Physics of Linear and Non-Linear Optical Waveguides
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Module - 05
Nonlinear Fiber Optics
Lecture - 55
Pulse Propagation in Nonlinear Waveguide

Hello student to the course for Physics of Linear and Non-Linear Optical Waveguide. And now, we will going to understand in this class, the governing equation of Pulse Propagation in Nonlinear Waveguide ok.

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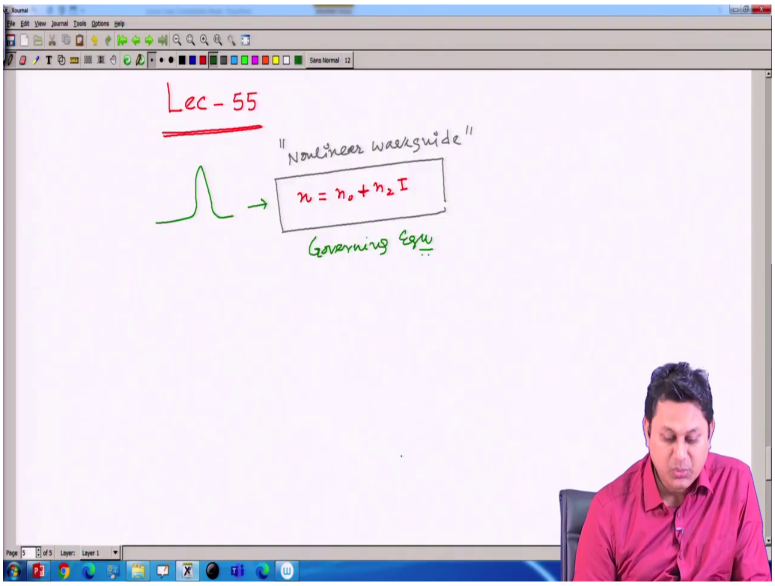

So, in the last class, we roughly understand what is the meaning of cell phase modulation. So, today, we will going to use that concept to understand what is the governing equation of a of an optical pulse propagating in a non-linear waveguide. So, this is, suppose this is a non-linear waveguide. So, this is a “Nonlinear waveguide”. Suppose and when I say this is a non-linear waveguide, the refractive index is now will be written in this form; n 0 plus n 2 I.

And when a optical pulse is launched here, and when it is propagating inside this, so there should be some governing equation which basically tells what should be the dynamics of this pulse inside this kind of waveguide. So, today, we will going to derive this governing equation.

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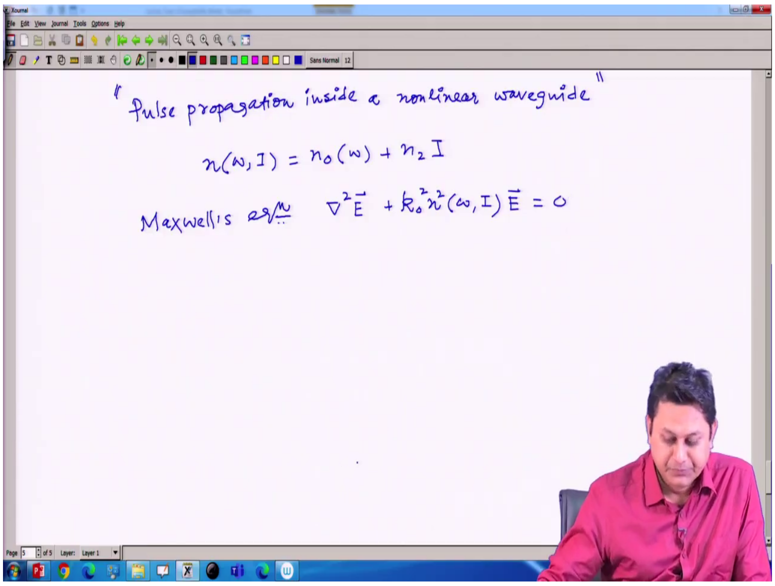

So, the topic is “Pulse propagation inside a non-linear waveguide” ok. So, I will start with the refractive index, because that characterize the nonlinearity of a waveguide. So, the refractive index if I write, it should be simply this at first. Now, the Maxwell’s equation if I consider, it is simply square E plus k 0 square n square which is now function of intensity as well, that we need to remember every time and then, E equal to 0.

