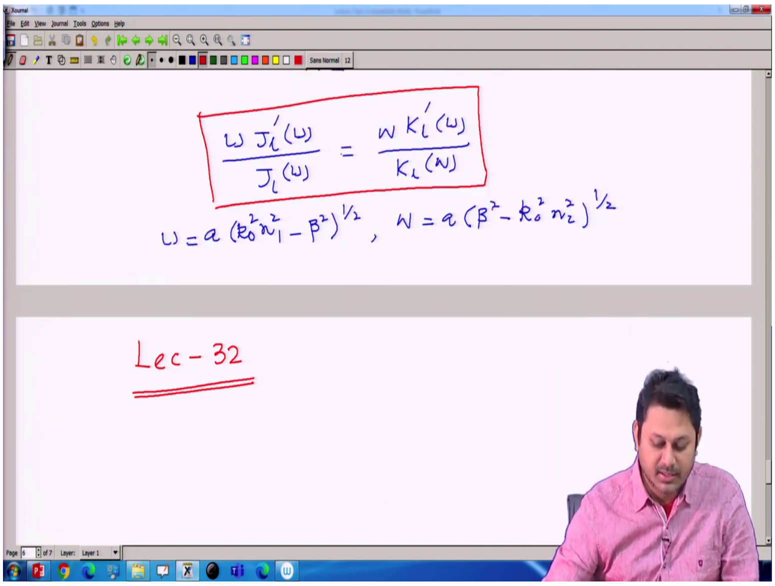
Physics of Linear and Non Linear Optical Waveguides
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Module - 03
Modes (Cont.)
Lecture - 32
Modes in an Optical Fiber (Contd.)

Hello student to the Physics of Linear and Nonlinear Optical Waveguide course. Today, we have lecture number-32. And we will going to continue the Modes in Optical Fiber. In the last class also we done few things. And today we will do further calculation to find out the propagation constants etcetera.

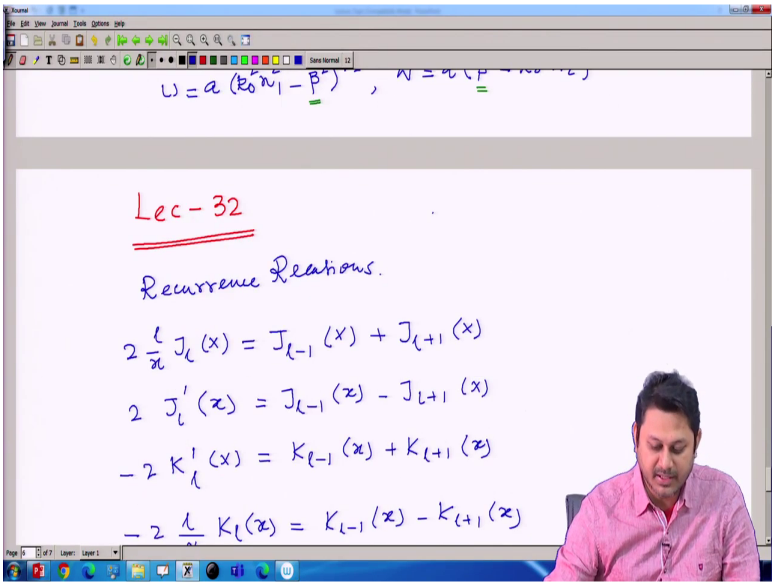
(Refer Slide Time: 00:36)



Well, in the last class if you remember, we had a transcendental equation like this after putting a second boundary condition. And then we mention that if I able to solve this equation, then I can find out what is the value of this beta which is important which is basically the propagation constant of a given mode.

