Solid State Physics

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Lecture - 63
Magnetic Property of Solid (Contd.)

So, we will continue our calculation.

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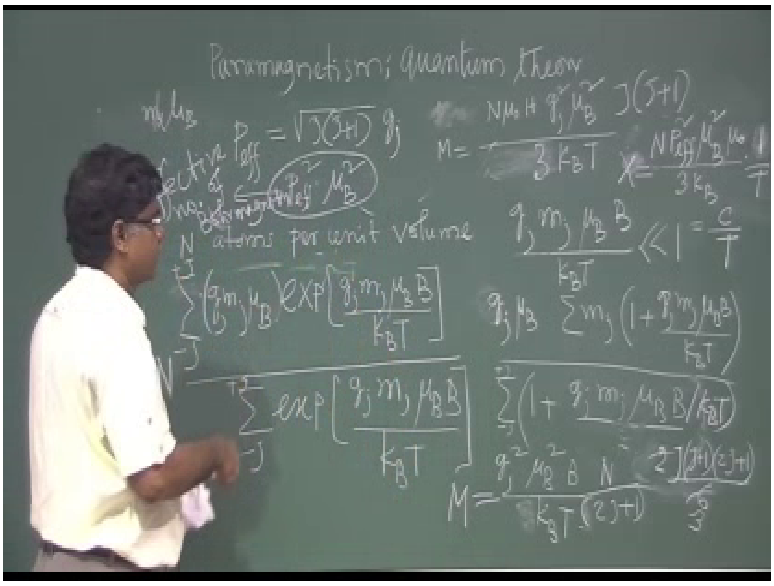

So, here for these condition when low field and high temperature; so, we have derived the expression for the susceptibility and these have the form of C by T where is the curie law and C is the curie constant; so now, actually for. So these, whatever condition we have taken this low field and high temperature. So, for classical theory whatever result, we got this from here also from quantum theory also, we got for similar result only slight different of few P effective that is effective number of Boltzmann's atom.

So, now if; so, this is the original expression where we have not considered anywhere; you have not considering any condition. So, if you without considering any condition, if you in general if you proceed and you do calculation; so, one can show probably I will not calculate, but I can tell you few step; I can tell you few step.

So, I think, I will just mention few step, then it will be helpful for you. So, let us take; let us take this g, this term; this term except m j except m j equal to X; right because this summation is over the m j.

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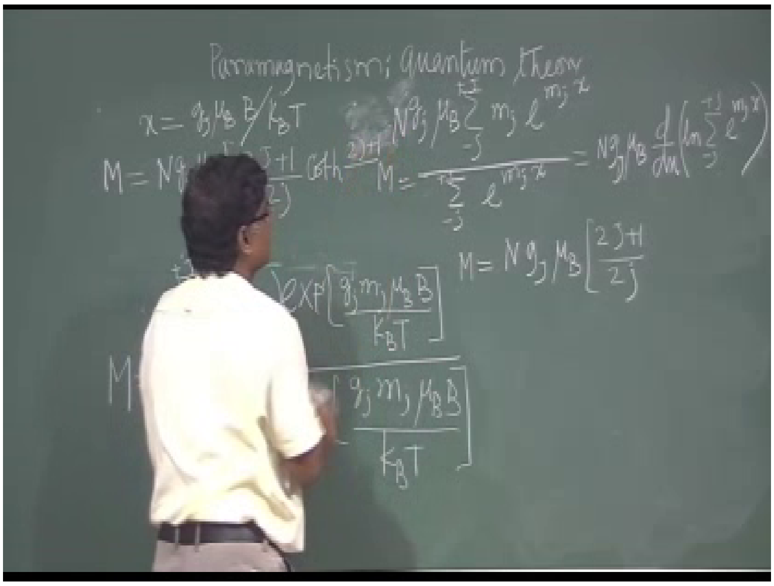

So, if I put x equal to X equal to g J; capital J; g J mu e B by K B T; K B T, then this I can write, then this I can write m equal to m equal to sum over i, this I will write slightly below m equal to sum over minus j to plus j that is there and this will give me this m j will be there. So, I can take out this.

