Solid State Physics
Prof. Amal Kumar Das
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture - 70
Magnetic Property of Solids (Contd.)

So, we have seen this susceptibility or paramagnetic material.

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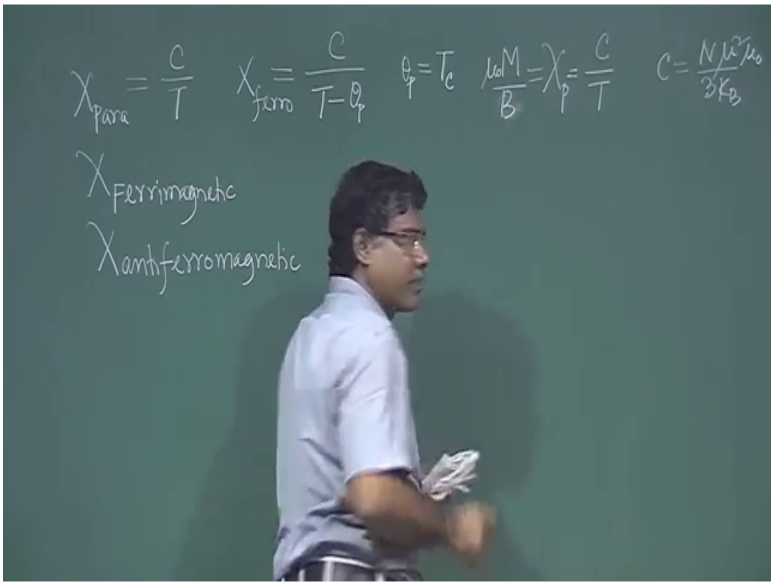

It is C by T; C is Curie temperature and C basically, C equal to N mu square by 3 KB, then here mu 0. For chi; for ferromagnetic that is C by T minus theta, so theta is base the sometimes we write theta P means at this temperature this ferromagnetic material becomes the paramagnetic material.

So, this theta P is basically we write theta P equal to theta C; although it is not true, theta P slightly less than theta this T C; Curie temperature, but its difference is very small. So, that is why we take theta P equal to T C. Now, today will see about this what is the relation between temperature and susceptibility of ferrimagnetic and as well as susceptibility for anti ferromagnetic magnetic.

So, see this chi is basically for in paramagnetic phase is C by T and chi is nothing but M by H or we write sometimes this way also mu 0 by B whatever.

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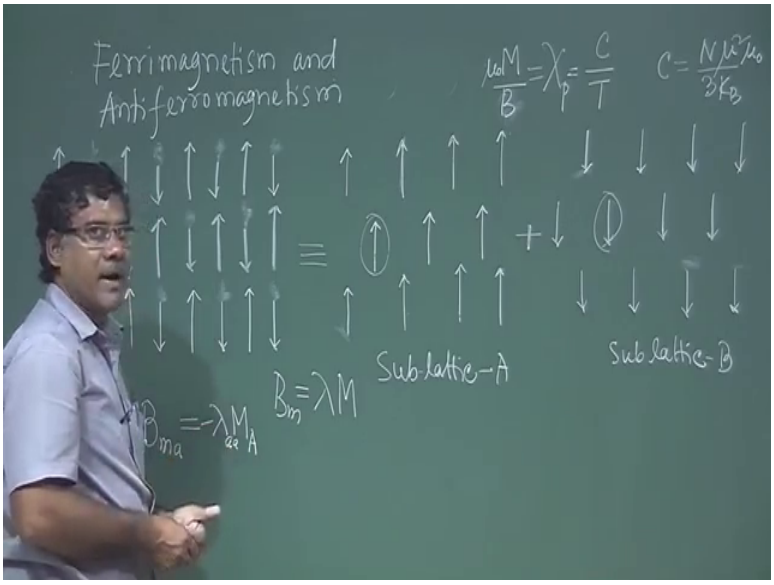

So ferrimagnetic or ferrimagnetism and anti ferromagnetism anyway; so, it looks clumsy; let me write here and anti ferro magnetism. So, both are basically this anti ferromagnetism is basically this special case of ferromagnetism; see know this alignment of this spins are anti parallel. So, in case of ferri; this spin magnitude or magnetisation are not magnetic moment are not same, they are different. So, will get resultant moment whereas, in case of anti ferromagnetism this moment of both parallel, anti parallel one these are same; so, net magnetisation become 0.