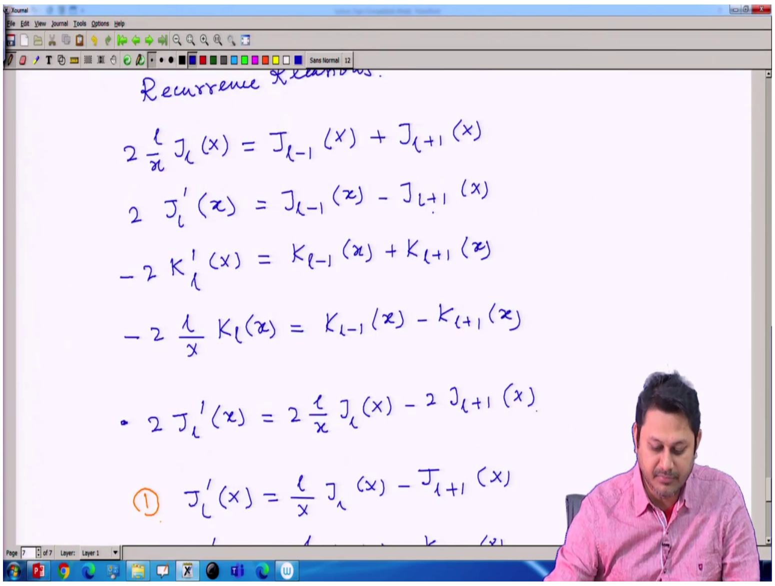
So, in order to solve this transcendental equation, we need to simplify a bit and that we can do with using certain relations using the Bessel functions, certain identities in the Bessel functions.

(Refer Slide Time: 01:20)



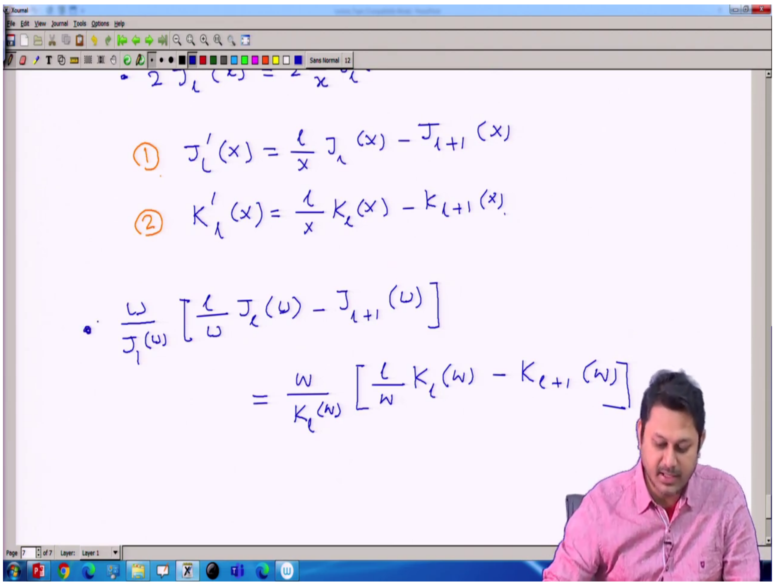
So, these are called the recurrence relations. And few recurrence relations I can show here using which you can simplify that. One is say 2 l by x J l X is equal to J l minus 1 X plus J l plus 1 X. This is one identity. Again I can have another identity.

(Refer Slide Time: 02:30)



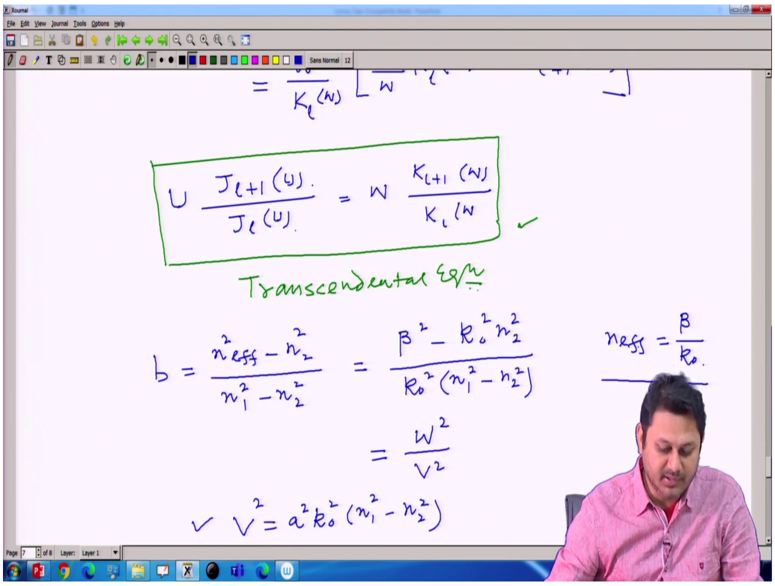
Also for modified Bessel functions, I have the identity like this. From these identities, whatever the identity is written, so I can write it as this. If I simply from this, this one, and if I use 1 and 2, then I can have this one. So, my J l x is simply l x J l x minus J l plus 1 x. This is say equation 1 I have. In a similar way, I can have another equation. Please try it out.