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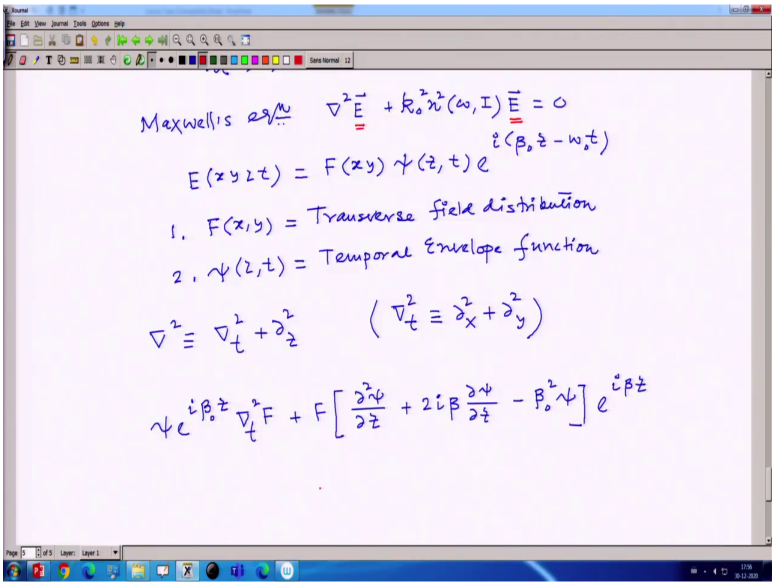

So, this is the form of the Maxwell's equation. Now, I form the electric field. I define the electric field which is now function of x, y, z and t. And I can write this electric field into 2 form; one is the spatial distribution of the electric field which is x, y and another is the propagation part with the temporal distribution and since, it is a propagating wave, I can have this form as well; beta 0 z minus omega 0 t.

So, I have a optical field, electric field rather, with this particular form where it is a spatial distribution. So, let me write it one by one. F x, y this is a Transverse field distribution; transverse field distribution. 2nd, psi z t and that is a Temporal Envelope function; temporal envelope function.

And then, I have the propagation part; beta 0 z minus omega 0 t. So, beta 0 is a propagation constant and omega 0 is a frequency of that. So, all these things are defined. After that, I will divide this operator which is in the Maxwell's equation into two part that we have done several time in the earlier calculations.

One is the derivative transverse derivative and another is the derivative with respect to z, where this transverse derivative in shorthand notation is equivalent to this. Now, when this operator is operating over this electric field as per the Maxwell's equation suggest, then we have psi e to the power of i beta 0 z and then, the transverse component will going to operate over F because F is a function of x, y.

Then, F and this operator which is that the second derivative with respect to z, will going to operate over this entire function and if it is operate over this entire function again, these things we have done earlier. So, it should be the second derivative of psi with respect to z. Then, the 2 of the first derivative of psi and first derivative of the next function; so, it should be 2 i beta, this and the second two derivative of the last function which will simply gives us this. And the final function e to the power of i beta z.

Please note that I just eliminate e to the power of i omega 0 t. Because if you put e in here and here, then e to the power i omega 0 t will cancel out from both the sides. Because here, there will be e to the power i in this part we have an e to the power of i omega 0 t. Here also, we have e to the power minus e to the power of minus i omega 0 t. So, these two terms will going to cancel out.

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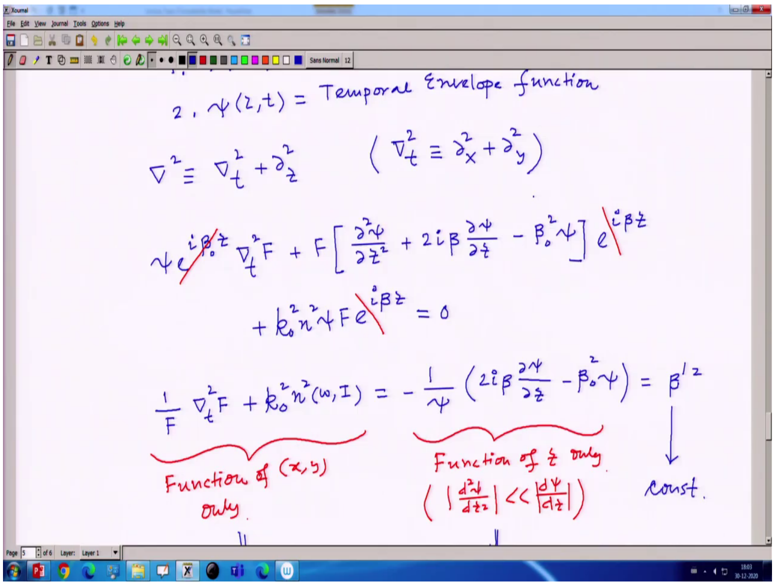