I can take out g j; g j mu B from here g j mu B and then put sum over sum over minus j to plus j of m j e to the power m j x right divide by this one sum over minus j to plus j this will give me e to the power m j x, right. Now here if you just write the series, you just write the series. So, minus j to plus j, it will vary; what we will get that you can just; I think it is not so difficult; one to write, but here one approach would be useful that is this I can write; this I can write; I think m i miss; I am missing 1 N; yes, I am missing 1 N, there should be 1 N and this I can write N g j mu B N g j mu B and here you see.

If I take log of this and then differentiate it with respect to x d by d x d by d x log of this. So, log of this difference is 1 by this. Now log function that is taken care, then differentiation of this one; so this will give me e to the power m j x e to the power m j x and this part differentiate with respect to x it will give me m j. So, this can be written as 1

N differentiation of l N of l N of this term minus j to plus j minus j to plus j e to the power m j x m j x; right m j x.

So now, you have to proceed just explicitly, you write them; explicitly you write them. So, this l N I can write this one e to the power e to the power j x plus e to the power j minus 1 x right. So, this way I can expand this series, we can expand that series and then do the differentiation. So, you have to just go ahead you have to just go ahead and then what I will get; N equal to N equal to yes m equal to N g j. So, I will not calculate full, it will take time that you can do the few steps of from here to here you can come N g j mu B that is your home task it is not difficult one I am telling the few step.

So, just fill up the gap. So, mu after differentiation after differentiation you will get you will get this type of expression 2 j plus 1; just let me 2 j plus 1 by 2 j cot hyperbolic better, I should write here it will take; so, m N g j mu B N j g mu B N g j mu b. So, if you do this. So, you will get basically 2 j plus 1 by 2 j, then I will get cot hyperbolic cot hyperbolic 2 j plus 1 by 2 (Refer Time: 10:04) cot hyperbolic 2 j plus 1 by 2 j, then minus 1 by 2 j cot hyperbolic cot hyperbolic cot hyperbolic 1 by 2 j. So, this just j is a.

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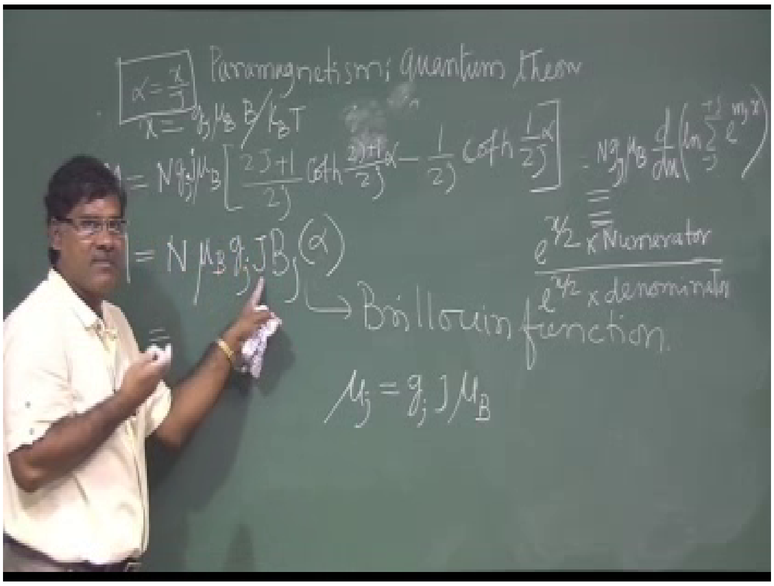

So, here is the; this either x; x should be there. So, that x here x x; I am writing alpha; I am writing alpha. So, this alpha equal to x by j x by j in between you have to put alpha equal to x equal to alpha j x equal to alpha j. So, then this place x was there. So, x was there right. So, that will come as a alpha. So, here this alpha term will come alpha term will come yes alpha in place of x alpha term will come.

So, this ultimately I will get and without any approximation. This will be the result please try yourself I think, you have to proceed only when you are from here; when you are proceeding to get. So, just you have to expand this series do the series summation standard summation I will find out and then in between, you have to multiply with by e to the power C by 2 you have to multiply by e to the power x by 2 in both denominator numerator; so this into numerator and then this into denominator.

