Physics of Linear and Non-Linear Optical Waveguides

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Module - 05
Nonlinear Fiber Optics
Lecture - 60
Concept of optical soliton (Contd.)

Welcome student to the course of Physics of Linear and Non-Linear Optical Waveguides. So, today we have lecture number 60, and probably today we have the last class and we try to understand Optical Soliton.

So, we did few calculation in the last class. Today, we will going to conclude this calculation and find the solution, ultimate solution of the optical soliton.

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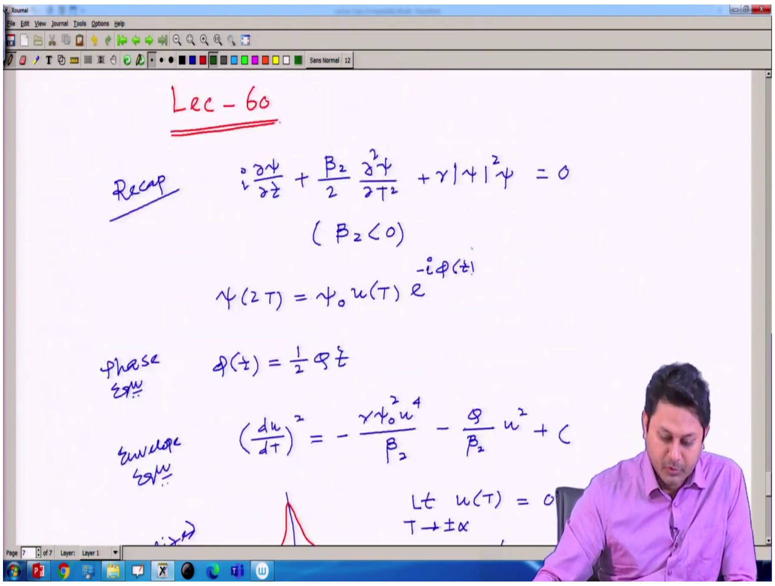

So, let me quickly refresh because last day we did few things. So, let me quickly recap. And the equation we try to solve is this. For beta 2 less than 0 in anomalous dispersion the solution we are looking for and this is the form of the equation which we call the non-linear Schrodinger equation so this is 0.

We tried to solve this. It is not easy to solve, but we put some approximation and then using certain boundary condition try to solve this differential equation. Numerically, normally people solve this equation and find what is the solution. There is another method called the inverse scattering method then we using that also people solve which a very very lengthy calculation to find the solution. But here we try to provide certain solutions and which one can realize that how the soliton is coming.

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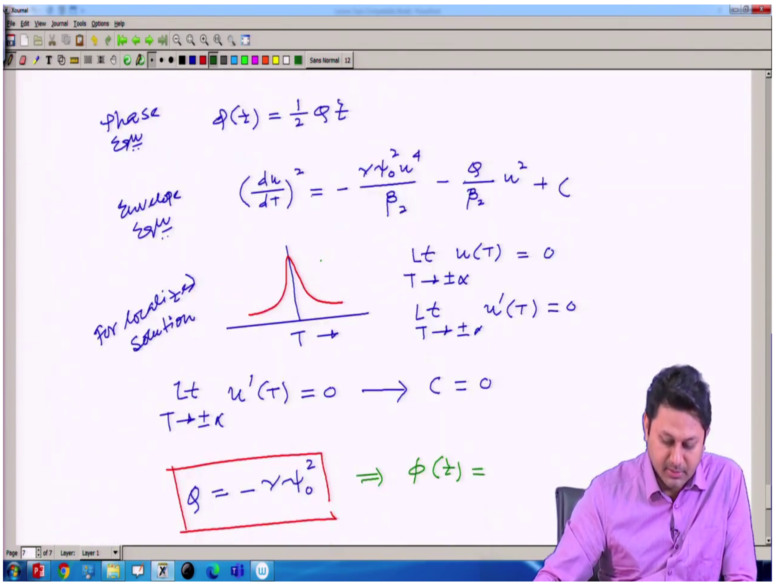

So, this is the equation we try to solve, and then we put this is when beta 2 is less than 0, in anomalous dispersion, that was that was the condition that in anomalous dispersion when beta 2 is 0 we had this solution. So, we had a negative sign here, but now it becomes positive because beta 2 is less than 0.