So, that is why I just remove that part. So, these plus I have k 0 square n square psi F and then, e to the power of e to the power of i beta z is equal to 0. So, this is the equation. Now, I can rearrange this equation and I can have 1 by F, F plus k 0 square n square which is a function of this, that is one part and I can write this as 1 by psi 2 i beta del psi del z and then, minus of beta 0 square and psi.

So, few things I have done here. So, I divide everything with respect to psi e to the power i beta 0 z and then, psi e to the power i beta 0 z and F. So, this term will be 1 by F e to the power i beta 0 z will going to cancel out. So, this term, this term and this term, we will going to cancel out.

And then, if I divide everything with psi multiplied by F, then, it should be 1 by F this and this F will be 1 by psi and this quantity will be k 0 n 0, k 0 square n square omega 0 omega I.

And I can put this one side here, which is a function of this portion is a function of x, y only; whereas, this portion is function of z only. Please note that when we calculate, we consider that change with respect to z of this quantity is much much less compared to the change of psi with respect to z, the first derivative. So, I can use the slowly varying approximation here.

The slowly varying approximation tells me that I can ignore the second order derivative ok; I have a mistake here, I should have a square. Second order derivative compared to the first order derivative of psi, applying the slowly varying envelope approximation. If I do then, we have a function of x, y in left hand side; function of z in the right hand side. So, these two things has to be equal to some constant and I write this constant as beta prime square. This beta prime is essentially a constant and why it is written beta? Then, we will going to understand here.

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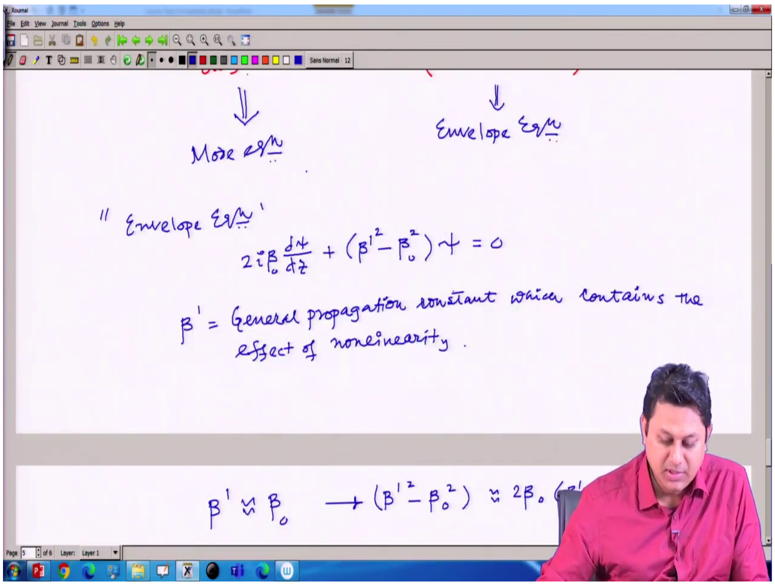

So, the envelope equation, if I ignore this equation, this portion because this is basically gives us a transverse distribution or how the mode is distributed. So, I will not going to I would not going to take account this right now. I only consider this envelope equation.

So, this is the mode equation and this portion is basically the envelope equation. So, the envelope equation if I just concentrate on the envelope equation. This will be simply 2 i beta, then del psi del z, then plus of beta prime square minus beta 0 square and psi equal to 0. Now, let me define what is let me explain what is beta prime.

So, beta prime is essentially, the general propagation constant. General propagation constant, which contain the effect of non-linearity. So, what is the meaning of that? That means, beta prime; so, beta 0 is a initial. So, when I launched the pulse, beta 0 was its propagation constant and beta 0 is a propagation constant, when there is no non-linearity.

