Physics of Linear and Non Linear Optical Waveguides
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Module - 04
Fiber optics components
Lecture - 42
Fiber Bragg Grating (FBG) (Contd.)

Hello student to the course of Physics of linear and non-linear optical waveguide. Today we have lecture 42. And in the last class, we started a very important concept called Fiber Bragg Grating, this is a fiber based component. And today we will going to continue on this topic.

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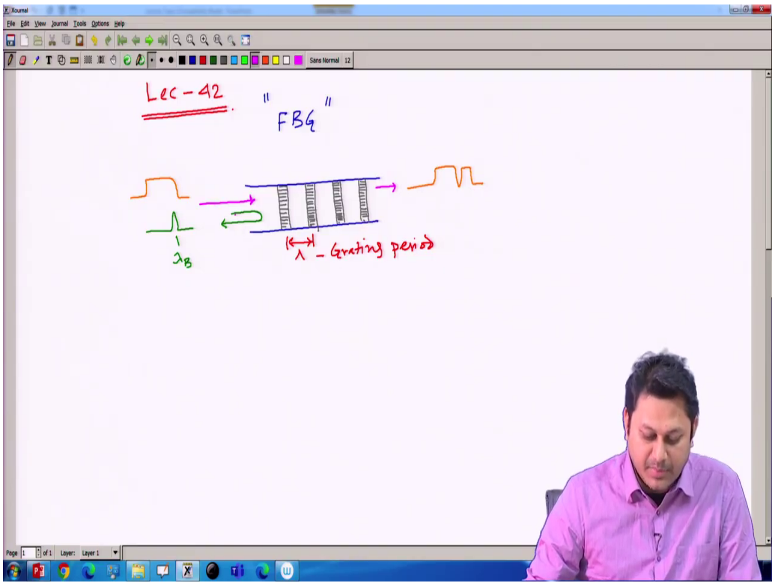

FBG in short Fiber Bragg Grating. So, quickly want to remind what is the principle of fiber Bragg grating that, we have a grating structure inside the fiber core that is the first thing. And how this grating structure formed? We also described in the previous class. So, these are the periodic variation of the refractive index inside the core of an optical fiber.

Let me draw this once again, so that you can understand what was the structure of the fiber Bragg grating. These are the refractive index which is periodically changing. From here to here, we had something called big lambda, which was grating period. And what is the phenomena? The phenomenon is something like that, if launch a light here; then a particular wavelength will going to reflect back.

So, here we had a broad spectra. And what happened, a certain wavelength will going to reflect, which we call Bragg wavelength lambda B. So, if look what is the output spectra; the output spectra will be simply like this, this wavelength will be missing here. So, that is the structure of the fiber Bragg grating and this is the phenomena we are discussing that, if launch a light what happened that; it behave like a mirror for a particular wavelength and that wavelength will be reflected back.

So, we will try to understand using the couple mode theory, what is the, I mean how these things is happening.

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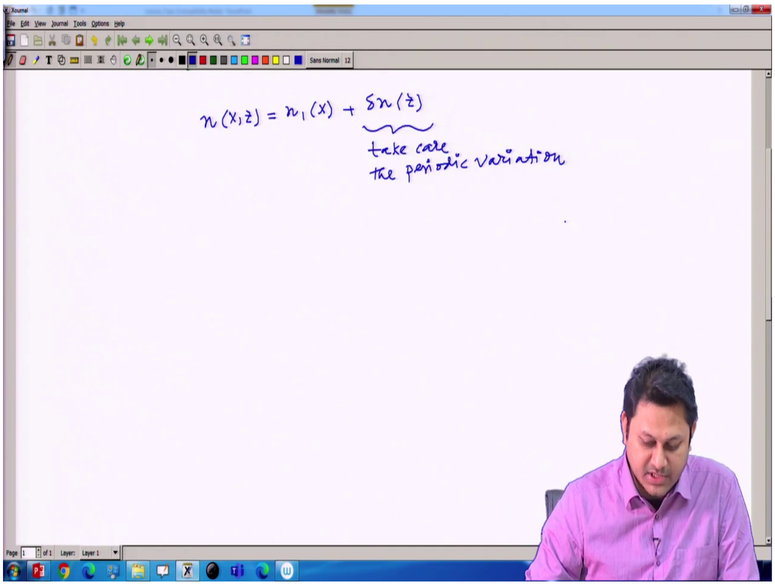

So, refractive index variation in the last class we defined, now it is a function of x and z. So, this is a refractive index of the core, original refractive index of the core without any kind of perturbation and delta n is a variation or this is the perturbation we have. And this is basically the, this basically takes care; this delta n basically take care the periodic variation of the refractive index.