So, this arrangement if it is; just anti ferromagnetic or ferrimagnetic arrangement, we can just draw like this; so, this say one can show the lattice of this ferrimagnetic or anti ferromagnetic spin alignment; only difference as I told this; in case of this say anti ferromagnetic; for ferromagnetic, this magnitude will be different; then it is we tell it is ferromagnetic.

So, now here this lattice; we can think as a interpenetrating of two sub lattice. So, this basically one can think about this one sub lattice is for parallel spin plus another one sub lattice for anti parallel for down. So, if it is anti ferromagnetic; so, length will be same now just for ferromagnetic, this length will be small, but arrangements are same.

So, then I can this, this, this, this; so, this, this if we tell this sub lattice sub lattice A and this sub lattice; lattice B. In A side; this, this spin up and this side spin down. So, we think this ferrimagnetic or anti ferromagnetic lattice crystal is basically where this arrangement of this magnetic moment is interpenetrating of two sub lattices.

So, one is for spin up another is for spin down. Now, you think that if you take any spin here. So, then according to Heisenberg model; so, there will be exchange interaction, this is spin; this is A side spin. So, from surrounding this from a side, there will be interaction with this A side other; other spins there will be a exchange interaction as well as from B side; with B side it will have exchange interaction.

Now, if you take from B side one. So, exchange interaction with this B side spins as well as this A side spin. So, interaction here; so, if you consider this Weiss molecular field; so, what was that? We have a spin and surrounding nears neighbour whatever the spins are there. So, they have influence on this; so, it is as if this some fields are there. So, interacting this spin with a field provided by the nearest neighbour.

So, that you know that molecular field that we took lambda M proportional to the magnetisation and this is the proportionality constant; molecular field constant. So, that we tell this molecular field say B M. So, B M is this; so, that Weiss field; mean field. So, here now we think; we can think that way. So for this; so molecular from this A side as well as molecular field; from B side.

So, A side spin will feel the molecular field from A side as well as from B side and vice versa. This will fill the molecular field from this side B side as well as from a side. So, here; so, in this lattice basically whatever this if you take any spin either up or down. So, it will fill interaction; there will be exchange interaction with the anti parallel one as well as interaction with the parallel one.

So, since anti parallel one is the nearest neighbour; so this exchange interaction will be stronger; then this from same side. So, this parallel; parallel they are not the nearest neighbour, so that that will be weaker. So, that molecular field for molecular field for a spin at A side. So, here this molecular field; we can write B molecular field on A side spin. So, that is from; that we will take magnetization from A side; magnetization of this A side and then this proportionality constant.

So, this I am writing lambda a; because it is A side and this interaction with this A side; magnetisation from A side. So, this constant I am writing lambda a; and I am taking negative sign because we are taking here all our anti ferromagnetic exchange interaction. So, that is what we are taking this minus sign; then plus on this A side interaction that exchange interaction from the B side.

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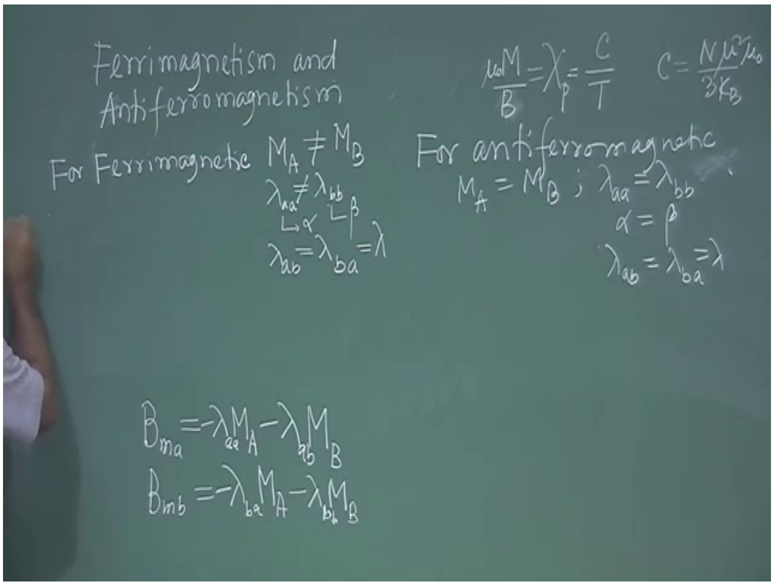

So, that is again this is whatever the magnetisation of B side; I can write M B; then this constant here interaction; so A from B side. So, let me write lambda a b and again this is the negative side and that again exchange interaction we are taking negative. So, this will be the molecular field on A side spin; and similarly molecular field on B side spin will experience. So, that will be again it is from A side; so magnetisation is M A; that will proportion to the magnetisation of this A side on B side.