Though the pulse is not going to experience any kind of non-linearity and is just a free medium and the pulse is propagating and I just assign a propagation constant beta 0 for that. But here, I introduce beta prime it is equal to beta prime. So, beta prime is a propagation constant that take account also the non, effect of non-linearity.

So that means, when the pulse is propagating inside a medium experiencing a non-linearity. The propagation constant will modify and it should be now beta prime. Well, in general, beta prime is very very. So, there is a small difference between beta prime and beta 0 and if I consider this small difference, then the equation simply becomes i del psi del z plus beta prime minus beta 0 psi equal to 0.

From these, I can write this beta prime square minus beta 0 square as nearly equal to 2 beta 0. And then, beta prime minus beta 0. This approximation, I can made, and this 2 beta 0 will going to. So, here, it I think it is beta 0.

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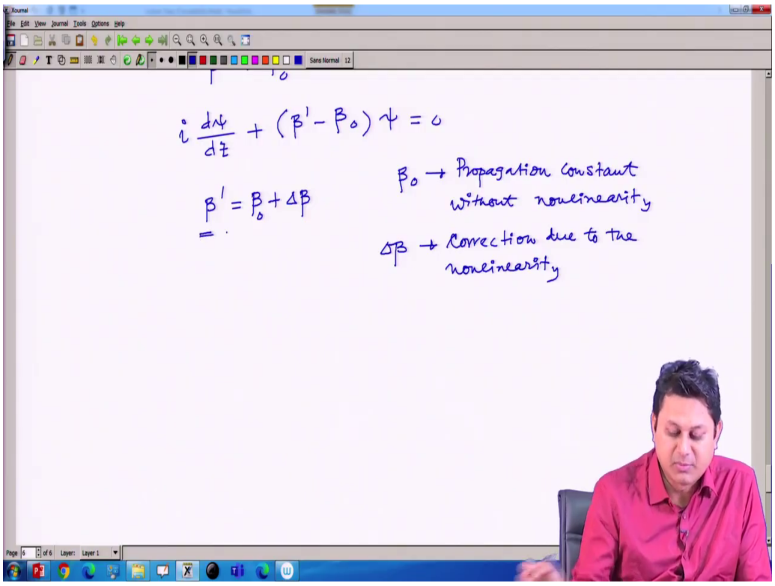

So, this beta 0 will going to cancel out, here from here; so, I have this. Now, beta prime which is a which is a which is a propagation constant under non-linearity is simply beta plus the change of beta 0 plus change of the propagation constant delta beta. So, propagation constant without any non-linear, it is defined by beta 0. So, let me write it clearly.

So, beta 0 is a propagation constant without non-linearity and delta beta is basic basically the correction due to the non-linearity. So, beta 0 is now modified with the correction term so that I can have my actual propagation constant beta prime, when the pulse is propagating in inside a non-linear medium, non-linear waveguide. Then, the equation simply turns out to be.

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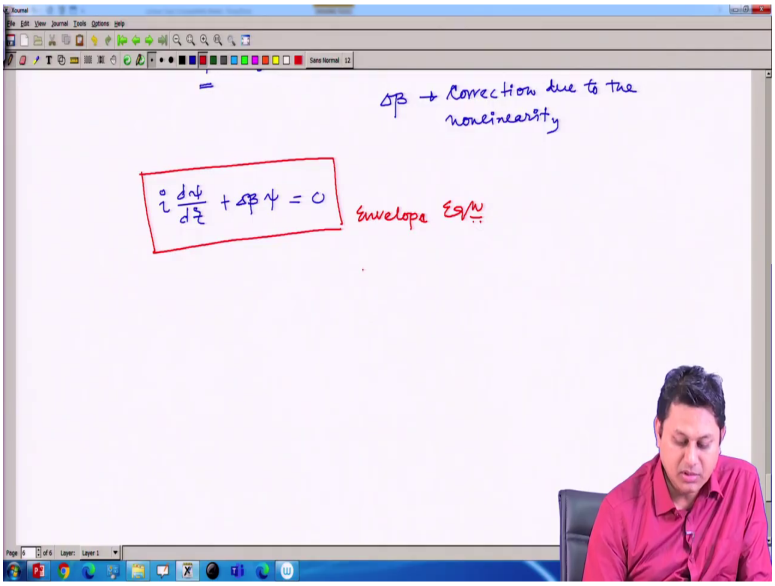

So, that should be the equation. Because beta prime minus beta 0 is simply delta beta. Well, this is in fact, our envelope equation or the pulse propagation equation. This is the envelope equation or the governing equation, that we are looking for that we are looking for.