So, I think is e to the power x by 2. So, when you are doing 3 step. So, in one step just you have to we have to take this one then proceed again proceed again and then you will get this type of format not except this j except the j here; except the j here. So, then if you take this alpha equal to x by j and then in terms of alpha just x you have to replace by alpha j; so, then you will get this final format. So, this you have to consider. So, I think this will just its just algebra straight forward only the some tricks one has to apply in this way. So, this is called basically N mu B N; N mu B g j N mu B g j and this. So, this B j is called Brillouin function; this called Brillouin function. So, yes; so, here just is m j; m j.

So, here when you will do step; I will multiplying with by some this factor right. So, here 1 j will come; here 1 j will come. So, that you have to include. So, in this final form here 1 j will come. So, here basically 1 j will come. So, this will be the final form without any approximation this will be the m this will be the m and this and you see here this i can write in principle this g j how; what was the mu j? What was the mu j? I had just read a mu j was g j j a mu B right, mu j was g j mu B right g j j mu b. So, along the magnetic field along the magnetic field m j equal to j because m j value is j to minus j j to minus j minus 1 j minus 2 j minus 3 2 minus j, right. So, when it is j value m j equal to j value; that means, it is aligned along the magnetic field; so, this g j m j mu B that is the component of this moment along the magnetic direction.

So, now, this m j is j; m j is j; that means, this it is this dipole moment is fully aligned along the magnetic direction; it is not component this fully aligned along the magnetic direction. So, as if this term this term means giving us the full alignment of this dipole moment along the magnetic direction; so, N number of them, right; so, all N number atoms all N number atoms having the magnetic moment having the magnetic moment and their fully aligned along the magnetic direction. So, that is nothing but the saturation magnetization that is nothing but the saturation magnetization.

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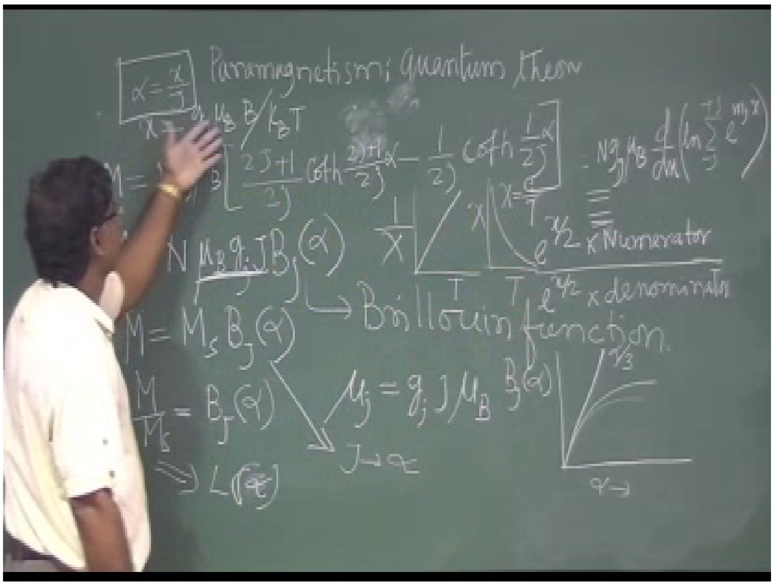

So, I can write this one saturation magnetization M; M equal to M S Brillouin function; Brillouin function alpha. So, this M by M S. So, here that your B is there, here your B will be there, this B magnetic field sorry N mu B j. So, I think these magnetic fields inside of this function alpha, you can; it is contains this alpha. So, this M S equal to. So, it is similar expression earlier from classical theory what we got this M by M S, it was Langevin function right it was Langevin function, right what is the Langevin function.

This cot hyperbolic alpha minus 1 by alpha, right and this Brillouin function is this Brillouin function is this. Now here just one thing you can see that if you take if you take in this function here your if you take that condition this term is this term is less than this term means x is less than or l is sorry alpha is very very less than one very very less than one. So, from this Brillouin function you can proceed you can you can simplify it and you can show that whatever before taking this one what is approximation we took condition we took approximation condition it took.