Then, we have a solution of the form like psi z T is equal to psi 0 u T, e to the power of say minus of phi z that was the form we are looking for the solution. And then making a separation of variable we had an equation for phi, and this equation tells us that the phase will be something like this. That was the phase equation.

On the other hand, the envelope equation becomes something like, where C is a constant. Now, we then we put a concept of localized solution, for localized solution we have a shape like this, and the condition for localized solution was limit T tends to plus minus infinity u T, this is 0 and limit T this is T tends to plus minus infinity u prime T which is a derivative of that is 0.

The second condition limit T tends to plus minus infinity u prime T equal to 0, basically gives us the value of C as 0 that we figure out. Then, another constant is still sitting here Q and we find this solution Q, we find the value of Q as minus of gamma this square. Mind it, this Q was sitting here also.

So, from that we can say that the phase, the phase from this, the phase will be changing this is the phase it is half Q z. So, it should be changing like minus of half in this way.

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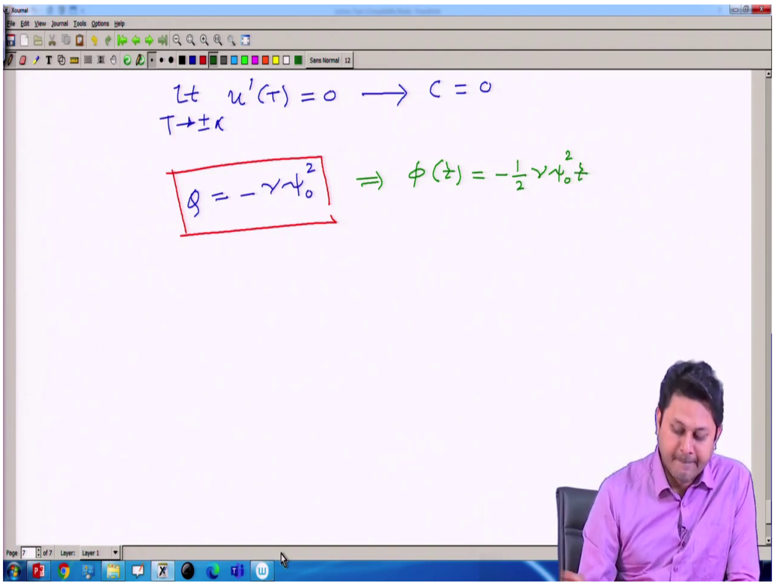

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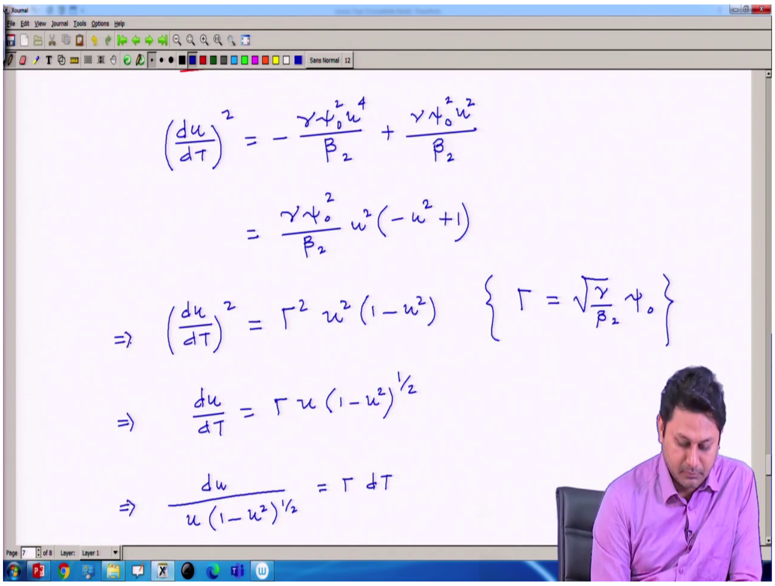

After that we find the Q and from this Q we can have the differential equation in this form, ok. So, now, that is the equation we derived till last class. So, today we are going to solve this and try to find out what is the form of u.