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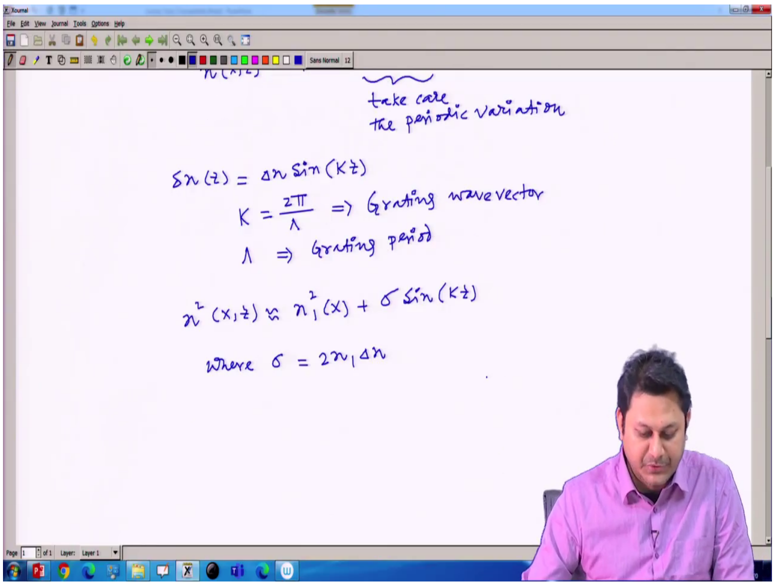

Now, these delta n which is a function of z; since it is taking care of the periodic variation, so I can write it as a periodic variation like that, where K which is equal to 2 pi divided by big lamb the period of the grating is called the grating wavelength, this is called the grading wavelength, we will define that later.

So, this is the value or grating wave vector rather. And so, this is let me write it here only; this is grating wave vector and lambda, big lambda was grating period. And after that we find the value of n square, which is a function of x and z; this n square value will be required in Maxwell's equation, it is like this sigma, sigma is some sort of modulation we have, like that. I approximate this thing, so I should put a approximate sign here.

So, it is approximately that, neglecting the higher order term of delta, where sigma is defined at 2 n 1 delta n, that was the definition of the sigma. Well, now we try to understand using the couple mode theory. So, what is our aim here is this.

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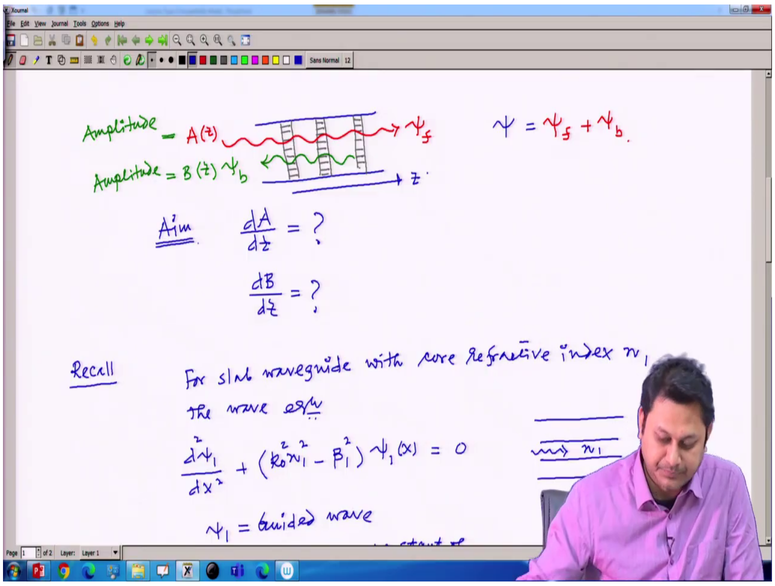

So, let me quickly draw once again the grating structure. So, this is the grating structure we had.