So, we these to the C by T Collies law from here also alpha is very very less than one if you consider. So, you can go to the that Collies law directly and another aspect is here magnetic here what is happening this the quantum theory this is the quantum theory and classical theory; these the classical theory right here Brillouin function and here Langevin function, right, but one can show that here this what we considered this dipole moment are free to rotate and in this case.

What we have considered dipole moment are not free to rotate there is a constant it can rotate only with 2 j plus 1 value 2 j plus 1 value this quantized value in space. So, that was the restriction. So, j equal to it means; it can rotate only in 5 directions right it can rotate only 5 direction 2 j plus 1 is 5 j equal to 2 right j equal to 1; only it has 3; it can have 3 rotation in space. So,; that means, if j value is smaller, it is more restricted, right. So, now, if you take j value is infinity j tends to infinity. So, then what will happen j tends to infinity means m j value tends to infinity 2 j also was infinity towards infinity.

So, then it is equivalent to the free rotation, right as in case of classical theory we consider. So, when j value tends to infinity then it is equivalent to the 3 rotation as you can concept for classical. So, if you from these in case of Brillouin function if we considered j tends to infinity j tends to infinity then you can get this Langevin function from Brillouin function it will convert to the Langevin function so that you can check yourself.

If you take j tends to infinity. So, from here; so, this Brillouin function will convert to the Langevin function. So, Langevin function these it varies; it varies Langevin function is varies not Langevin function this Brillouin function this Langevin function; I have shown you. So, Brillouin function as a function of alpha it varies also like this. So, basically we are more interested about at the variation at the lower temperature higher temperature and lower magnetic field instead of lower temperature because alpha is by T na alpha is this by T means this side is higher temperature the other side is lower temperature right.

So, we are interested basically at the higher temperature why because at lower temperature at lower temperature and higher field mostly you will get this system will go towards the saturation filed. So, there is the much interests on this side at low temperature and high magnetic field. So, it goes towards the saturation value in billowier function this is varying like this and in Langevin function also we have seen it varies like this right it varies like this.

But at higher temperature low files approximation we took the we took the tangent at higher temperature and this straight line as actually we have shown that alpha it is alpha by 3 under this approximation; so, this alpha by 3. So, these at higher temperature this way this function varies or in that function is nothing but M by M S. So, basically M magnetization varies with temperature at higher temperatures linearly. So, that in terms of chi in terms of chi we have seen in terms of chi in terms of chi you can see if I plot 1 by chi 1 by chi as a function of temperature. So, you see it is a straight line. So, in parametric it; this is for higher temperature at lower temperature we are not much interested as I told that it goes towards the saturation. So, that is why we are always conceive, we are always considering the; this higher temperature and lower temperature.

So, it is straight line or if you just plot this chi and T chi and T; so, it is a high profile hyperbola kind of mediation because chi T equal to C chi T equal to C. So, this basically this law chi equal to C by T chi equal to C by T if you plot. So, these the verification of parametric material; if you measure the magnetization and then subset it will be plot as a function of temperature right then it starts follows like this it curves follows like this then we tell that us the we tell that us the parametric material and from this one can calculate the one can peaceful calculate the p effective p effective p effective that is mu.

So, it is not just 2; B 3, B 5 B mu B not B 5, mu B. So, it is something fractional value and this fractional value one can find out this what is the average magnetic moment of each dipole of this for this atom for each atom. So, that one can find out from because from this loop one can get the C one can get the C or 1 by C right since it is 1 by chi and C contain this all this things C contain all this things, I have deleted that one anyway you know. So, from there we can calculate the effective number of board magnate ion. So, and this is important because as I told this probably I will tell most about this.

So, this about the quantum theory of this parametric; so, let me tell something more interesting from this; so, this are there are very nice application of parametric solve you know one application is cooling of that is that is called basically cooling of cooling of this of a system by adiabatic demagnetization adiabatic demagnetization; what is the meaning of that cooling by adiabatic demagnetization.

