Physics of Linear and Non Linear Optical Waveguides
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Module - 04
Fiber optics components
Lecture - 43
Reflectivity Calculation

Hello student to the course of Physics of linear and non-linear optical web guides. So, today we have lecture 43 and we started some calculation regarding Fiber Bragg grating. Today, we will be going to continue with that calculation which is eventually the reflectivity calculation.

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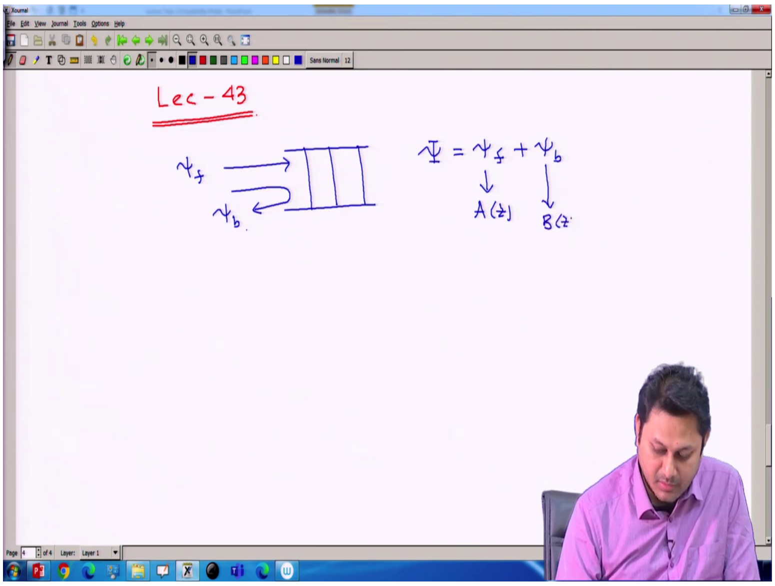

So, let me write down so far whatever the expression we got. So, our aim here is to this was the Bragg grating structure and I had a wave psi forward and then, psi backward; my big psi was psi forward plus psi backward and this psi forward is associated with the amplitude term A z and this backward is associated with another amplitude term B.

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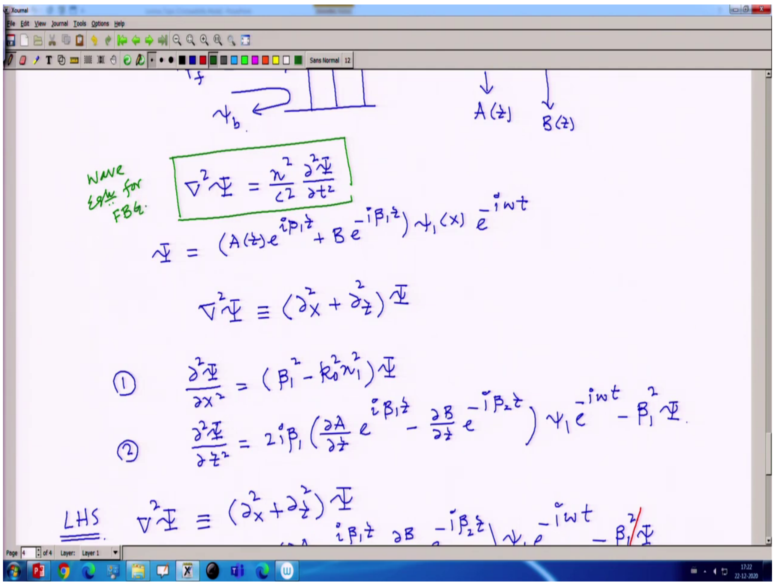

We wanted to find out this wave equation for fiber Bragg grating and psi was in total A function of z e to the power of i beta 1 z plus B e to the power of minus of i beta 1 z psi 1 which is a function of x and then e to the power of minus i omega t, that was the by definition that was the psi.