(Refer Slide Time: 04:29)



And you will find that another equation one can also write in this way.. So, from this 1 and 2, I can have, using this 1 and 2, I can write it as this, replacing X to U, because my U is my variable here that should be equal to this one. Why I am writing this? Because this equation already I derived in the last class – this one. I just replace this prime which is a derivative using equation 1 and 2, and put that, and I am getting this.

(Refer Slide Time: 06:10)



Once I am getting this, I can simplify I can have something like this, U U l J l J l this term will be 1 – the first term. Here also we have the first term one, so that will going to cancel out. So, I will going to have something like this. This is my new modified transcendental equation which so this is a tran scen dental equation.

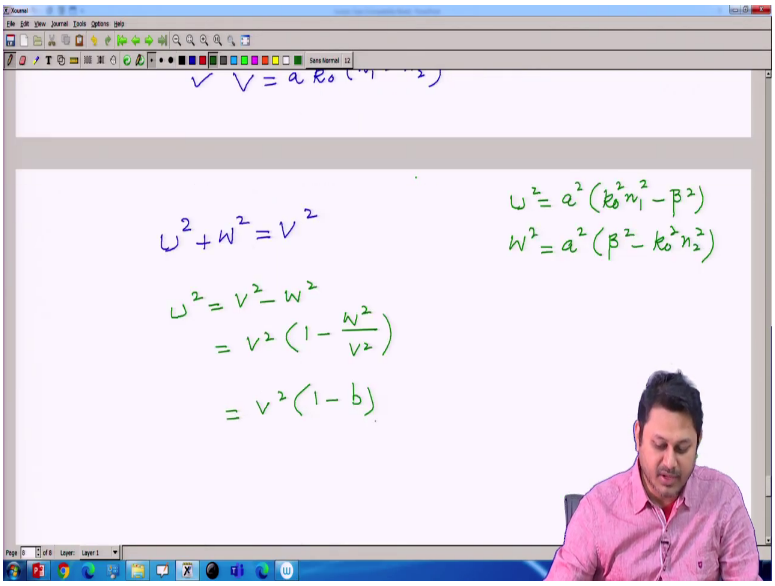
And I modified this accordingly; so that I can have in the previous equation I have the prime which is the derivative of the Bessel functions. So, I just remove this derivative by using the recursion relation that is all. So, you please check this recursion relation and try to do this exercise, and you will find a equation something like this.

Now, like the previous case that we have done for planar wave guides, we will going to introduce this normalized parameter, normalize propagation constant b which you remember is defined is this way.

Mind it, the n effective is beta divided by K 0. So, this quantity beta square minus this might b is simply W square because the this part is W square divided by V square. What is V by the way? The V is a V parameter. And V square I can write it as a square K 0 square n 1 square minus n 2 square that is the value of V.

So, if I now divide W omega W square divided by V square, I will going to get this one. So, I just write my b in terms of terms of W and V, so that I can use this V this b parameter in this equation. And what is U? Because in this equation you can see that the W and U is sitting. So, I will going to replace this in terms of b.