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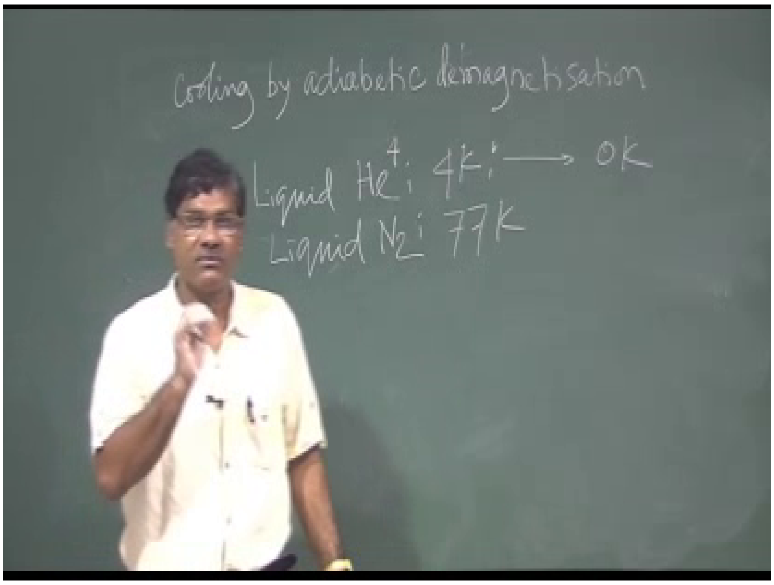

So, if you take this parametric solved; so by adiabatic demagnetization I can cool it, I can lower the temperature that is very important application of this parametric material you know this using liquid helium liquid helium liquid helium we can go up to 4 Kelvin, I do not know whether.

You have seen or not, but we use in the lab this liquid helium to get temperature 4 Kelvin and I think in many places this liquid nitrogen used liquid nitrogen used right and that is 77 Kelvin. So, these are the discovery basically and because of this liquid helium because of this 4 Kelvin temperature one can reach. So, that discovery of super conductivity because of this discovery of super conductivity because of the discovery liquid helium which gives the temperature of 4 Kelvin, right liquid nitrogen its gives 77 Kelvin. So, it is easier to produce and many labs it is produced and used for experimental purpose.

But now question is how to go to the lower temperature from 4 Kelvin towards 0 Kelvin how to go there is a challenging there is different way people try. So, liquid helium will be this helium 4 basically. So, people use helium 3; helium 3. So, some different mechanism to go lower than 4 Kelvin lower than 4 Kelvin, but one efficient method is this cooling by adiabatic demagnetization of parametric solved. So, that I want to just discuss. So, what happens you know the entropy?

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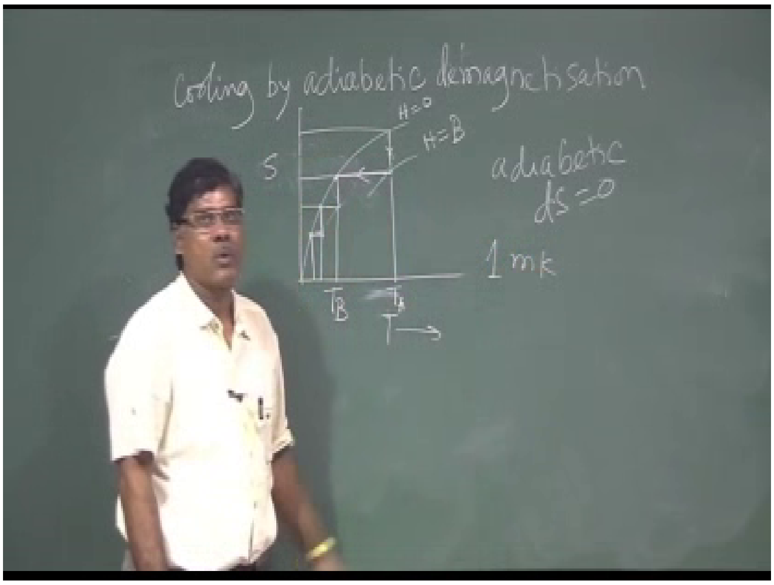

Entropy is the measurement of the dis-orderness for any system E P T is disordered then its entropy is higher E P T is ordered its entropy is lower.

