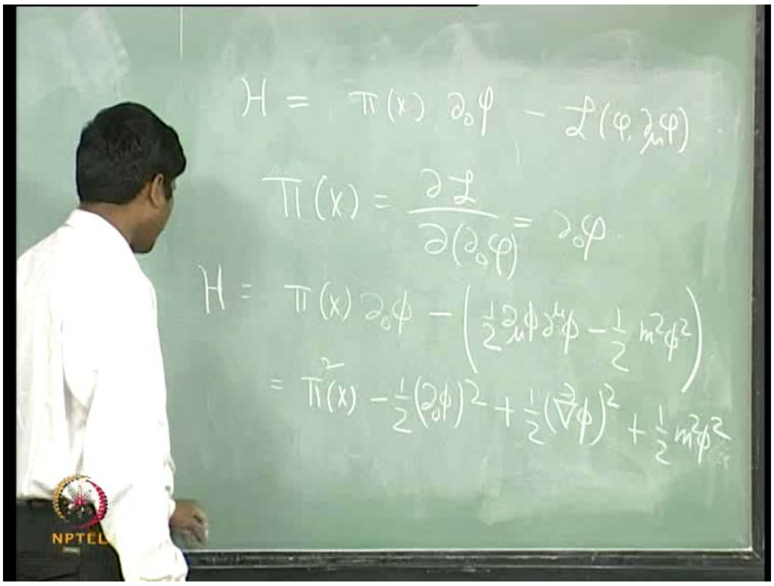

So, you have already introduce the Hamiltonian density H equals to del pi of x del L over del of del 0 x pi minus L of pi del mu pi. So, this is equal to pi of x and we can see from this expression that del L over del del 0 pi is again del 0 pi. So, you can use this expression here; no no sorry this is simply del 0 d I q I dot del 0 pi minus L. So, pi of x is del l of del del 0 pi this quantity is again del 0 pi. So, we can use we can substitute this for pi of x. Then what we get is Hamiltonian density H equal to pi of x del 0 pi del 0 pi minus half del mu pi del mu pi minus half m square pi square; this is equal to pi square of x minus half del 0 pi square plus half grade pi square plus half m square pi square; we can again substitute this for pi of x.

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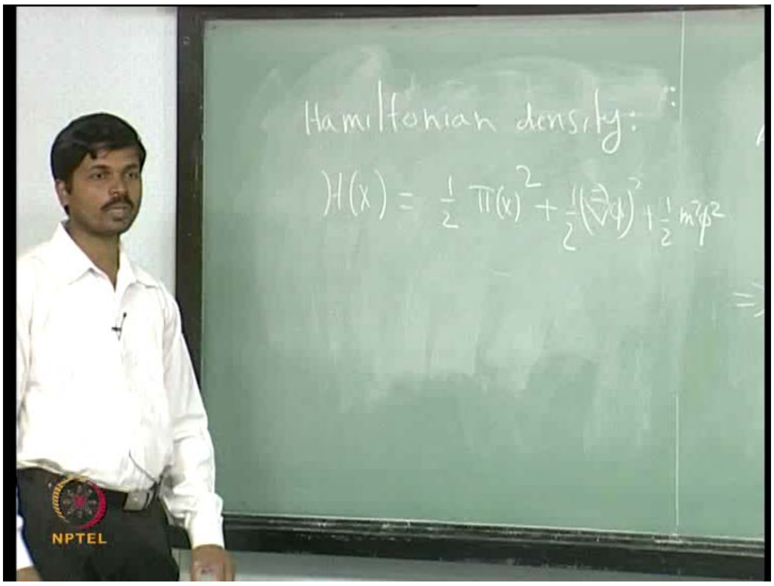

And, then what we get is that Hamiltonian density H (x) is equal to half pi of x square plus half grade pi square plus half. As you can see this very simple system actually satisfied all the criteria that we have a specified in the beginning of this lecture; in the sense that the system will actually Laurence and variant. The action is invariant in the Laurence transformation, the equations of motions are co variant under Laurence transformation it is local because the equation of motion only is a 2 derivative it contains on the 2 derivative terms. Therefore, itself local field theory and the energy density as a lower bound; all the terms here are positive definite. So, this satisfied all the criteria.

So, what we will do this what we will do in this subsequent lecture is that; we will start quantize this very simple system. And then we will study more and more complex field theories. So, with this we will close today's lecture; tomorrow I will discuss some of the symmetric and conservation loss. And then we will actually start quantization ((Refer Time: 37:02)) theory.

Thank you.