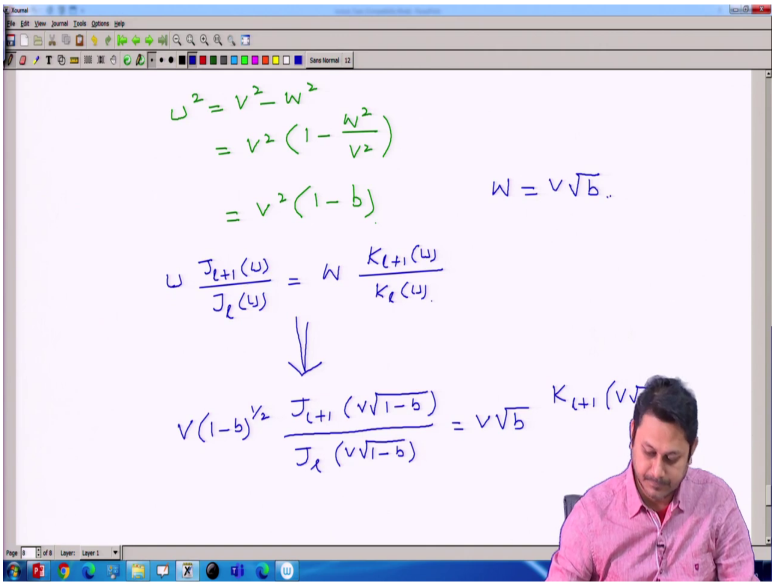
(Refer Slide Time: 09:51)



So, one can find that U square plus W square is V square. Let me write it here. What was the U square once again? So, U square was a square K 0 square n 1 square minus beta square that was my U square. And my W square was a square beta square minus K 0 square n 2 square. So, now, if I add these two things, you will going to find this V square.

So, my U square is V square minus W square. And if I take V square common it should be 1 minus W square divided by V square which is V square into 1 minus b, because b already I calculated here as W square divided by V square. So, now I have everything in terms of V and b. And I know what is V; and b inside the b we have the value of beta.

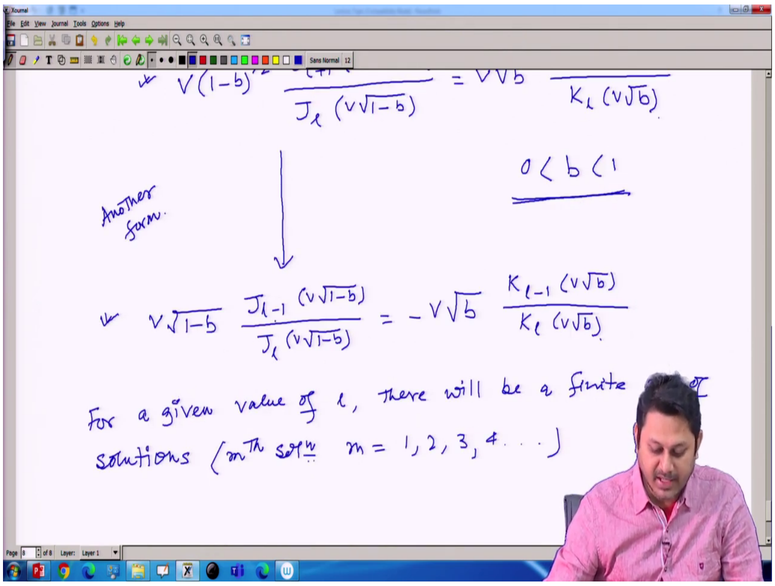
(Refer Slide Time: 11:18)



So, I can write down my transcendental equation once again. So, my transcendental equation was U is equal to sorry my transcendental equation was U J l plus 1 U divided by J l U is equal to W K l plus 1 U divided by K l U that was the transcendental equation.

And now I replace this U and W to b, and then U I replace as V 1 minus b whole to the power half J l plus 1, then I write V in the argument it is 1 by V, then J l again I have V minus b equal to V root over of b then K l plus 1 in the argument I should write V root over of b. Because my W if you so U is this quantity and my W from here, it is V root over of b and K l V root over of b.