So, this is a lambda; it is on B side from A side. So, I am writing lambda b a. So, negative sign and then from B side on B. So, M B; proportional to M B and that I can write lambda b b. So, that will be the molecular field on A side and B side. So, we can notice now that that M A will be equal to M B. So, I think I do not need now this.

So, see this B equal to this lambda a for say ferromagnetic; M A; magnetisation of this A side is not equal to M B; because their magnitude are different, their density may be different also and for, but lambda a is exchange interaction that strength here is not equal to lambda b b because their magnitude are different so, strength of interaction exchange interaction may not be same. So, this say I am writing lambda a; a is alpha and lambda b b is beta. And on this lambda a b will be equal to the lambda b a; because this is the inter exchange interaction; so, they will be same, so this I am writing say lambda.

So, for anti ferromagnetic case; for this is for ferrimagnetic case; for anti ferromagnetic say anti ferromagnetic. So, M A will be equal to M B, then lambda a a will be equal to lambda b b. So, say this we are writing say alpha or whatever beta you can write. So, this so; that means, alpha equal to beta; that means, alpha equal to beta in this case and of course, this on again same; this lambda a b equal to lambda b a; say it is lambda say it is lambda.

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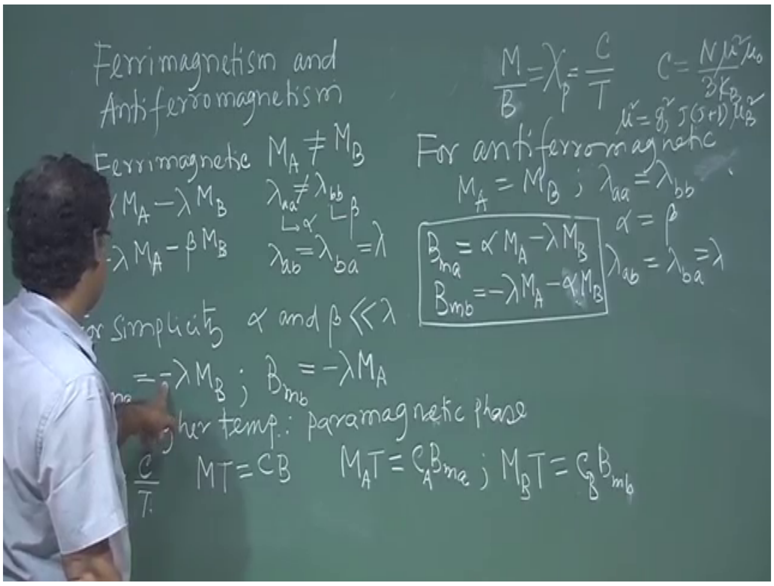

So, for ferrimagnetic case; So, I can write B M on A equal to; so now, if you apply magnetic field; so, then this total field on a side. So, that will be B 0. So, applied field if it is B 0; so I will put later on; so, this one will be minus alpha M A; minus lambda M B and B; M A molecular field on B side sorry this will b; So, that will be this will minus lambda M A minus this is lambda b b means this is beta I have written beta.

So, beta M B; on the other hand for these case; I can write B M A equal to; in this case lambda a a; lambda will be both are equal. So, I can use just alpha or beta whatever alpha M A; minus lambda M B and B; M B will be equal to minus lambda M A; minus lambda b b. So, lambda b b is is again alpha; alpha M B.

So, one can write M A equal to M B; one can put also M A equal to M B one can put, but when we need we will put it there. So, now let us consider that this in paramagnetic phase both are in paramagnetic phase; means at higher temperature T is greater than that transition temperature that we see the what is the transition temperature. So, there it is they will be in paramagnetic phase right; in paramagnetic phase what will be the chi? That we want to find out what will be the transition temperature that you want to find out.

So, in this case; so, for simplicity for this ferromagnetic case for simplicity; let us ignore the alpha and beta is very very less than basically they are very weak; experimentally also it is found, they are very week than lambda. So, lambda is very strong interaction between; because as I told you they are nearest and they are neighbour; anti parallel one neighbour. So, if you consider this one; so, what we will get that B, you will get in this case B M A equal to minus lambda M B; and B; M B, you will get minus lambda M A.

So, this in paramagnetic phase; T is very high at higher temperature; that is paramagnetic phase. So, in paramagnetic phase; so, chi equal to C by T that is valid chi equal to C by T; that is valid means M equal to C. So, we can write in; that is M equal to M by B.