So, these I can write it like gamma psi 0 square divided by beta 2 I take this common from both the term then u square also, and I have something like minus of u square plus 1. So, this term is there you can see. So, if I take this common then you will be getting this term.

So, d u d tau square is equal to let us take these as constant because gamma psi 0 beta 2 all are constant. So, I put a constant here, say gamma big gamma square and then the rest of the term u square and then 1 minus u square. So, here gamma is equal to root over of small gamma beta 2 psi 0. So, that is the big gamma the constant, , ok.

After that what we do? Try to find out del u del T. And if I take only the positive part there will be plus minus, but I am ignoring the minus sign then I will have a form like this, ok. Then, I can have d u u 1 minus u square whole to the power half is equal to gamma dT.

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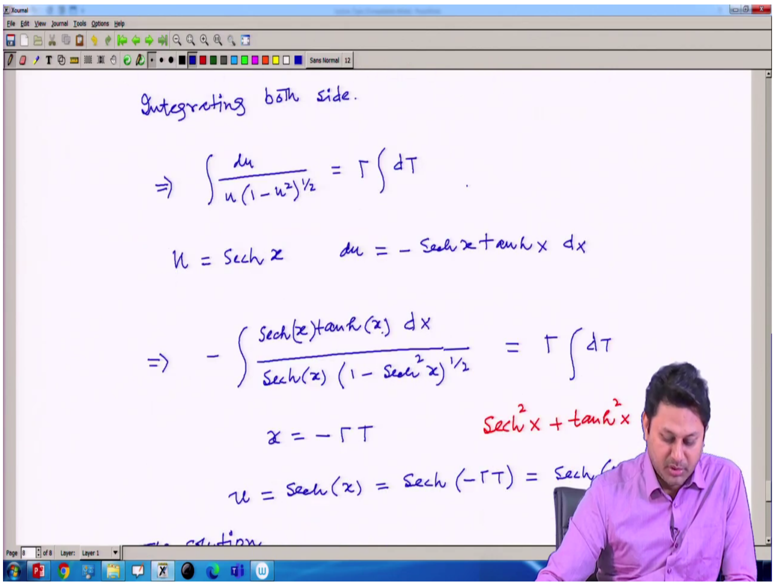

Integrating both side, integrating both side, we have integration d u this. Now, this integration we need to solve. So, if you take u as sec hyperbolic x, then d u should be minus of sec hyperbolic x, the derivative of this quantity, then tan hyperbolic of x, dx. I just change this u which is a variable as sec hyperbolic x, then d u will be this one.

If I put here in this integration if I put this integration, then I will have minus integration of sec hyperbolic of x tan hyperbolic of x let us put a bracket here. And then d x, in place of d u, I put all these values and in the denominator it should be sec hyperbolic of x and then 1 minus sec hyperbolic square of x whole to the power half. In other side, we have integration of dT.

Well, if I take, so this is gamma multiplied by T. So, x if I take minus of gamma T then u which was sec hyperbolic of x will be simply sec hyperbolic of minus of gamma T which is equal to sec hyperbolic of gamma T. And then the solution. So, here we can have this after doing this integration. So, you can see this tan hyperbolic and 1 minus sec hyperbolic square will going to cancel out because this is tan hyperbolic square.

So, I use this identity here and after using this identity this becomes x equal to because these things will going to cancel out, x equal to minus of gamma T; after doing the integration on the right hand side. And u originally was sec hyperbolic of x, so eventually I find x, and u is sec hyperbolic of x which is sec hyperbolic of minus of gamma T, and sec hyperbolic of gamma T because this minus we are going to absorb because this is the sec hyperbolic term.