So, one way we have which is moving forward, forward, the wave that is moving forward and another wave that will be reflected back along this direction, which we called psi b. This is the backward wave; one wave that is going forward and another wave will be backwards. The total wave, total field I can define as the combination of these two; forward moving wave plus backward moving wave.

So, we will define in detail. Here our aim is to find out some amplitude will be associated to the forward moving wave say A and this amplitude is a function of z. And some amplitude will be related to the backward wave also, so this amplitude I can define B; these are the amplitude; these are the amplitudes of the forward and backward moving waves.

So, using the couple mode theory, we will try to find out what is the evolution equation of that. So, our aim here is to find out, how this things will evolve over z? So, we want to find out what is this one and what is this one; it is exactly like a couple more theory that we have done in the previous classes.

And here also we try to do the same thing; only things that here the wave is moving, one wave is moving in the forward direction and another wave is in the backward direction.

So, in order to do that, let us recall for slab waveguide with core refractive index n 1; the wave equation was this, where psi is the guided field. So, this is for a slab waveguide, so simple slab waveguide. So, I have a slab waveguide structure like this and here we have refractive index n 1. So, whatever the wave it is moving here, can be modeled with this wave equation, where psi 1 is guided wave or guided field.

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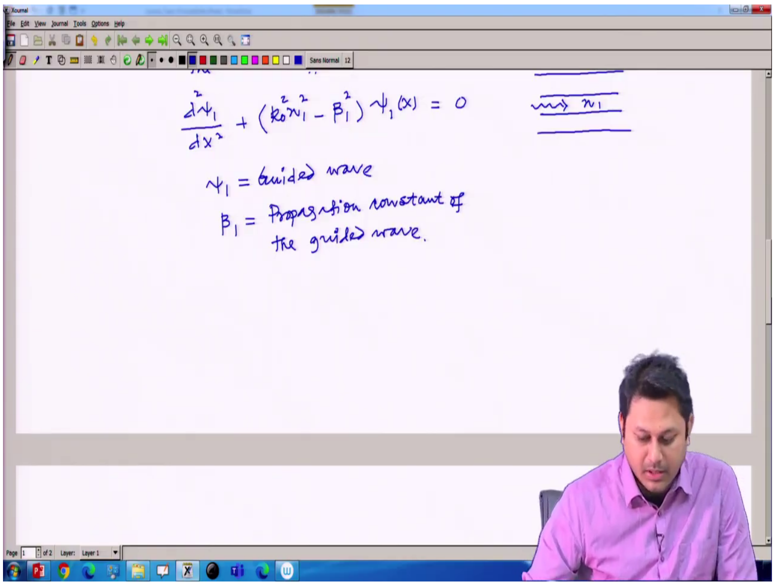

And beta 1 was the propagation constant of the guided wave, ok. Now, for FBG what happened that we have some kind of modulation here in the core region, it is not n 1; but on top of that we have a modulation and this modulation we already wrote here, n 1 square plus some modulation is there and that modulation is due to the fiber Bragg gating, because the grating is now there.

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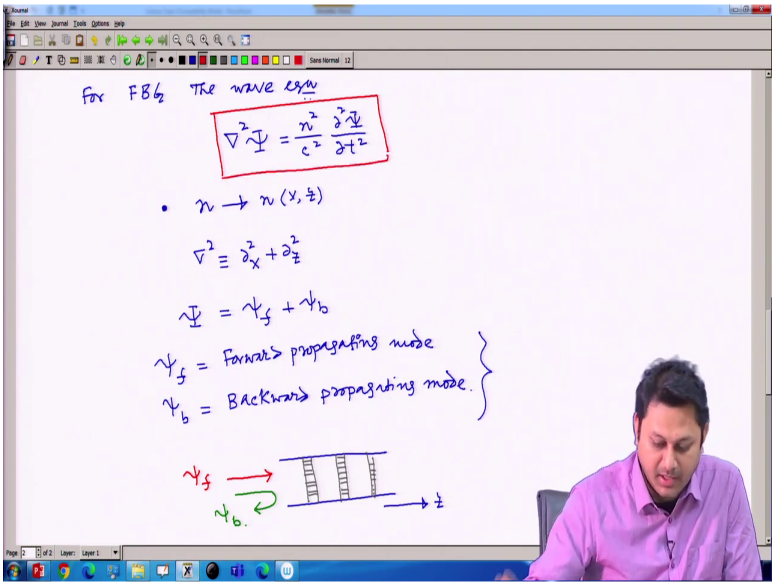