So, I have magnetic system parametric system this parametric system had the dipole moment. Now if I apply magnetic field, I can get; I can align them along the magnetic field so; that means, before applying any magnetic field they are disordered magnetic moment was 0 magnetization was 0. Now if I apply magnetic field, then they are aligned; that means, they are becoming ordered. So, their entropy when I will apply magnetic field their entropy will decrease, right or with respect to temperature I have aligned system under say magnetic field. So, entropy is lower right and temperature is lower now I am increasing the temperature giving thermal energy to the system.

So, it will try to disorder this dipole moment right. So, increasing the temperature it will be disordered more and more is a entropy will increase, right. So, entropy for this is parametric system in terms of dipole moment or in case of dipole moments. So, this entropy if I plot entropy versus this temperature entropy versus temperature; so, this will vary like this; this will vary like this; this will vary like this. So, this lower temperature entropy is lower right without applying any magnetic field without applying any magnetic field just as a function of temperature the dipole moment of parametric solve this moment or oriented.

So, it is at lower temperature oriented along the particular direction and now I am increasing temperature. So, it is disordered entropy is increasing and entropy is becoming highest right now this for this; this curve for without applying magnetic field without applying magnetic field now take a point here take a point here at temperature say T A at temperature T A. Now at this point what is the entropy? So, this is the entropy value right these; the entropy value right. Now entropy this entropy this is the very high value. So, system is disordered at this temperature as this temperature.

Now if I apply magnetic field at this temperature. So, then what will happen? So, dipole moment, they will try to align along the magnetic field. So, they will go towards the ordered state. So, its entropy will decrease right. So, its entropy will decrease, but at same temperature we are considering say entropy is decrease it is taking this value. So, if I apply magnetic field. So, my curve basically for this see f you take this whole curves to be like this. So, this will be for H equal to some value magnetic field H equal T B.

Now, this point where entropy is this entropy is this entropy is this, right. So, entropy decrease earlier it was this. So, before applying magnetic field at this temperature this are the entropy now after applying magnetic field; it is in some extend this is ordered see entropy changes come down here, but at the same temperature. So, to keep that temperature same we take the help of some resolver some conducting source we keep the temperature fixed. So, this isothermal power isothermal case right and adiabatic is this we do not allow thermal energy to go out from the system. So, its temperature will change for isothermal case we allow heat to go from the system to either go out or to come in.

So, that is temperature will remain constant of the system. So, this change is isothermal change isothermal change right. So, just I applied magnetic field system becomes order keep the temperature same temperature then entropy will be this it will decrease, but temperature is same. Now if I we do the magnetic field now at this point now at this temperature. Now I have applied magnetic field. So, that is why it has come here. So, now, I we do the magnetic field then what will happen I will withdraw the magnetic field and then. So, it will go to this curve because it is this curve for H equal to 0. So, what it will do.

So, it from here it came here after applying magnetic field when I withdraw magnetic field. So, from here it will go here because in case of adiabatic change of entropy 0 change of entropy 0 in case of adiabatic d q equal to T d s from there I can say anyway say in case of adiabatic in case of adiabatic d s equal to 0. So, this now changing temperature or just it will come back to its or H equal to 0. So, this is the point. So, this I can say this; the temperature is T B.

Now, T B is lower than T A. Now just I have applied magnetic field and then withdrawn now the systems temperature is entropy is decrease and its temperature decrease you know then again from here I will apply magnetic field. So, it will it will be ordered. So, it will come here. So, again I will withdraw magnetic field it will come back here system see temperature of this system will decrease gain; I will apply magnetic field here. So, it will come down here.

So, this I will withdraw magnetic field I will withdraw magnetic field sorry it will come here I will do magnetic fields it will go back here. So, this just applying magnetic field we do not need temperature of the system are decreasing. So, that is is the very effective technological very effective and this is used for getting temperature to the up to I think towards 1 milli Kelvin, 1 milli Kelvin from 4 Kelvin to milli Kelvin; 1 milli Kelvin is 10 to the power minus 3 Kelvin, it is towards 0, it destroys 0.

So, this the application of this parametric material paramagnetic solved. And there are other things also, but I will stop here. I will discuss in next class.

Thank you.