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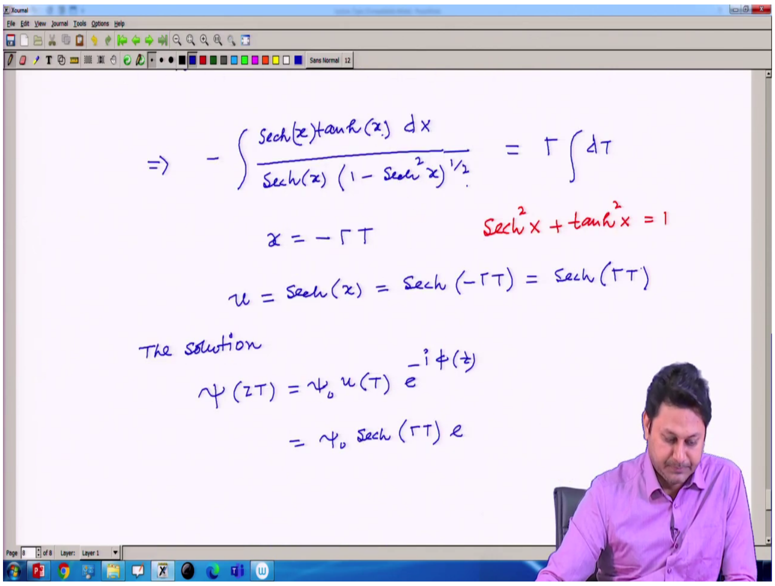

Now, the solution we assume in this form. This solution becomes psi 0 sec hyperbolic of gamma T because u T is now this one, then e to the power of what was my phi, my phi we calculated here. So, phi is minus of half of gamma psi 0 square z. So, that I should put. So, it should be of i by 2, then gamma, then psi 0 square.

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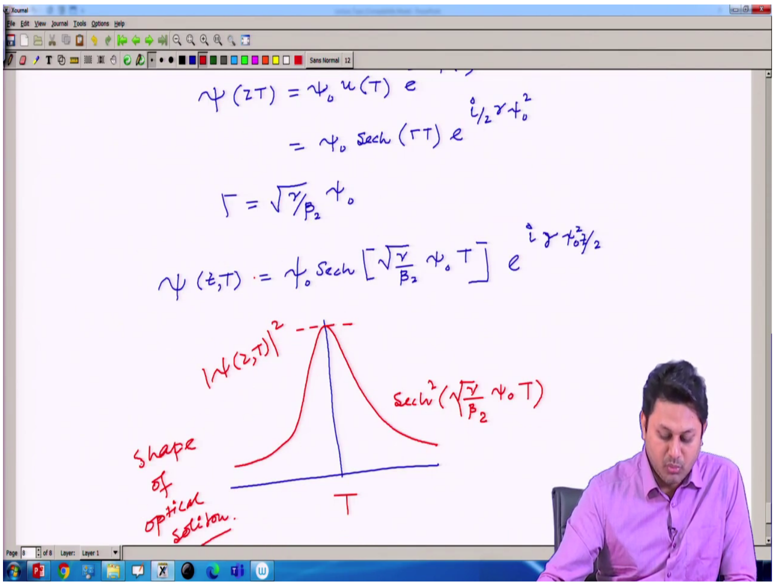

And what is the gamma? Gamma is root over of gamma divided by beta 2 psi 0. So, finally, the solution that we obtained is this. So, this is the solution and the solution suggests that the envelope will going to if it is optical saliton, the envelope will have a sec hyperbolic kind of shape. And the phase, one z will be there because this is a function of z, z divided by 2.

So, the shape of the optical soliton is a sec hyperbolic type. This is a very very important. So, if we plot the sec hyperbolic square because normally we put the mod square of that. So, if I put the mod z t square the phase term will no more. So, this will be the form sec hyperbolic square of, in the argument we have root over of gamma beta 2, then psi 0 and then T. So, over T that is the envelope and this basically the shape of optical soliton, shape of optical soliton.