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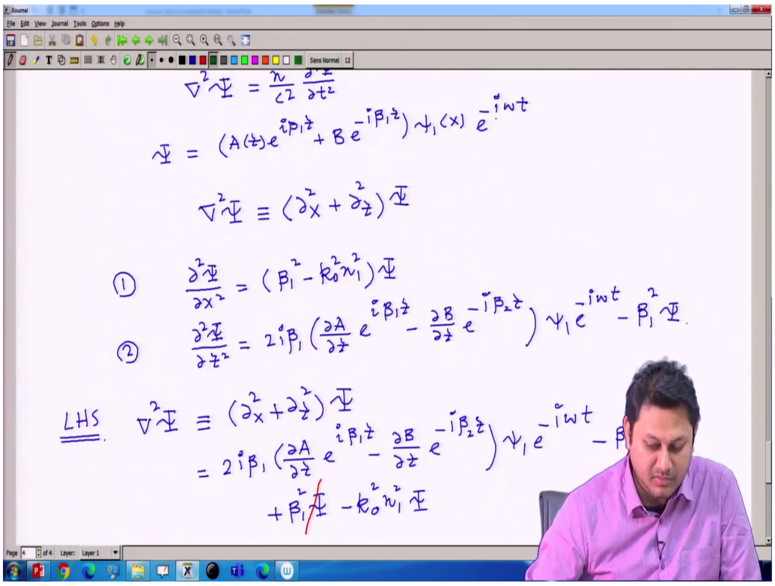

Then, I try to execute these, these term which is sitting in the left hand side, this is equivalent to of psi and we derive equation 1 as this was we derive as beta 1 square minus k 0 square n 1 square big psi that was one equation we derived and the second equation that we derived is this; d z e to the power of i beta 1 z minus del B del z e to the power of minus i beta 2 z, then psi 1 e to the power of minus i omega t and then, minus of beta 1 square big psi that was the 2 equation, we derived last time.

So, now, we execute the left hand side because two terms, the partial derivative of x and partial derivative with respect to z, these are now evaluated. So, we will just put it.

So, so, the left hand side which is equivalent to this is simply 2 i beta 1 just add these two terms; then, del A del z e to the power of i beta 1 z minus del B del z e to the power of minus i beta 2 z. Then, psi 1 e to the power of minus i omega t and then, this term plus beta 1 beta 1 square rather big psi minus k 0 square n 1 square big psi and another beta 1 is here, so I missed that. So, minus of beta 1 square psi I should write here only.

So, these two term will going to cancel out; beta 1 square plus and beta 1 square, these two term will going to cancel out. So, this is roughly the left hand side 2 i beta 1 del A del z e to the power of i beta 1 z minus del B del z e to the power of minus i beta 2 z and rest of the term. So, this is the left hand side.

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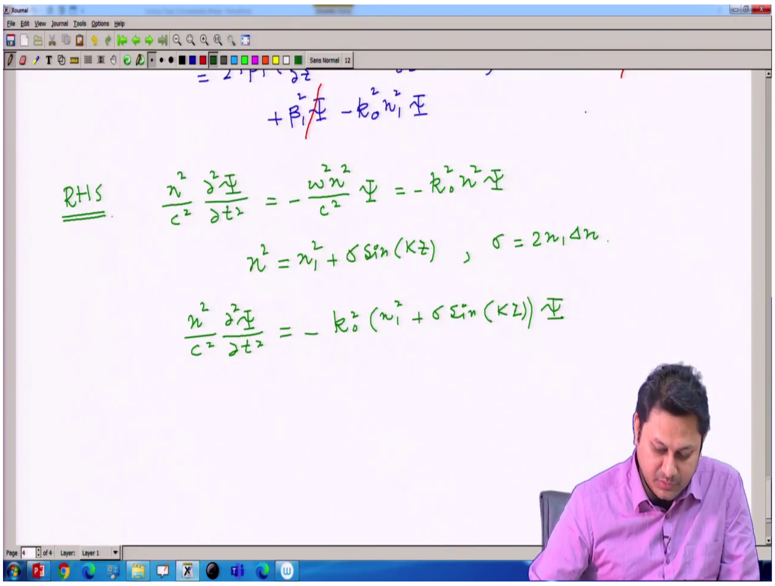