So, I think one can just forget this mu 0. So, mu 0 one can take inside this C. So, I can write M B by or M H; M by H by C by T. So, this will give me M T equal to C B; M T equal to C B. So, now, I have two field; one is M A; T equal to C; B M A and another one another one M; B T equal to C B; m b. Now, here I have to write basically this C will not be same; in this case for ferrimagnetic in this case C will not be same.

So, because this moment are different for A side and B side in case of ferrimagnetic, but in case of ferromagnetic this C A will be equal to C B; because moment are same or both. And this mu here this I have not written; so, mu you know that I think here I can write mu equal to or mu square equal to g j square; j j plus 1 mu B square mu B square. So, in this case it is a Curie constant will not be same C A and C B; we have to use. Now, from here this you can put this B M A; is what is this B M A is minus lambda M B; so minus lambda M B.

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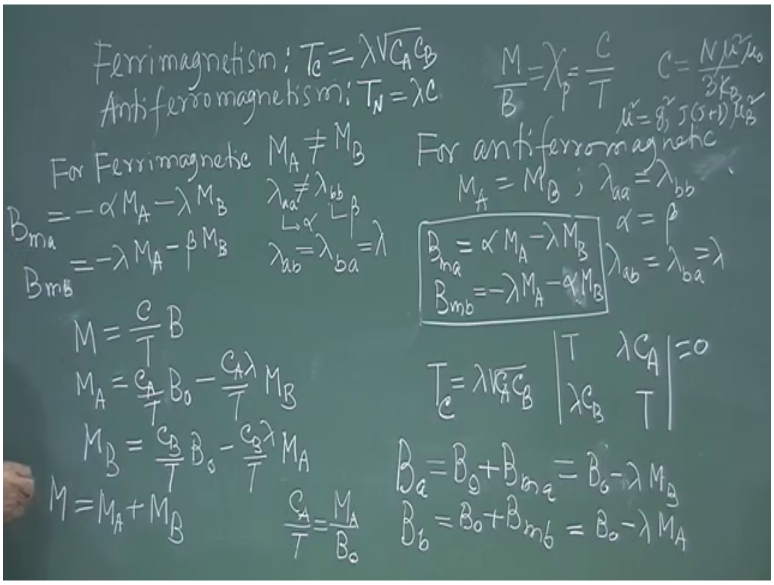

So, I can write here; so, this side plus minus lambda M B. So, here I have yes, but this that is M B. So, I have to write here M B. So, I can replace this B M A by this and B M B; I can replace with this. So, minus; so, this side I can take I can write plus; then B M lambda. So, this lambda and that M A; this I can write M A. So, here we can see; so, this I have to write equal to 0 and this equal to 0.

So, now to get the non (Refer Time: 30:15) solution for M A and M B; so, determinant has to be 0. So, coefficient of this M A; M B what is here? T. So, determinant will be T of M A and lambda C A and this case M A; this lambda C B and for M B that is T. So, these has to be equal to 0; so, from here what you can see. So, I can get here T square minus lambda square C A B; square equal to 0.

So, I will get; this will give me T equal to square root of; equal to lambda square root of C A; C B; from here I will get this T equal to; so, you cannot see. So, I think I should write; so this will give you T square equal to this square; so T equal to lambda square root of C A; C B.

So, that will be the basically this is Curie temperature for ferrimagnetic case. So, for ferrimagnetic case; what is the Curie temperature I got? T C equal to lambda square root of C A; C B. Now in case of anti ferromagnetism; so, as I told that C A equal to C B. So, square root of C square; so, lambda c.

So, for this case; whatever this transition temperature we will get that is; we tell this nil temperature. So, it is T n; T n will be equal to lambda C. So, it depends on the exchange interaction constant as well as Curie constant. So, for both cases we are able to find out the transition temperature.

Now, one can show that if we do not neglect this alpha beta. So, one can show that this will be; it is I think I will show you. So, let me if alpha beta is not neglected. So, for anti ferromagnetic case; we will show this; what is the result, but for this case; for ferrimagnetic we will not. So, it will be slightly complicated.

So, that I will come later on; so, here why I want to show you that basically if you; so, that will be more clear, if we calculate the susceptibility. So, for susceptibility what we have to do? So, you have to apply; so, susceptibility in paramagnetic phase, we have to apply magnetic field. So, in that case; so, this we can write B; so, this if we write that magnetic field, the total field on this A side; that will be, if it is B a; so, that will be B 0 applied field plus B m a.