Well, normally this is the way one can do, one can find the shape of optical soliton. The mathematical formulation of optical soliton is quite difficult, but this is some way one can understand the how the optical soliton will have a sec hyperbolic kind of shape. So, this is the procedure; I try to, with this procedure I try to make you understand that what should be the shape of optical soliton.

So, next thing, there is a final thing that I like to do also in today's class since today is the final class. So, that is the normalization of optical non-linear Schrodinger equation. In many books, you will find this normalization form which is useful.

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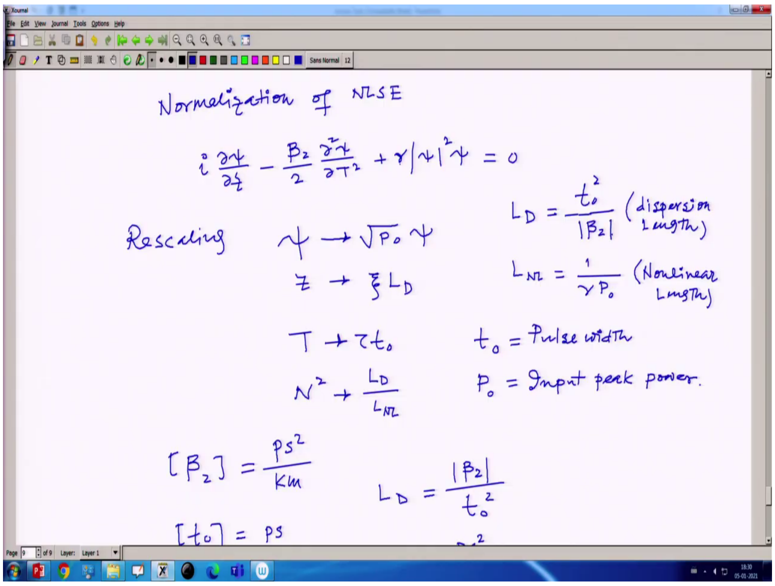

So, normalization of non-linear Schrodinger equation. And we had that we had that equation of the form like this, this equation with dimension because beta 2 has some unit which is the unit of dispersion coefficient, pico second square per kilometer that we know. So, everything with units here. Gamma is a unit of nonlinearity, non-linear coefficient which is having the unit 1 per meter Watt. So, these are the units we have. So, now we normalize that.

So, rescaling, we make the rescale like psi is equal to say root over of P 0 psi, z i scale as xi multiplied by L D, L D is something which is defined in this way which is mod of beta 2 divided by T 0 square. This is called the dispersion length. This is called the dispersion length.

And another quantities also we have accordingly it is called the non-linear length. This is one divided by gamma P 0. This is called the non-linear length. So, T, I can rescale as tau T 0, where T 0 is a pulse width and N square it is called the soliton order is defined as the ratio of the non-linear length, dispersion length and non-linear length. T 0 is the pulse width and P 0 is the input peak power. These are the few terms that I can put here in this equation to make it normalize. This is called the rescaling.

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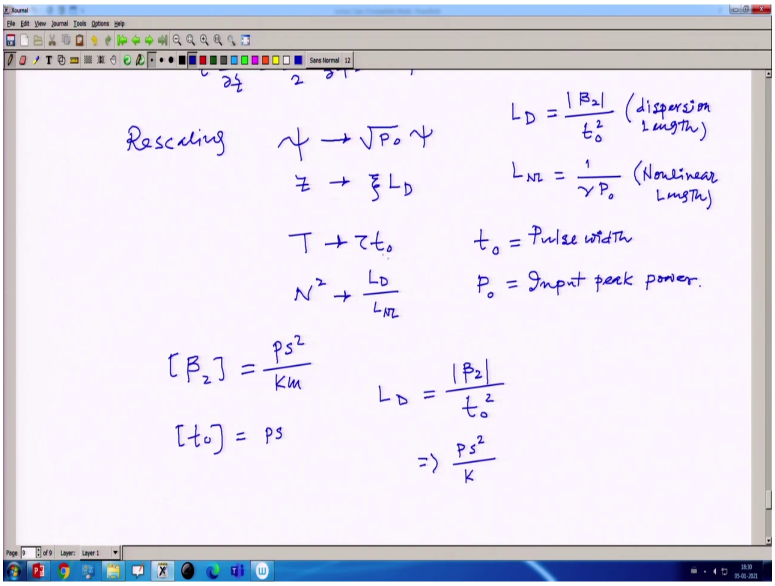