Now, let us concentrate what in the right hand side. So, right hand side; so, in the right hand side, we have n square divided by c square del 2 psi del t square that was the term. Now, in the psi if I look, what is my psi? My psi here this t dependence is here with minus i omega t. So, only minus omega square term will come out and then, we have n square, then I have c square and simply big psi.

Now, omega divided by c square, I can write it as minus of k 0 square and n square psi. Now, we should remember what was the n square we derived for Bragg grating. So, let me write it once again; n square was n 1 square plus sigma sin of K z that was the n square we derived earlier. So, this quantity is simply n square divided by c square d 2 psi d t square, this quantity is simply minus of k 0 square and then, n 1 square plus sigma sin K z bracket close and then, I have big psi.

So, here you should remember, the sigma was 2 n 1 delta n. So, this is the roughly the right hand side. So, I execute the left hand side of the equation and then, right hand side of the equation. So, these two things should be equal because according to the wave equation, xi square; so, this is the wave equation we had. So, now, I equate this left hand side and right hand side; left hand side I know and right hand side is also I know right now.

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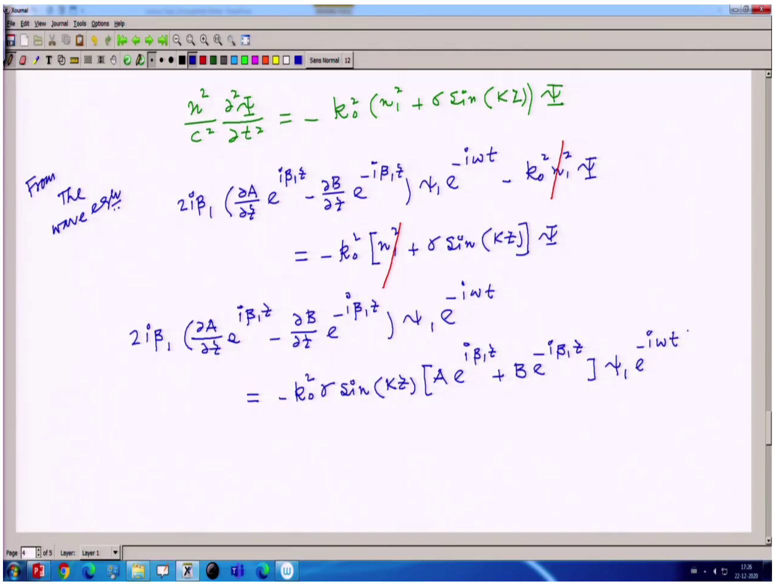

So, I can write as the wave equation from the wave equation. We have 2 i beta 1. So, whatever the term I have here I should write it. So, 2 i beta 1, then del A del z e to the power of i beta 1 z minus del B del z e to the power of minus i beta 1 z, then psi 1 e to the power of minus of i omega t, then minus of k 0 square n 1 square big psi. That is in the left hand side, I just write this one. The right hand side that should be equal to this should be equal to this one, whatever we derived here.

So, minus of k 0 square n 1 square plus sigma sin K z bracket better to put a third bracket and then, big psi. Kindly note that this k 0 n 1 square psi and this k 0 n 1 square psi will going to cancel out. So, this term and this term will going to cancel out. So, eventually, we have eventually we have 2 i beta 1 and then, del A del z e to the power of i beta 1 z minus del B del z e to the power of minus i beta 1 z.

Then, psi 1 e to the power of minus i omega t as usual and then, in the right hand side, I can have minus of k 0 square then sigma, then sin of K z and I have a big psi. So, I just write down the big psi once again with this form. So, it should be A e to the power of i beta 1 z plus B e to the power of minus i beta 1 z bracket close psi 1 and e to the power of minus of i omega t.