And similarly for B side; B will be B 0 plus B m b. So, now one has to put B m a and B m b; here it is there. So, in this case also if you consider that this B m a is lambda M B; B m b is lambda M A. So, this will get basically B 0 minus lambda M B and in this case you will get B 0 minus lambda M A; so, I think I can show you simply this anti ferromagnetic case.

So, I have to go few step more; then I can show you the calculation. So, these guy is your total field that is this; now susceptibility, to get the susceptibility. So, that is basically chi equal to M by M by H or B. So, not B; it will has to be B 0; so, we have take to proper form. So, what we should do? So, B a equal to B 0 this fine.

Now, what is; so, B A; what we will do? I can write this one, I can write chi equal to this is fine M. So, we think in case of M A; so, this chi A. So, this and this chi I will write chi 1 can write C by T because it is in paramagnetic phase. So, this is C A by T; so, I can write M a T; equal to C A; my applied field. So, C; so let me I need space; so, let me use this space. So, chi equal to M by; so, I can write this M equal to in general C by T in paramagnetic phase.

So, for M A equal to C A by T; so, I am doing mistake. So, this is M A I can write M equal to C by T; this is chi is C by T. So, into B; magnetic field because M by B equal to C by T. So, then I can write M A; so, for paramagnetic phase I can write chi equal to C by T. So, chi is M by B; so, M equal to C by T into B.

So, for M A; I can write C A by T and B is; I have to write B a; and B a is I can write this B 0 minus lambda M B; B a I can write B 0 minus C A by T; this lambda M B. Similarly M B I can write; C B by T; B will be B b; this one, so that is B 0. Again this is B 0 minus lambda M A means C B by T; this; it is a lambda M A, this I can write.

So, from here if you add them; so, you will get M A plus M B; M A plus M B that will be basically total magnetisation on the system M.

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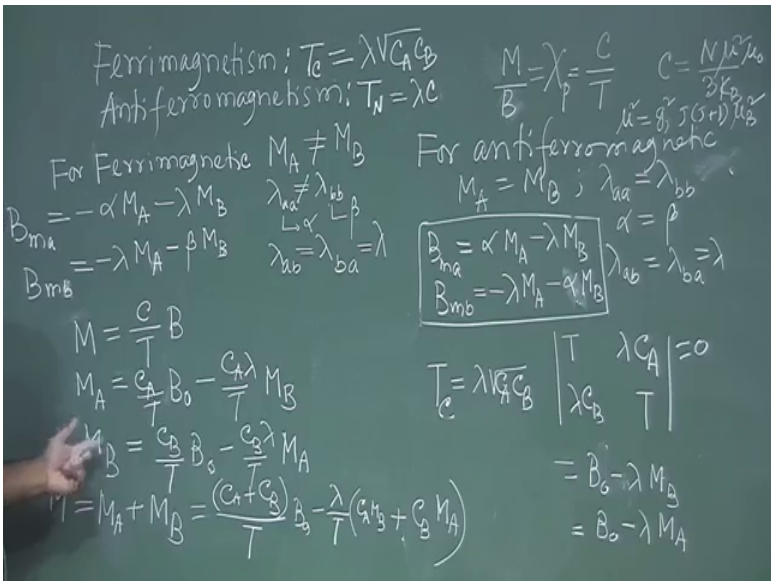

That is equal to 2 C A. Now it will be C A plus C B by T; B 0 and here C A lambda by T minus lambda by T; C A here C A; M B and here C B; M A. So, I have to write this lambda by T; then C A; M B.

So, this I have to; I need space C A; M B plus C B; M A. Actually, I think what we should do; just we can replace this M B by this M B. So, we can put this value here; this we can put here. So, we will get M A. Similarly, we will get M B; if you put this one M A here.

So, I think that way I think I will just write the result; I have calculation just I will show you. Then you; I think here I need this one to show you its calculation; I think this I want to show you that.