And if you look carefully the dimensions that, so beta 2 is having a dimension the unit of pico second square divided by kilometer and L D is the dispersion length and it should have certain units. So, T 0 which is a pulse width should have the unit like pico second or femto second etcetera. So, let us put it at pico second.

So, what is the unit of L D? L D has to be unit of length. And if you see L D is mod of beta 2 divided by T 0 square. So, unit wise it should be pico seconds square divided by, ok, this L D is beta 2 I making a mistake here it should be L D inverse is this quantity actually. So, pico second square per kilometer. So, I am making; so, this is it should be T 0 square divided by beta 2. I make a mistake here.

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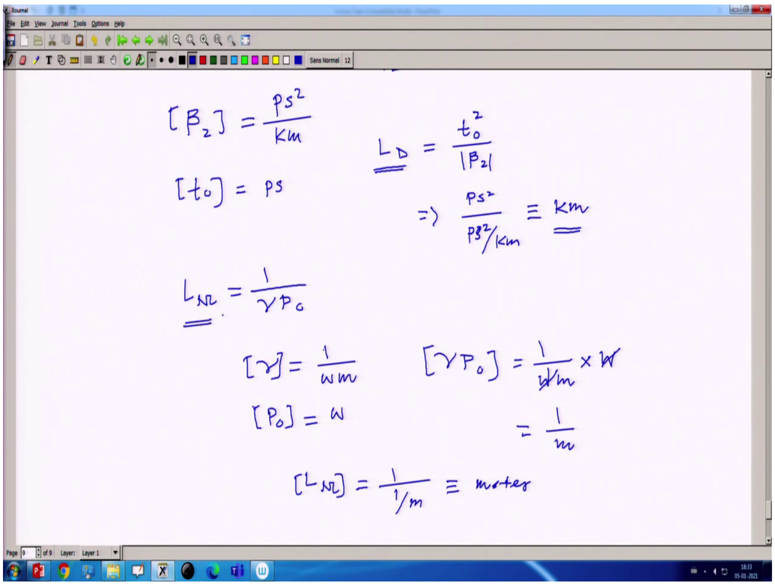

And now L D should be in the form of length. The unit of this things should be length, ok. Let me erase completely. So, it is T 0 square divided by mod of beta 2, unit wise it is pico second square and denominator we have pico seconds, pico second square divided by kilometer, so which is eventually the unit of length or unit of kilometer, unit it is kilometer, so kilometer; so good.

It is a length dispersion length and it has to be in unit of kilometer and whatever the rescaling we have, whatever the definition we have, from this definition we can see it is coming like a kilometer. Also it is better to check the L nonlinearity. So, the non-linear length L NL which is 1 divided by gamma P 0. Now, gamma we know it is Watt per meter.

So, it is, so gamma is 1 plus Watt meter and P 0 is Watt in in unit it is Watt. So, when we multiply gamma P 0, this quantity is 1 plus Watt meter multiplied by Watt. So, this Watt we going to cancel out. So, it should be 1 per meter. Or L NL which is 1 divided by 1 by meter, so it should be equivalent to meter, the unit is equivalent to meter length. Again which is ok, because this is basically the non-linear length.

So, dimensionally you please check that everything is or not. Now, after putting this dimension you can see that this z which was initially the dimension of length is now having a new coordinate xi which is a dimensionless coordinate. In the similar way, tau is a dimensionless coordinate.