Now, again, this quantity psi 1 and psi 1 will going to cancel out here and e to the power i omega t and e to the power i omega t also going to cancel out. So, these two term will going to cancel out and I can write this expression as by dividing, now I what I do? I divide 2 i beta 1 from left hand side and right hand side and also, I divide 2 e to the power i beta 1 z, so that I can free I can have a free term like del A del z.

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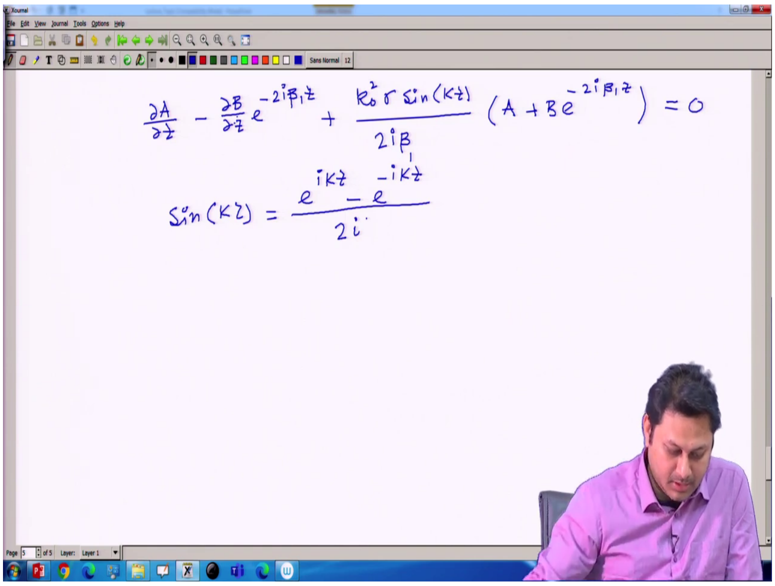

So, if I do, I can have del A del z. I want to free this term minus del B del z e to the e to the power of minus of 2 i beta because I divide everything with e to the power of i beta z. So, e to the power of minus 2 i beta 1 z plus I put everything in the right hand side plus k 0 k 0 square sigma sin K z divided by 2 i beta. Because I divide this term as well. 2 i beta 1 was there, so I divide this 2 i beta 1 again.

And rest of the term which is A because I divide e to the power i beta 1 z here also. So, it should be A plus B e to the power of minus 2 i beta 1 z equal to 0 that is the total expression. Next thing is interesting. I write this sin K z, I can write as e to the power of i big K z minus e to the power of minus i big K z divided by 2 of i.

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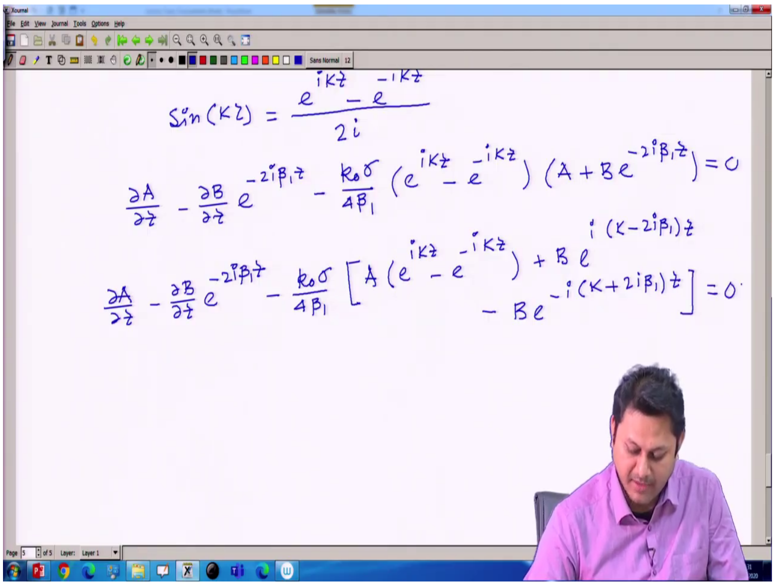