(Refer Slide Time: 13:18)



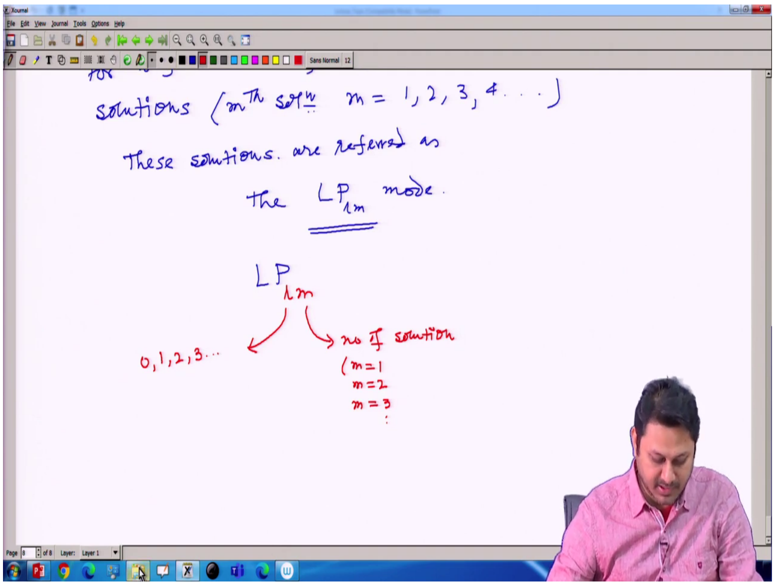
Please note that my V my b is restricted with this limit that is the most important thing that is why we define the normalized propagation constant b. It is restricted to minus 1 to 0, so that will make our life easier.

By the way, one can also have another form I should write this another form because in some books this form is also been used. A similar looking form only thing is that instead of having J plus 1 they have J minus 1. Using the recursion relation also it is possible to find out that this transcendental equation is forming in this way. One can derive this equation using the transcendent using the recursion relation that was given earlier ok.

So, after having this two equation what basically we try to do? So, for a given value of l, there will be a finite number of solutions. And this solutions we write as mth solutions m th solutions such as m is equal to 1, 2, 3, 4 and so on. So, in this equation which is given here either this one or this one, what I do that for a given value of l, that means, I put a value of l, and then I plot the left hand side, and I also plot the right hand side.

And this left hand side and right hand side these two curves should cut at certain points, not only in single point there is a possibility that this they should cut in different points. So, this individual points for a given l is a single solution, and we called as a mth solution if m 1, 2, 3, 4, I defined the solutions as 1, 2, 3, 4 and so on.

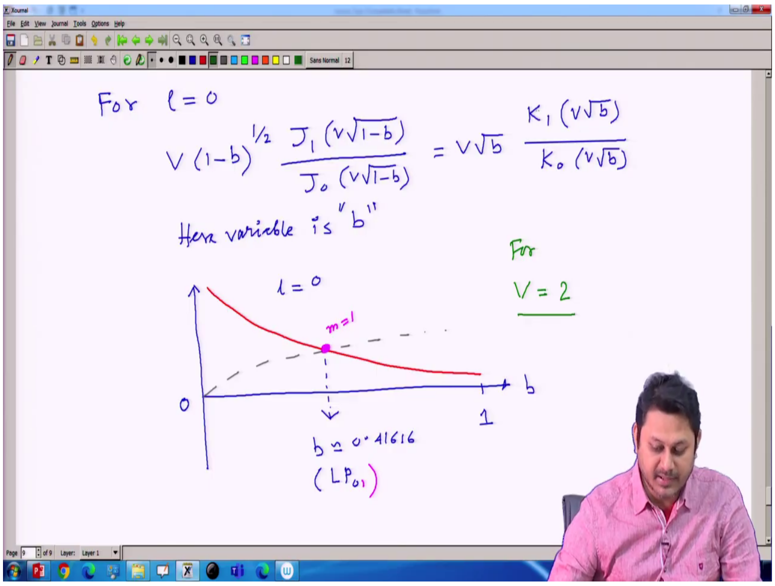
(Refer Slide Time: 16:33)



And each these solutions these solutions are referred as the very important term L P l m modes. L P transfer linearly polarized. And this l m gives the idea that which mode I am talking about. So, L P this subscript is important l m. So, l we know that it can be 1 0 1, 2, 3, etcetera, and it is a number of solutions.

If I am dealing with first solution, then it should be m equal to 1; if I have a second solution, if this will be 2; if I have a another solution, it will be 3 and so on. So, we will going to see that how this basically looks like. So, let us do one example.