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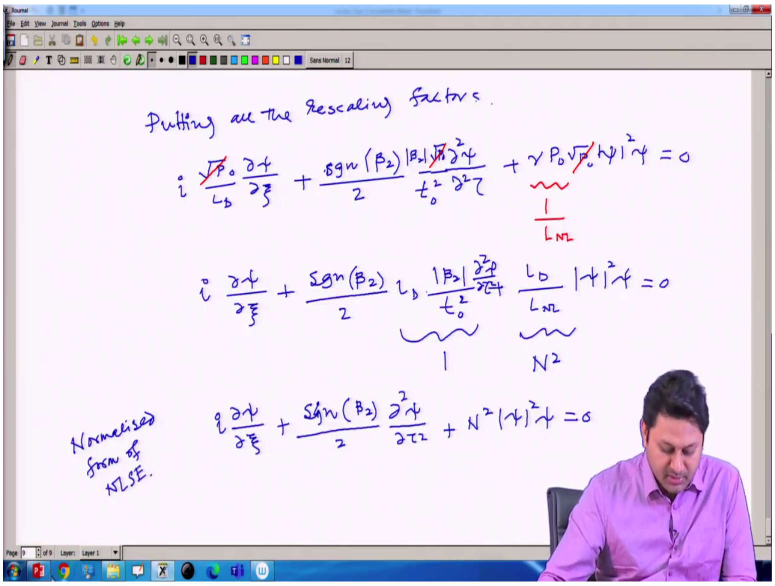

So, if I put all this together, then I can have putting all this putting all the, or rescaling factors. Let us put this one by one. So, psi is equal to root over of P this quantity. So, if I put i and this is root over of P 0 del psi and here it is z. So, z I can write it as divided by L D del xi plus, beta i just put the plus sign and then I put then I put the mod of beta, so that means, I can put a signum function here.

Signum means only I am taking the sign of that quantity divided by 2. And then this derivative, so I should have root over of P 0 d 2 psi and again for t I have t square here. So, sign of beta multiplied by mod of beta. So, 1 mod of beta it should be included it should be included like this. So, just a minute. It should be mod of beta and then root over of P 0, divided by t 0 and del 2 tau.

And the final term which is gamma mod of these things. So, I should have the gamma as usual and then P 0 square term will be there, not P 0 rather P term will be there because it is root over of P the scaling is root over P. And then I have a root over of P and then psi mod square and psi that is 0. Putting all the rescaling term I am having this.

Now, this P 0 root over term will going to cancel out from here, here, and here, and these things is, this things is 1 divided by L NL. So, I can now write the equation like i del psi plus this signum of beta I can put, then I multiply L D into mod of beta 2 divided by T 0 square plus L D divided by L NL. So, this quantity again I already defined N square which is called N is called the soliton order there is a specific name for that. So, N is called I should write here is called soliton order.

So, finally, I have. So, here also you can see L D is T 0 square divided by mod of beta 2. So, T L D multiplied by mod of beta 2 divided by T 0 square simply becomes 1. So, eventually I have an equation which is normalized and this normalized form if I write signum function of beta 2 divided by 2 and then, ok.

I make a mistake here because the second order derivative term should be there tau square. So, del square psi del tau square plus N square mod of psi square psi equal to 0. So, in this equation this is a normalized form non-linear Schrodinger equation.

Now, this equation should have a solution. So, let me write down the solution here and this solution already we derive, but I want the student to please check the solution.

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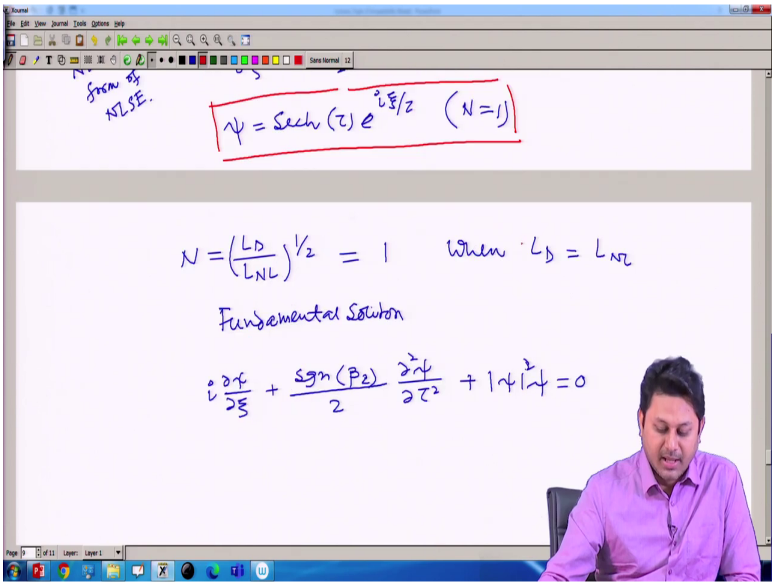