So, for fiber Bragg gating, the wave equation I should write from the beginning; the wave equation is this general wave equation rather is this say, this is big psi is equal to n square divided by c square del 2 big psi del t square. Now, please note that n is a function of x and z, that is something which basically characterize these waveguide; because along this direction; along this direction, we have z.

So, n is now function of x and z. So, this operator I can now write as this. Now, the next thing we need to construct the big psi, already we all already mentioned that this psi can be constructed into two part; one is the forward moving wave and another is a backward moving wave. So, precisely we will do that.

So, big psi can be constructed as forward moving plus backward moving wave. So, psi f is the forward propagating mode and psi b is a backward propagating mode. So, inside the grating this is psi f and the wave that is reflected back is psi b. So, this is the structure we have.

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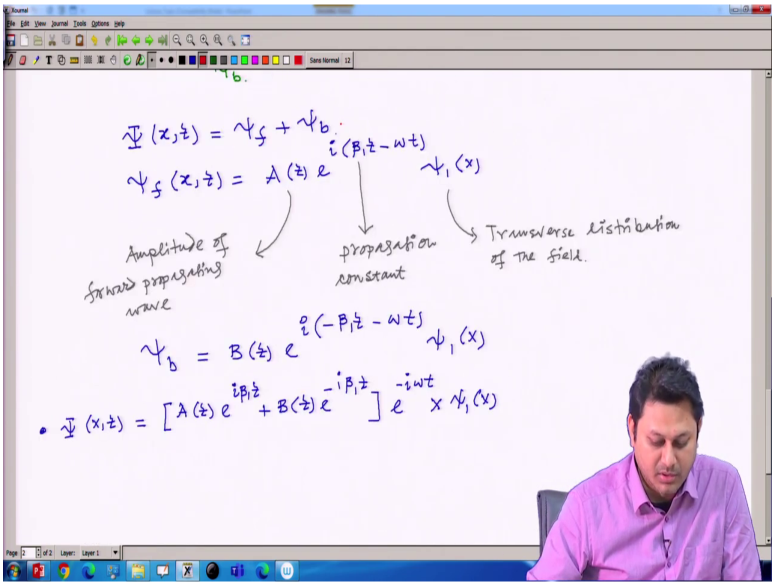

Now, I can construct in detail that, big psi which is a function of x and z can be written as psi f plus psi b already mentioned; this psi f, psi b also function of x and z. So, psi f which is a function of x and z is defined as a amplitude term; we already mentioned that, this amplitude term should be function of z, should be function of z, mind it this is z direction. And then e to the power of i forward propagation constant beta 1 z minus omega t and the transverse field.

So, A is the amplitude of forward propagating. So, this is amplitude of the forward propagating wave, which is a function of z; beta 1 is the propagation constant and psi is a transverse distribution of the field. In the similar way, I can construct also psi b, where psi b is B z e to the power of i minus of beta 1 z minus omega t and psi 1 x.

So, this is the forward propagating wave, that is why beta 1 will be replaced by minus beta and I put a new amplitude as well; because this this amplitude is important in our calculation. So, we want to find out what is the reflectivity at the end of the day. So, then this amplitude, evaluation of this amplitude is important and already I mentioned that this is our m in this calculation, ok.

So, next I can write the entire psi, which is a function of x and z as A z e to the power of i beta 1 z plus B z e to the power of minus i beta 1 z. Then I have e to the power of minus of i omega t multiplied by psi 1 x. Now, I will going to put this into the equation we already defined here; that this is the wave equation. So, we will going to introduce this wave equation now once again and put the value of psi and try to find out what is the evolution.