(Refer Slide Time: 18:12)



So, for suppose for l equal to 0, what happened? I have the transcendental equation V 1 minus b whole to the power half J 1 V 1 minus b divided by J 0 V 1 minus b. This is my left hand side of the transcendental equation. And the right hand side, I have V root over of b K 1 V root b divided by K 0 V root b.

Now, what I do? I will change V is a variable. So, the variable is here, variable is b. So, b will going to change every time. And I will try to find out what is the solution. If I now plot these two solutions, left hand side will go like this. By the way this is b. So, it should be restricted to 0 and say 1. This is 1 and this is 0. The right hand side will show something like this.

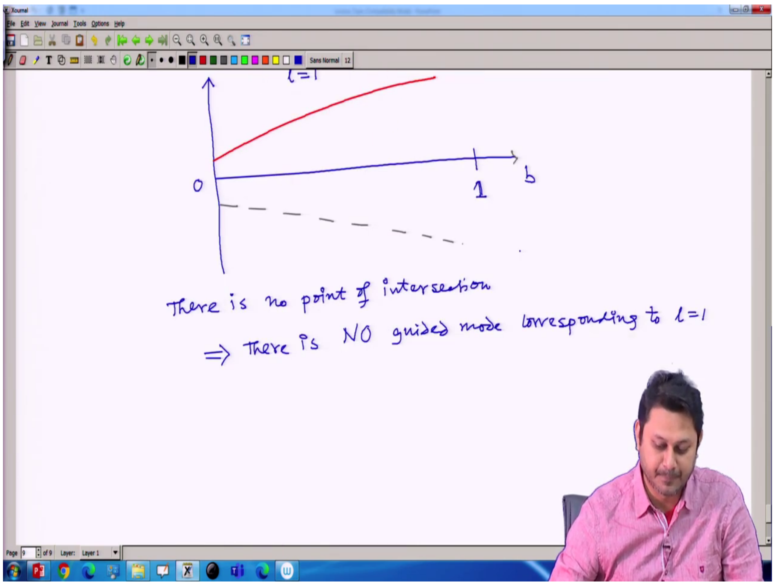
And I will have a cutting point here some, so this is for l equal to 0. And I am having a value solution b which is nearly equal to 0.41616. This is l equal to 0. So, I should have L P 0. And this is the first solution. So, this is the m th solution. So, m is 1 here. So, this is m equal to 1. So, this is L P 1 solution. And for which value I need to put the value of the b as well, because b is a parameter that need to be defined. So, this is for V equal to 2.

So, for V equal to 2, so what we do? I put V equal to 2 in this equation. And then I vary my b to 0 to 1. And when I vary my value b 0 to 1, I can plot the left hand side but everything is known this is a Bessel function of first kind with one. And this is Bessel function 0 0 J 1. So, you can computer you can use the computer to plot this portion, and also in the right hand portion for a given value of V which is 2.

When you do, you will find that these two solutions is like that these two functions is changing with respect to b like that, and there is a cutting point here somewhere. So, this is the only cutting point where single cutting point. And this cutting point is around b nearly equal to say 0.41616.

And this is a cutting point for l equal to 0, and m equal to 1. Why m equal to 1? Because we are having the one cutting only one cutting point, so that is why it defined as m equal to 1. If I have another cutting point, we should write m equal to 2 and so on. And this is basically corresponds to L P 1, L P 0.

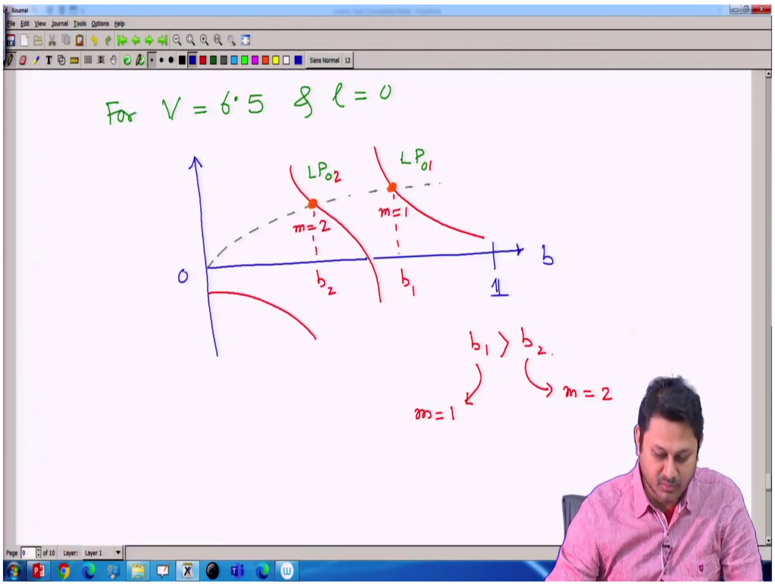
(Refer Slide Time: 22:30)