That the solution I directly write should be in this form, sec tau e to the power of e to the power of i and then xi by 2 with the fact when N is equal to 1, N is equal to 1 is a special condition. N which is L D divided by L NL whole to the power of half is 1 when L D is equal to L NL, we called this fundamental soliton.

For fundamental soliton, what happened? The equation, the non-linear Schrodinger equation will have a form like this because my N is now 1. And for that we have the solution this one. So, I am not putting this solution directly, but I request the student to please check whether this solution is working or not.

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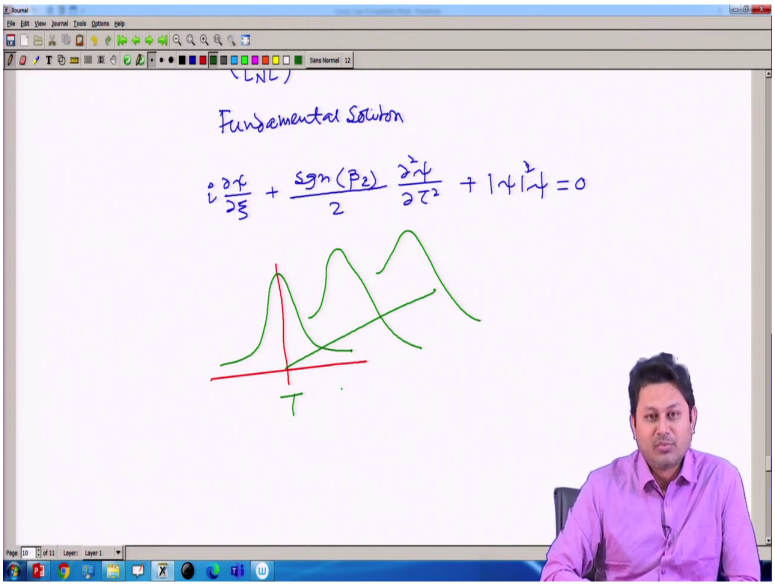

And this is the structure of the sec hyperbolic pulse I already mentioned and it behaves like a soliton. So, when it is propagating over the z, so there is no change. So, it will be propagating like that. There will be no change of this pulse and it will propagate without any kind of distortion over time and this is basically the concept of optical soliton.

So, in this course we just briefly try to understand the concept of soliton, it is the vast subject, but since we are dealing with non-linear waveguide also. So, we need to know that in non-linear waveguide we have this kind of solutions which we called the optical solitons that is propagating in a long distance that can be propagated in a long distance without any kind of distortion.

And the major reason behind that is there is a nonlinearity. And there is a dispersion, they counter balance each other. When they counter balance each other the overall effect of the distortion is gone. So, because of this counter balance they form a very very stable structure called optical soliton.

We try to understand today qualitatively, quantitatively by calculating, by solving the differential equation and we find that it should have a shape of sec hyperbolic kind of distribution.

So, with that note I like to conclude today's class. Today is the last class and I do hope that you people enjoy this course. Those who are taking the optics in your PhD, as your PhD topic for them especially those who are working in optical fibers for them this course may be very very useful and I hope that you enjoy this course and this course will be helpful for all of you.

So, thank you very much for your attention. And see you in the next, I mean this is the last class. So, see you again when I have some other courses.

So, thank you and see you.