And I will going to put it here so that I can I can have an expression like d A d minus del B del z e to the power of minus 2 i beta 1 z minus K 0 sigma divided by 4 beta 1. And these term, I write as e to the power of i K z minus e to the power of minus i K z this and then, A plus B e to the power of minus of 2 i beta 1 z equal to 0. This is the complete equation now. Next I write del A del z minus del B del z e to the power of minus 2 i beta 1 z minus k 0 sigma divided by 4 beta 1 and I multiply these things there.

So, I will going to have a term like A into e to the power of i K z minus e to the power of minus i K z plus B into e to the power of i K minus 2 i beta 1 z that is one term I am having. And then, I have minus this is I have beta e to the power A i plus and then this term, second term will be minus B e to the power of minus i, then big K plus 2 i beta 1 then z bracket close equal to 0. So, I am having a lot many terms.

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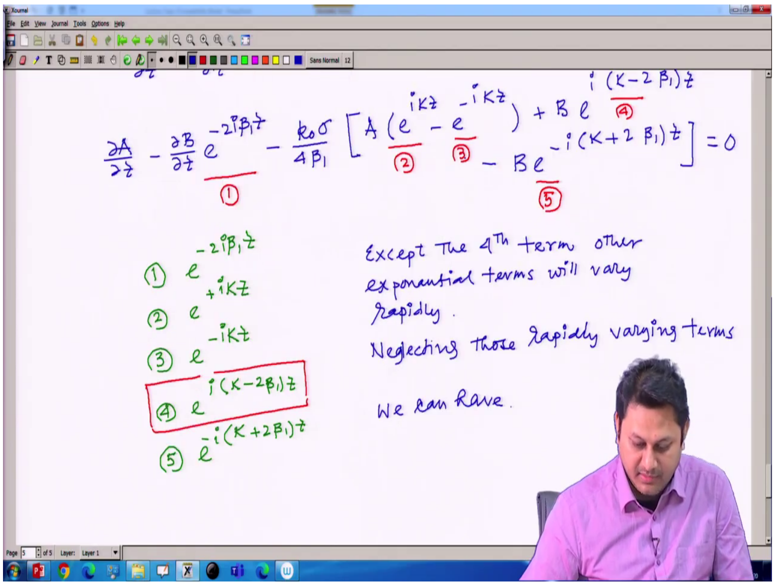

Now, let us try to understand how many exponential terms are there. So, here we have first exponential term; here we have second exponential term; here we have third exponential term; this is having fourth exponential term and finally, I have fifth exponential. So, there are 5 exponential term.

So, let me write down all these 5 exponential terms. So, first term is e to the power of minus 2 i beta 1 z; second term is e to the power of minus i K z; third term is e to the power of sorry this is plus, so minus i K z; fourth term is very important term, e to the power of i K minus 2 sorry here since I take i common, I have a mistake here. So, this I should be here.

So, 2 beta 1 z and finally, I have fifth term with e to the power of i with a negative sign kappa sorry K plus 2 beta 1 z. Now, you can see among these 5 terms, only these fourth term which I need to highlight. These term will vary slowly and other term will vary very rapidly. Why it is slowly varying? Because there is a condition that K can be equal to 2 beta 1. If this is the this condition is valid, then we can have a Bragg condition.

So, among all the five terms, so I can write it here; except the fourth term, other exponential term will vary rapidly. With increasing z, this term will vary very rapidly compared to this one because I have K. So, there is a possibility that kappa is equal to 2 i beta; if that is the case, then this term can be 1. So, there will be no rapid variation on that term.