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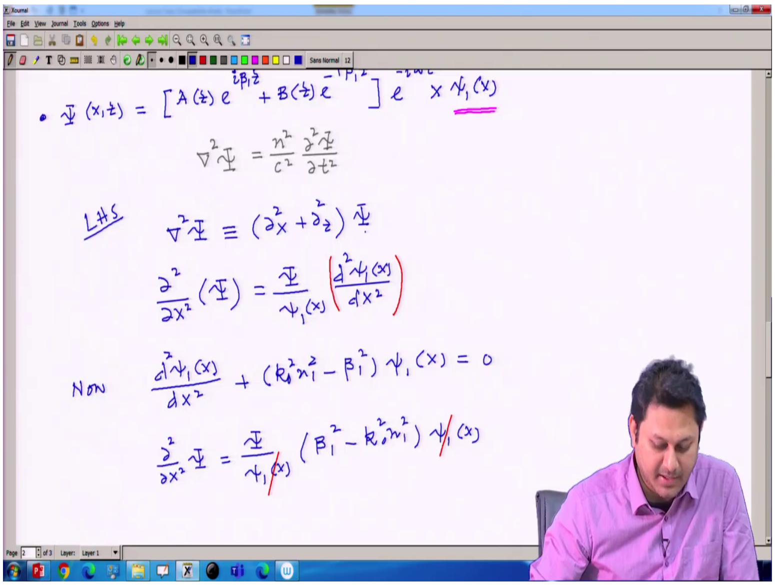

So, my wave equation is grad square psi is equal to n square c square d 2 big psi d t square. Now, one by one we calculate, where grad square big psi; let us calculate the left hand side first, left hand side, which is equivalent to this, d 2 x plus d 2 z, this one. D 2 d x square if I now calculate one by one big psi, so this is the first term.

If I look carefully in this expression of psi, the x is sitting here, the x is sitting here. So, when you make a derivative of this entire quantity with respect to X, that the derivative will be only for this function, the other function will remain constant, because ok this is the partial derivative. So, I need to write the partial one.

So, it should be del 2 del x square. So, this quantity I can write it as big psi divided by psi 1, which is a function of x and the derivative d 2 psi 1 x divided by d x square. Note that when we make a derivative of this quantity, the rest part will be there and if I multiply again psi 1;

then it should be big psi. So, big psi I write and since I multiplied psi 1, I make a I just put this in the denominator, I put another psi 1. So, that this will going to cancel out and we will have the result.

Well this quantity is known, this is for I already wrote this quantity; this is having the equation. Now, we already have this, this we know; because for slab waveguide when there is no perturbation, this equation satisfy and I had mentioned that, that is why it is called recall. So, I mentioned that for transverse field these field psi 1 will going to satisfy this equation, that we already used in several cases previously.

So, I can have, from this I can have del 2 del x square big psi is equal to big psi divided by psi 1 x and this quantity I can replace as beta 1 square minus k 0 square n 1 square multiplied by psi 1 x. I just replace this quantity here, let me put it in the bracket to this one, whatever wrote here beta 2 minus.

Because I have an equation that del 2 psi 1 del x square, that should be equal to minus of k 0 square n 1 square minus beta square; I just absorb this minus sign by writing beta 1 square minus k 0 square n 1 square multiplied by this. So, this quantity, this one and this one will going to cancel out.

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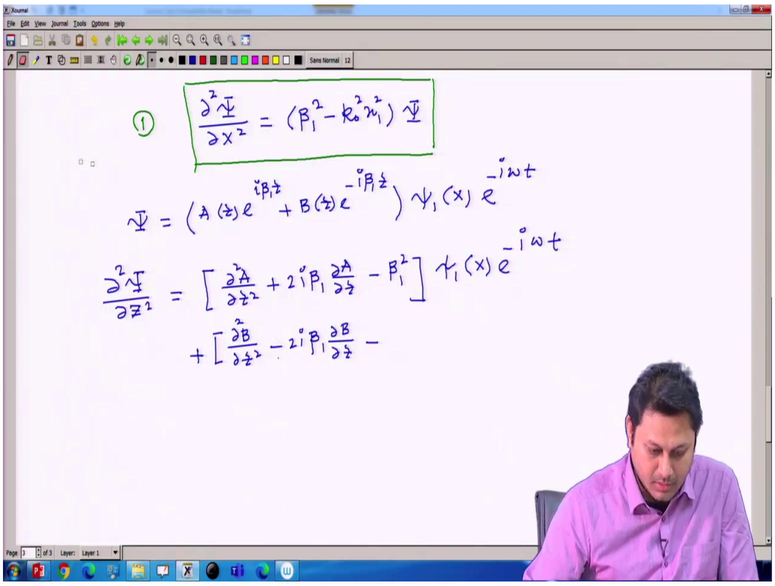