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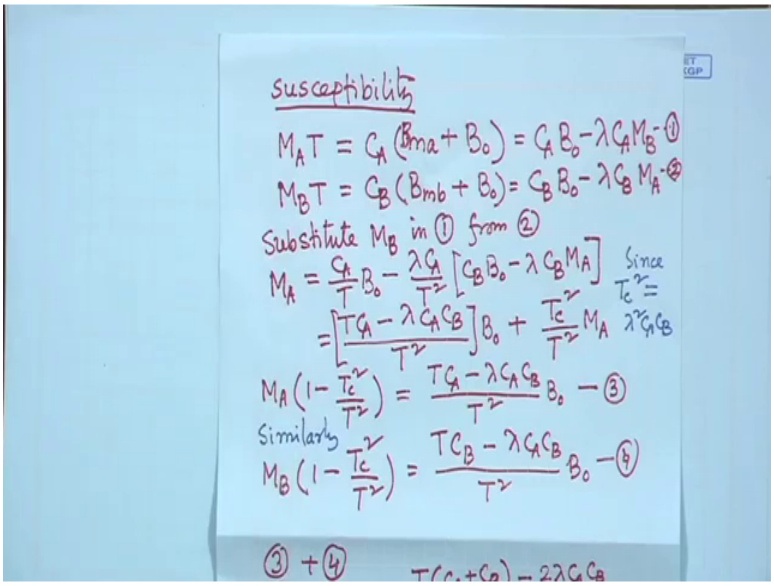

So, if you replace that one then you will get basically; so, you see this in M A; so, this is multiplied by T. So, this both T are there; so, M A; T. So, that is why I write M A T or just here you can see M A equal to; C A; M A equal to C A; T; B 0 minus lambda C A by T square; lambda minus lambda C A by T square; from where this T square T is coming? So, here M B; I am replacing by this one taking out T.

So, M B; I have replaced by taking out T; C B; B 0 minus C B; M A minus lambda C B; M A. So, just here I have shown you the calculation in every step, I have shown just you check it or you can do it yourself.

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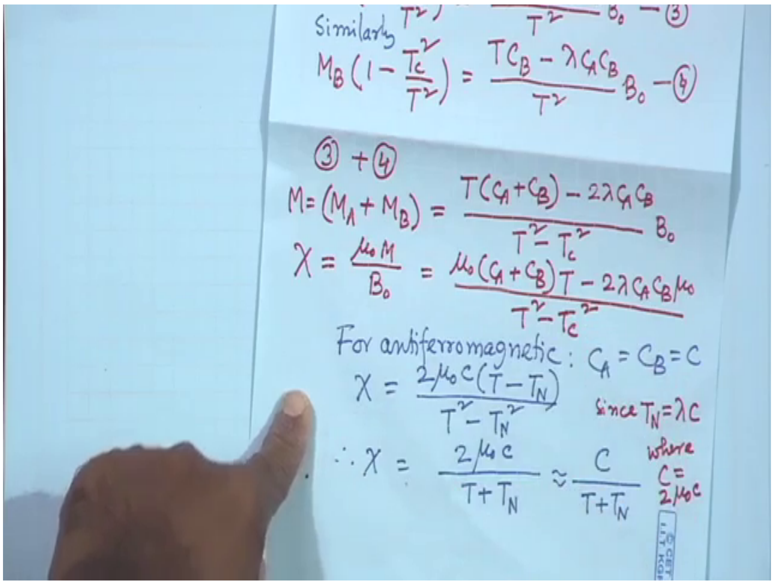

Then you will get this M A and M B; so, M equal to M A plus M B; equal to this. So, chi is mu 0 M by B 0. So, sometimes we write M by H or M by B 0; that depends on the unit, whether it is a C g s unit or S I unit. So, this you are getting equal to this; so, this chi equal to this. So, this is for the ferromagnetic case. So, here you can see this chi equal to it is not C by T minus theta or minus T C in case of ferromagnetic. So, it is dependence on temperature is different.

So, I will show; if we plot chi now 1 by chi as a function of temperature; then you can see this, it is different; its curve is towards the axis. So, that I will show you and for anti ferromagnetic case; just you replace this C equal to C B; equal to C, then you will get from this formula; you will get this. So, here just I replaced this with Curie constant, but its value is twice of this; normal Curie constant. So, some constant I have put to keep the similarity.

So, this is the susceptibility for anti ferromagnetic case and ferrimagnetic case. So, for ferrimagnetic case; you see this temperature behaviour is slightly different than this anti ferromagnetic and this paramagnetic, where it is 1 by chi versus temperature is linear.