Now, for this same V value if I plot for same V value, if I plot this equation for l equal to 1, initially it is for l equal to 0, now I am plotting for l equal to 1. I am going to have two function like this. This side I am plotting b which is 0 which is restricted up to 1, and this is 1. And when I plot, there is no cutting point at all.

So, that means, so the conclusion is no point of intersection. So, that means, there is no guided mode corresponding to l equal to 1. So, for l equal to 1, there is no guided mode because there is no solution at all for beta. I can do also for higher value of V, then it is interesting.

(Refer Slide Time: 24:19)



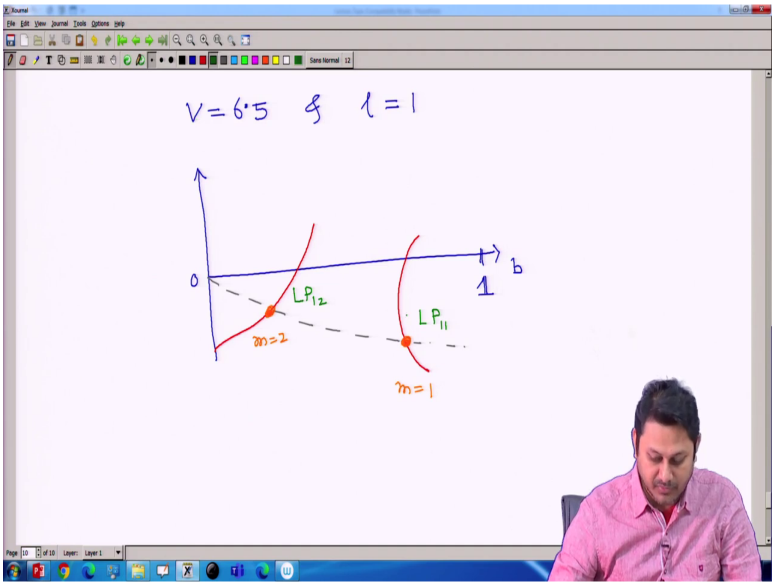
So, let us put say for V equal to 6.5 and try to find out l equal to 0, that means so l equal to 0 and, so this is b, so 0 to this point is 1, this is b. And if I plot this transcendental equation, I will have the right hand side as curve as this one, but something like this for sorry the dotted line is seems to be the left hand side, right hand side. And the left hand side, sorry, left hand side correct, and seems to be something like this these are the.

So, here interestingly one can find that I am having two solutions – here one and here another. So, this solution I can write it is for m equal to 1 and this is for m equal to 2. And this is for l equal to 0, I am plotting that. So, that means, this is for L P 0,1 and this solution can term as L P 0 2. These are the two solutions I sorry this ok. I, I have a mistake here ok.

You have to be careful. So, I cannot put m equal to 1 here and m equal to t 2 here, this is wrong. Because the first solution, so this is m equal to, so very important thing, this is m equal to 1 and this is m equal to 2. This is L P 0 1 and L P 0 2. Why because for this value I have the highest P. So, this is say let me write this is the solution beta 1 and for this the solution is beta 2 which corresponds to b 1 and b 2.

So, b 1 is greater than b 2. So, b 1 corresponds to m equal to one solution the first solution, and b 2 corresponds to m equal to the second solution. The highest value of b correspond always corresponds to the first solution which is related to the fundamental mode. Well, I can extend this I can extend these for l equal to 1.