And eventually I have and the derivative as beta 1 square minus k 0 square n 1 square big psi. And say this equation I write as equation number 1. Again need to calculate the next, this is the one part, this is only the one part of the left hand side; I just evaluate del square x psi, next I need to calculate del square z psi. So, let me write down once again what was my big psi. So, big psi was A which is a function of z e to the power of i beta 1 z plus B e to the power of minus i beta 1 z.

Then psi 1 x e to the power of minus of i omega t, that was the total field. And now the next thing we will going to calculate is this quantity, del 2 psi del x; del z square. So, I need to make a derivative of the entire thing, make a double derivative of the entire thing the right, which is in the right hand side, so that we can do.

So, this and this two functions are there. So, I can write it as del square A del z square plus 2 i beta 1 del A del z minus beta 1 square; this is the double derivative of these two functions and we know that when this kind of functions are there, it should be like a square plus 2 a b plus b square kind of thing, where square means double derivative and this is the single derivative and again I have a B square like a double derivative.

So, this multiplied by psi 1 x e to the power of minus i omega t. And next equation, next part I can have plus exactly similar way; I have del 2 B del z square plus 2 i beta 1 del B del z. Here I have a minus sign, because this quantity is minus; so be careful, I should have a minus sign here.

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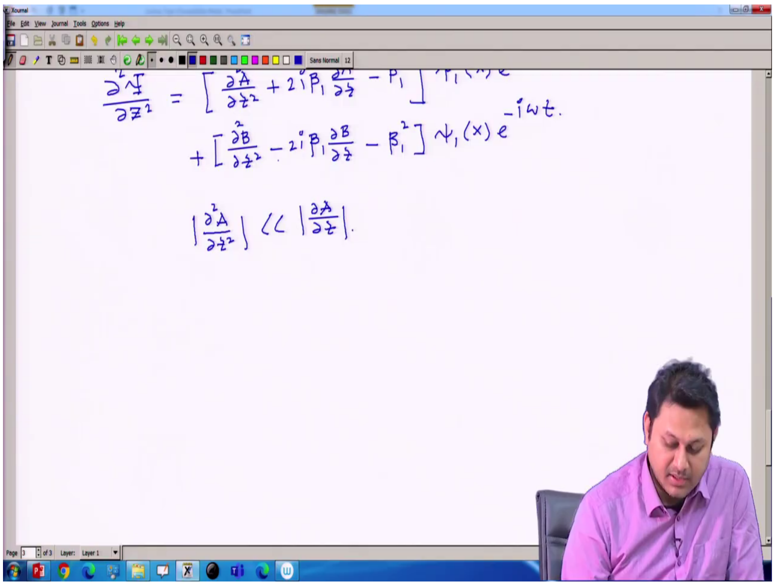

And then minus as usual beta 1 square, rest of the term will remain same.

Now, I will going to apply something called slowly varying approximation that we have been doing. So, second order derivative with respect to z is always much much less than the weightage of the first order derivative. So, I can safely neglect the second order derivative term to make life simple.