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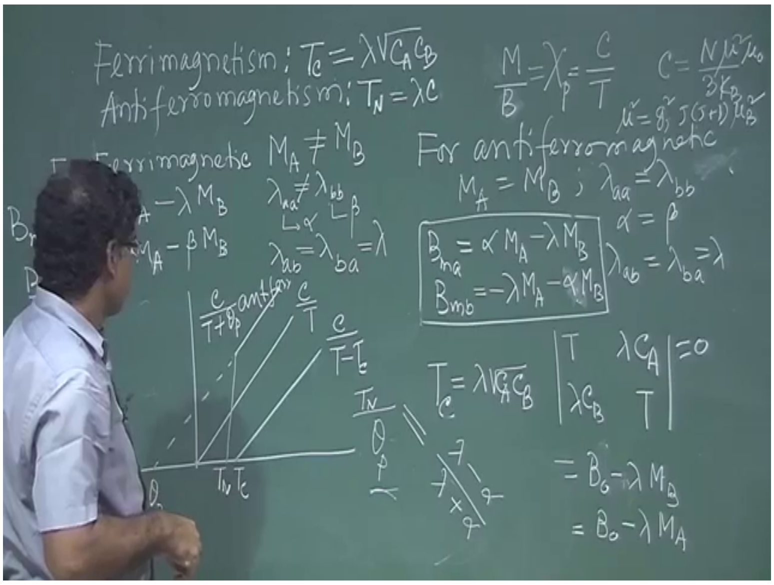

So, if I plot So, I will; so, basically; if I plot. So, this is the C by T for paramagnetic and then; this is C by T minus T C. So, this is T C and this; so, this is basically T N; and this we write theta sometimes P we write theta P. Now its T N and theta P; they are what is theta P; that I will show they are not same, they are different. But sometimes they are same depending on the condition.

So, if I ignore this alpha and beta then theta P equal to T N; if we do not ignore then one can show that this theta P is basically theta P; T N and this theta P this ratio; one can show T N by theta P; it is equal to, I guess lambda minus alpha divided by lambda plus alpha. This kind of things; if we ignore alpha then they are same, if you do not ignore then this T N is; so, minus alpha.

So, it is smaller; so, T N will be what is this? So, T N will be different from this theta P. So, that calculation; if we do not ignore this alpha and beta, so in that case you will get; I will show you, if I do not ignore, so what will happen?

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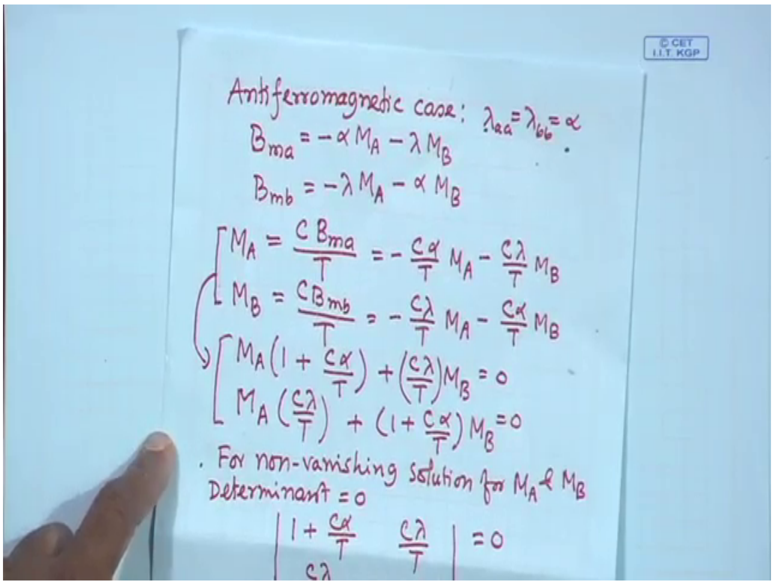

Just I will; so, if I do not ignore; if I keep in case of anti ferromagnetic case. So, I am keeping alpha equal to beta; so, alpha I am keeping. So, if you just proceed same way; just I have shown here calculation you can do yourself.

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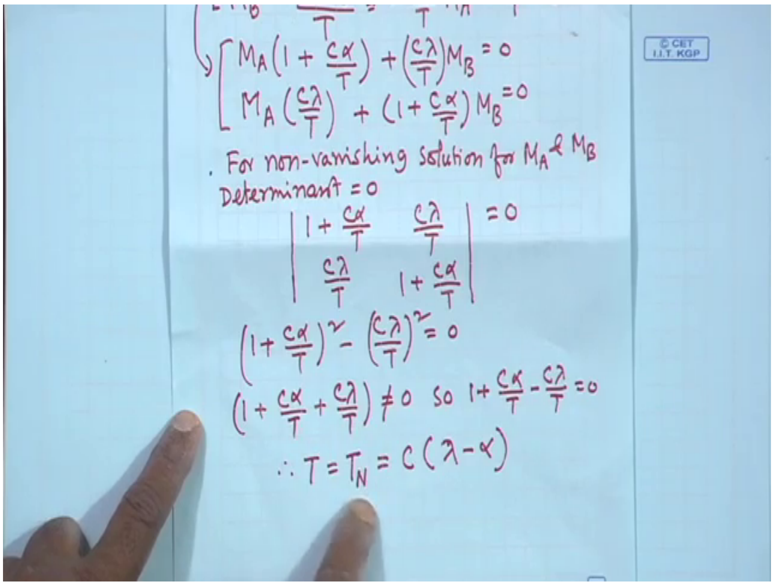