(Refer Slide Time: 27:19)



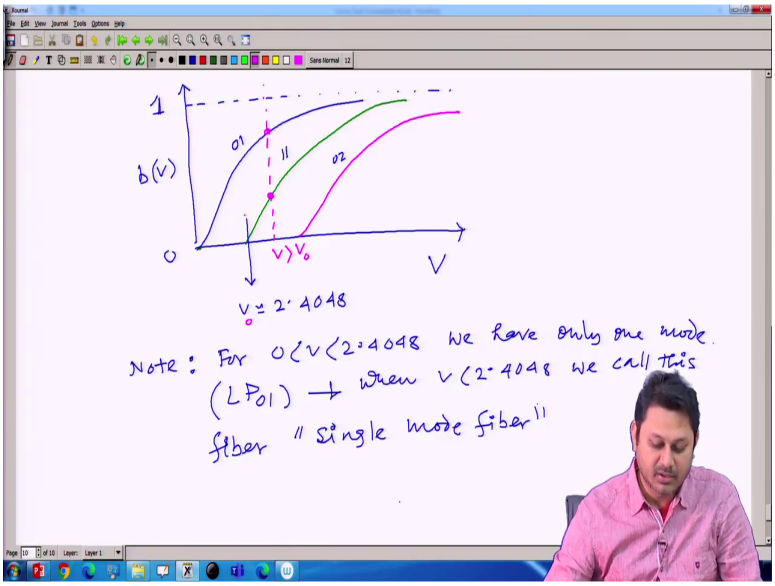
If I do, so say V equal to 6.5, and you can see the value of V is much greater than 2.40 this cut off. So, we are going to expect more than one mode to be supported by the fiber and l equal to 1.

And eventually I am getting this when the value of V is higher, I am expecting more and more modes and already here we find two solutions. And now if I go to higher value of l, then I can excite more higher order modes. This is so I can have the right hand side like this is changing like this.

This is 0 up to this it is say 1, this is along this direction I am plotting b. And I have two solutions like here and here. And this two solutions corresponds to again this is for higher value. So, this is my m equal to 1 solution, and this is equal to m equal to this is second solution.

This solution corresponds to L P 1 1 mode; this solution corresponds to L P 1 2 mode. After having all this, so for if I change my value of b, I will going to get the cutting points for a given value of V.

(Refer Slide Time: 29:21)



So, I can have the value of b. So, I can plot that like we have done in the previous case, we can plot this. This is again a well-known plot. So, 0, this is 1. Along this direction, I plot b as a function of V. And I am going to have this kind of curves. So, along this direction, I am plotting V, increasing V.

The first case I am get I am gathering all the solutions for say 0 1. So, l equal to 0 and m equal to 1, this solutions I will take for all the values for different Vs. In the second case, I can have the solution like this 1 1; third case suppose I am having the solution like 0, 2 and so on.

So, I should have different this kind of curves. It basically gives the solutions different solutions of different values of l and m. In the first branch, I am having for l equal to 0 and m equal to 1 solution for all different values of V.

This value is around 2.4048 if I calculate for step index fiber. So, this is a very meaningful thing note for V in between this values we have only one mode, and this mode is L P 0 1 mode. So, when V value is less than 2.4048, we call this fiber a special name called single mode fiber. Why it is that? Because if I look carefully that if the value of V is less than 2.4048, there is only one mode present. And this mode is L P 01, no other modes is there.

However, if I increase the value of V, you can readily find that if I have some point here which is greater than that value I may have this solution here one and here also. So, not only LP 0 1 I can have a I can have a solution corresponds to L P 1 1 also. So, I have instead of having one mode, I am having two mode because the value of V here is greater than these value. If I write V 0, so it is greater than V 0.

So, V 0 is a cut off value for which I am only having one fundamental mode. This is called the fundamental mode. And if I increase the value of V, I am basically getting more and more number of mode. When I am having only one mode for a fiber, then this fiber is called the single mode fiber which is a very, very important component special for the communication. Normally we like to have a fiber which can only support one mode.

With this note I like to conclude my class here. In the next class, we will learn more about mode and how the mode is distributed over the fiber we try to understand with proper mathematical descriptions.

Thank you for your attention. See you in the next class.