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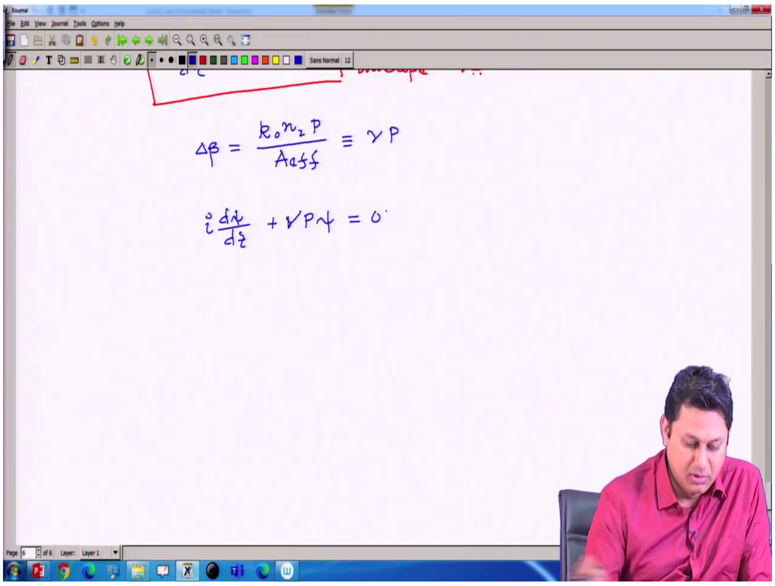

So, now quickly, try to write what is delta beta. Delta beta, we already defined we already figure out the last class, last to last class I guess is P divided by A effective that was our delta beta, which is simply gamma P. So, my equation is further modified and I can write it as gamma P and then, psi which is equal to 0 ok. This is gamma.

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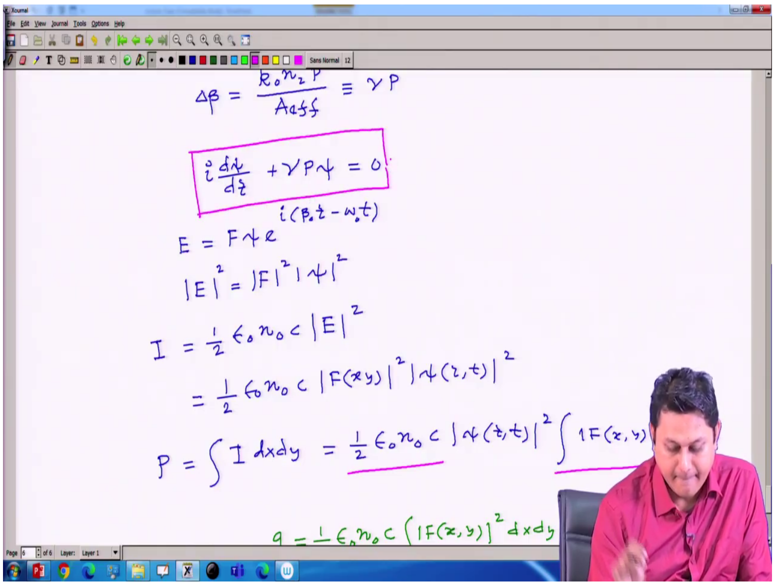

So, let me clearly write it, Gamma. Now, my E the electric field was F psi e to the power of i then beta 0 z minus so omega 0 t. So, mod of E is simply mod of F; mod of E square is equal to mod of F square and then, mod of psi square. Now, the intensity can be related to this amplitude and intensity is simply half epsilon 0 n 0 c mod of E square, that we have already used in earlier classes.