Then you can show that; T N equal to C lambda minus alpha. So, earlier it was C lambda; now it is C lambda minus alpha. So, if you put alpha equal to 0; so, it is the other case.

Similarly, if you see calculate the susceptibility without ignoring this alpha; in case of anti ferromagnetic case. So, just you proceed keep it; so, it is not do not; it is not this, this one in paramagnetic phase.

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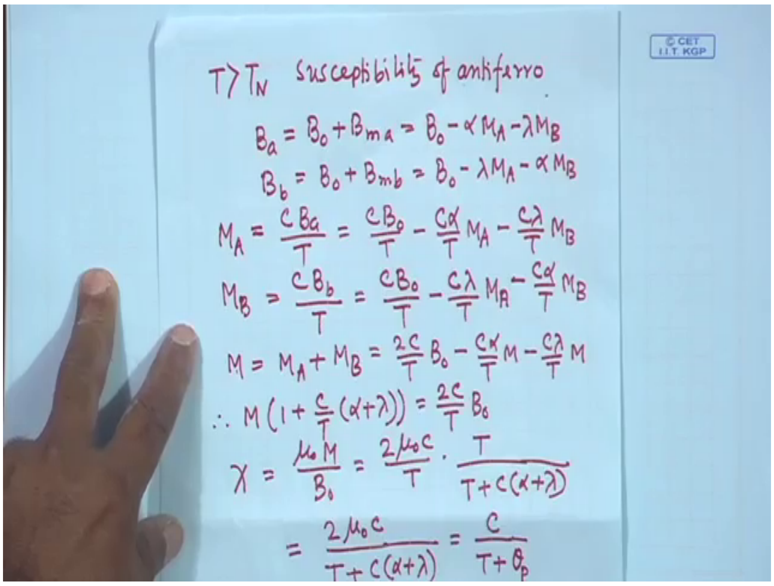

So, if you do not ignore alpha; keep it alpha and same way you proceed; then what we will get? Chi equal to you will get C by T plus theta P; now where theta P is C lambda plus alpha or alpha plus lambda.

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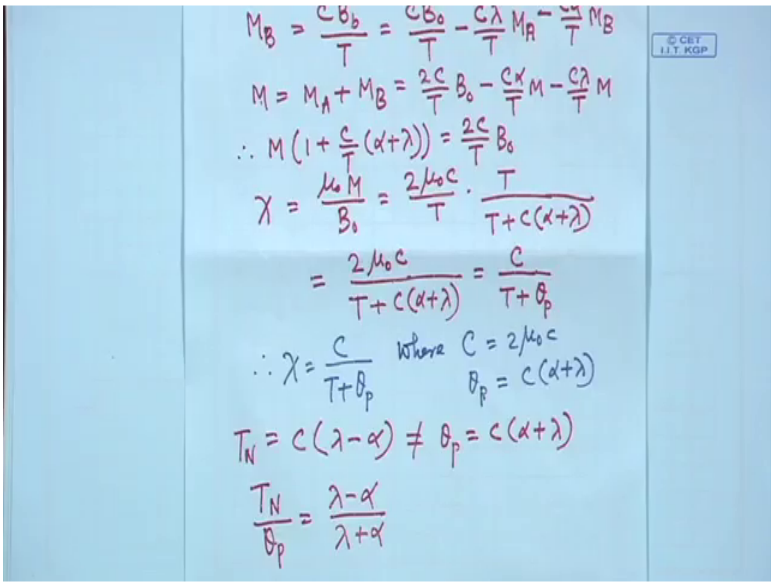

So, T N whatever we have seen that is C lambda minus alpha is not equal to theta P; equal to C lambda plus or alpha plus lambda. If we ignore alpha then T N equal to theta P; C lambda, both are same. If you do not; if you consider this alpha, then they are different. So, that is what I told you. So, this is for anti ferromagnetic case. Where, C is T minus; this plus theta P.

So, I think I will stop here, I will continue in next class.

Thank you.