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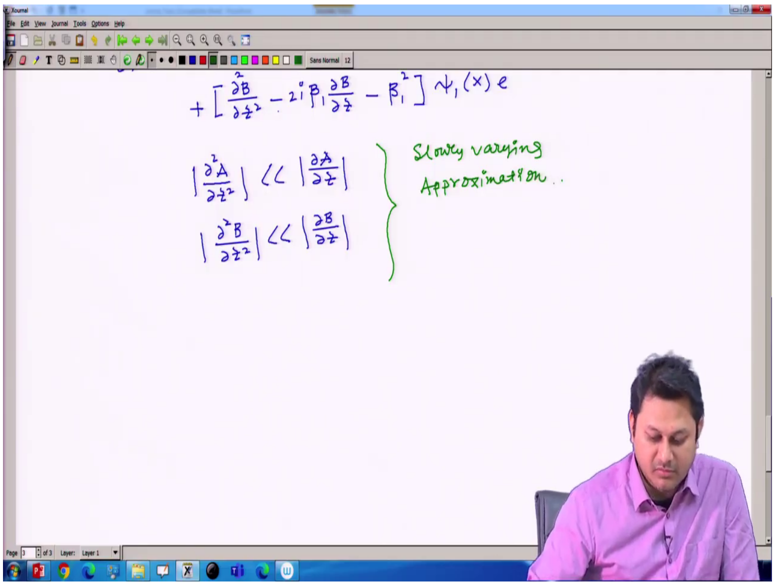

And also del 2 B del z square. So, we considered that the amplitude is varying over z and this variation is a slow variation; that is why the second order derivative with respect to z I can neglect compared to the first order derivative. So, this is called the slowly varying approximation. So, if I make a slowly varying approximation, then what happened; this equation will be little bit simplified.

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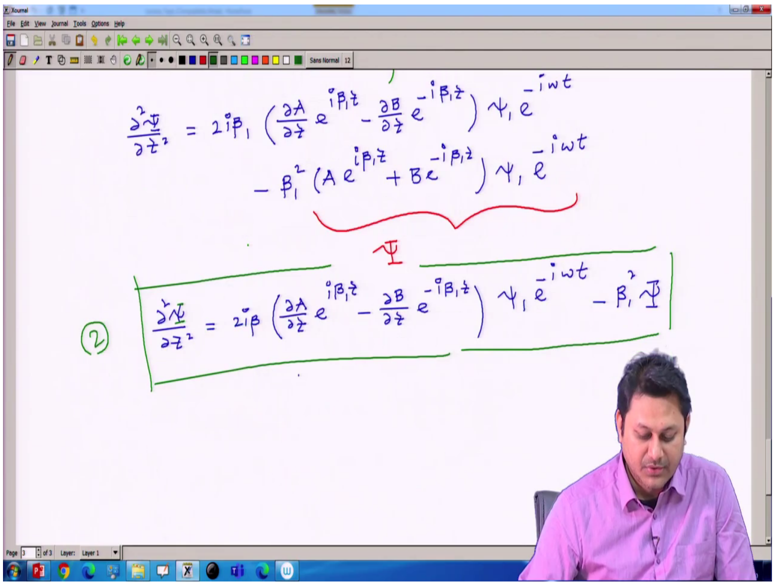

And let me write it as d 2 psi del d z square is simply 2 i beta 1; I take 2 i beta 1 common from these two terms, then del A del z e to the power of i beta 1 z minus del B del z e to the power of minus i beta 1 z. And then I have psi 1 e to the power of minus i omega t. So, that is one part.

Apart from that I have this beta 1 square term. So, I can have minus beta 1 square if I take common; then I have an interesting term here, which is A e to the power of i beta 1 z plus B e to the power of minus i beta 1 z bracket closed, then psi 1 e to the power of minus i omega t. So, this quantity is nothing, but our big psi; by the way in the left hand side I should write it is a big psi. So, let us put this properly.

And now if it is a big psi, I should rewrite once again the expression and this expression is this. I am writing the same thing once again, that was the first term. And so, let me write in single line and then I have minus of beta 1 square and big psi. So, I have another equation, let me block this two equation.

So, I write this equation 2. So, so far we have calculated two the left hand side and right hand side, left hand side of these two part; one is this one the partial derivative with respect to x square, and another is the partial derivative with respect to z square. So, that we already evaluated. So, today I do not have much time to conclude to complete these calculations.

So, in the next class, we will start from here and try to find out what is the evolution of del A del z and del B del z. By that time you already note that this del A del z and del B del z term is already there, already appearing. And if I go back and you can see that what was our aim and our aim is to find out this d A d z and del B del z that was our aim. So, we will do that in the next class. So, with that note I like to conclude today. So, see you in the next class and thank you for your attention.