So, this quantity is half epsilon 0 n 0, then c, then mod of F which is a function of x, y by the way and mod of psi which is a function of z and t like this. So, the power if I want to calculate in terms of this, it should be I and the area I can write like this. So, I can have something like half epsilon 0 n 0 c and then, mod of psi z t square and integration of that part; mod of F x, y square d x d y that I have.

Now, you can see these portion, it is integrate over x, y and this. So, this underlined portion, I can write it as a separate constant. So, this separate constant, say I write say some constant say h or g because I already put this notation g earlier. So, let us put this at g. So, g is simply half of epsilon 0 n 0 c and then, integration of F x, y square and d x d y.

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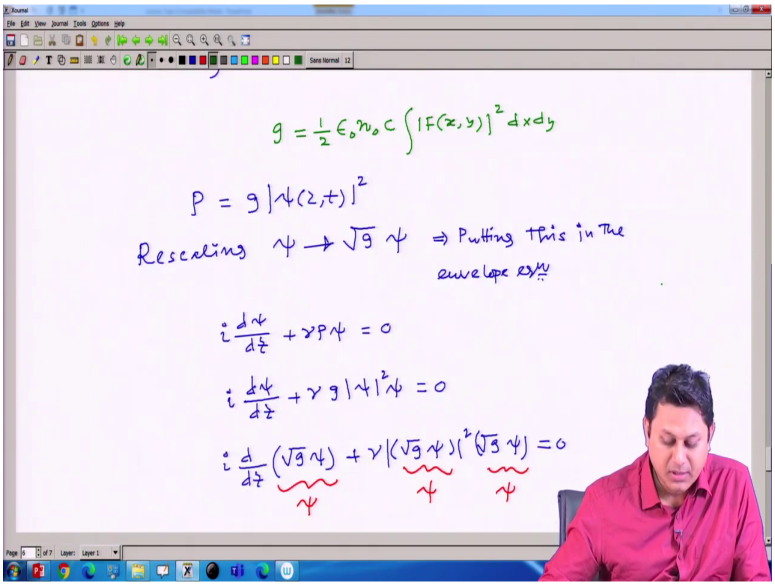

So, my P is now g some constant g and then, psi which is a function of z and t mod square. This is my P. So, I just rescale everything and after doing all this rescaling, I have P is a I mean these things we already used earlier. So, I am now getting the same thing that P is proportional to the envelope square mod of envelope square and this proportionally constant g, can be written in this way.

After that, I now rescale psi to root over g psi, I just rescale whatever the psi I have. So, I have this equation here. Let me highlight this. So, this is the equation and I rescale, if I just rescale this quantity here with psi to root over g psi so that I can have this.

So, putting this; so, putting this in the envelope equation, what was the envelope equation? The envelope equation was i. So, let me write down once again, what was the envelope equation. So, i del psi del psi del z plus gamma P psi equal to 0. P is this quantity. So, let me write down this P.

So, i d psi del z plus gamma and P is something like g mod of psi square psi is equal to 0. So, I multiply root over of g to entire equation so that I can have i del del z one term root over of g psi here plus gamma. I can write this quantity as root over of g and then, psi and mod of square and another root over of g and psi equal to 0.

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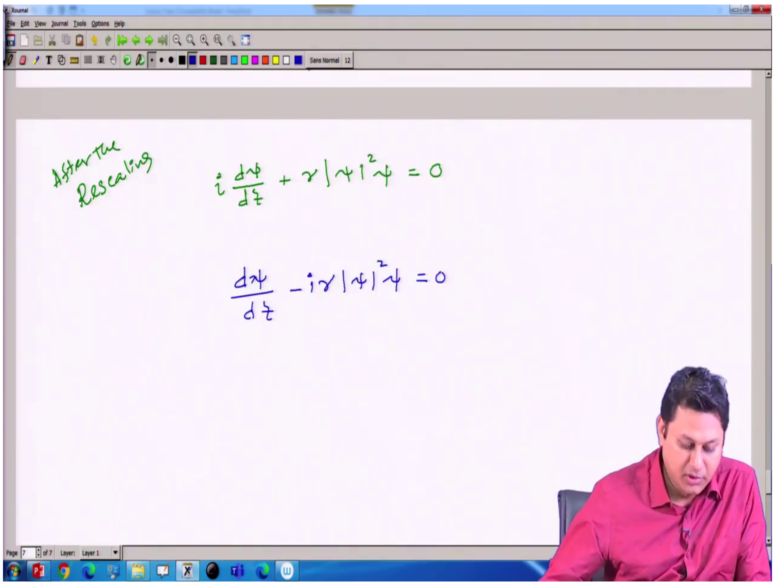

So, I just multiply root over of g both the cases, both the sides and after multiplication of that quantity, now I can have this thing as. These things is now after rescaling, I write these as psi, these as psi and these as psi. After having all this rescaling, I now have; so, after the rescaling, I can have one equation like this; new psi which is a rescaled one plus i gamma.