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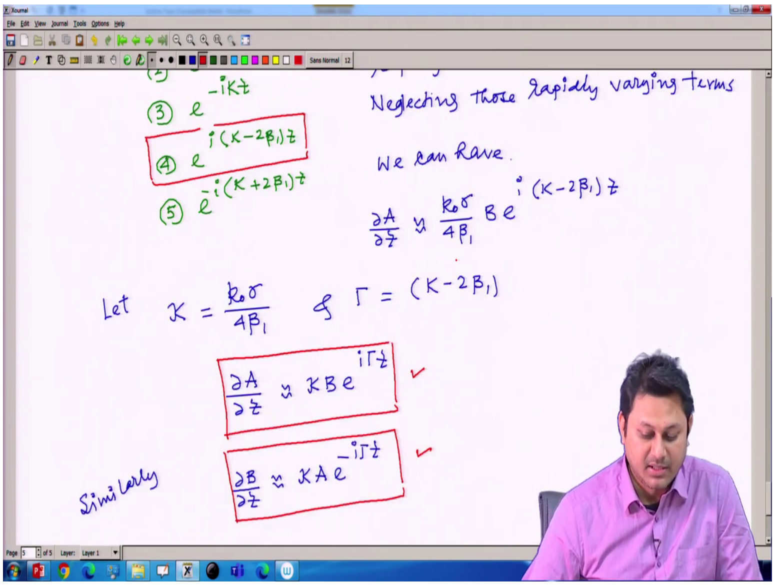

So, neglecting those rapidly varying terms, neglecting those rapidly varying terms we can have. So, you can see we can have a very simple expression and that expression is delta A delta z is nearly equal to k 0 sigma divided by 4 beta 1 then b the phase term i big K minus 2 beta 1 z. So, I am having a simple expression which after doing all these things, which tells me how d A d z is varying that is the aim from the very beginning.

Now, let kappa is equal to k 0 sigma divided by 4 beta 1. I just rename all these things and gamma is equal to this phase term big K minus 2 beta 1, just renamed. Then, I can have an expression like del A del z is nearly equal to kappa B e to the power of i gamma z. So, this is the expression, I am looking for. I am having the expression of the variation of the forward amplitude of the forward propagating wave.

In a similar way, exactly in the similar way, in the similar way, similarly, we can have a term corresponding to B and this will be like that kappa B sorry here it will be A because it is coupled with A now. A e to the power of minus i gamma z that is the another equation, you can have and how you get this equation? If you go back to this equation, the equation we had here, this is the equation I am talking about.

In the next line, what I do? I just divide this equation with respect to e to the power i beta z and 2 i beta 1.

I divide this entire equation both the side with 2 i beta 1 multiplied by e to the power i beta 1 z. I can also do the same thing; but I can divide 2 i beta 1 e to the power i beta 1 z minus i beta 1 z. If I do that, then instead of having del A del z free, I can have another term which is del B del z del z which is free. Then, I can write this equation in a exactly similar way and do all the procedure that we have done here and we can end up with this equation; del B del z is nearly equal to kappa A e to the power i gamma z.

So, whatever the aim in our in this calculation that how to find out the evolution of A z and B z will happen when the when the wave is propagating inside this Bragg grating. We managed to find these two equations that these are the two expression which guide us that how the A and B will going to vary inside the fiber Bragg grating. And these are related to certain phase term, very important phase term which is K minus 2 beta 1.

So, today, I do not have that much of time to expand more what is going on. So, in the next class, we will start from these two equation and then, study the reflectivity. The we complete the calculation of the reflectivity that because in the fiber Bragg grating what happened? I launch a light and a particular wavelength is reflected from these Bragg grating.

So, this Bragg grating behave like a this Bragg grating behaves like a mirror. So, since it is behaves like a mirror, I can calculate the reflectivity of this system and that that is the calculation precisely, we are doing in this course, in this particular in this couple of classes. So, in the next class, we will start from this equation and try to understand how A and B, one can evaluate by solving these two coupled differential equation. So, with that note, I would like to conclude.

Thank you very much for your attention and see you in the next class.