This quantity is simply mod of psi square and then, psi equal to 0. These things, I rewrite once again in a more convenient, not more convenient rather more well-known form and this is i gamma psi square psi equal to 0 ok. Here, I should have a negative sign.

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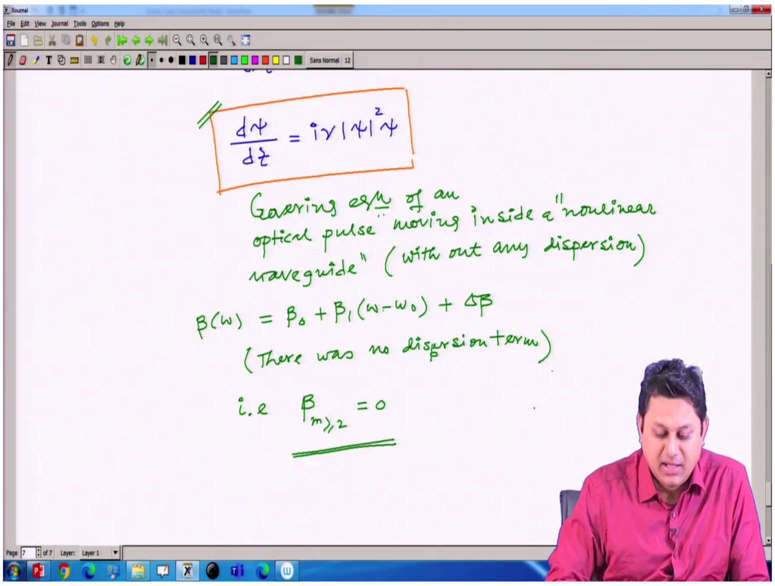

Because I am multiplying minus i. So, minus 0 or d psi d z is equal to i gamma square psi. So, these equations is very very important and this equation basically tells that when there is no dispersion, how the pulse is moving inside a waveguide and what should be the dynamics of this pulse. So, these basically the governing equation of the pulse. So, this is the governing equation of an optical pulse moving inside a waveguide, non-linear waveguide; very important term non-linear waveguide.

So, waveguide has to have a non-linear property and when it is moving, when the pulse is moving, then it should be this should be the governing equation. So, this is a very very important equation and this happens without so without any dispersion. Because if we go back to the equation, the way we derive this equation, if we go back.

Then we will find that we just expand the beta here, we expand the beta up to first order. So, that means, beta is expanded which is a function of omega. If you remember that we expanded is like this in the earlier classes; beta 1 omega minus omega 0 and then, we add this delta beta term.

So, when we expand, there was no dispersion term. Dispersion mean I am talking about group velocity dispersion; this. There was no dispersion term. So, that means, these beta 2, so we assume in this Taylor series expansion, we assume that beta m greater than equal to 2 is 0, that is the condition we assumed here.

And that means, we did not assume any kind of dispersion and if there is no dispersion, only non-linearity is there and the pulse is moving inside a medium, experiencing that non-linearity, but not experiencing any kind of dispersion. Then the equation, the governing equation should be this one; del psi del z is equal to i gamma which is a non-linear coefficient mod of psi square mod of psi square psi. So, I will going to conclude this class here.

So, in the next class, we try to find out the solution of that, this equation and check what happened, when the governing equation is in our hand; when the pulse is propagating in a non-linear wave guide and what will be the effect. Already, we find the effect that if there is a non-linearity when the pulse is moving, there is a change in its phase. So, a similar expression will find, we will get a result from which we can infer that indeed there is a change in the phase of the pulse. With that note, thank you for your attention and see you in the next class.