Solid State Physics
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Lecture - 57
Thermal Properties of Solid (Contd.)

So, we will continue our calculation.

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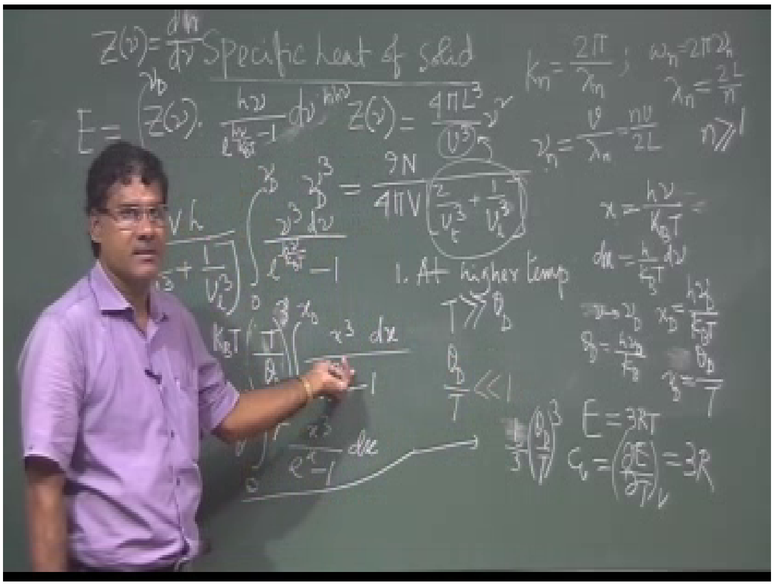

So, at higher temperatures that means, temperature is very, very high compared to Debye temperature. Theta D is defined. So, this theta D by T is very, very is less than 1. So, our x is very, very less than 1; so e to the power x equal to basically if x is small less than 1. So, one can write 1 plus x, plus x square. So, higher term one can neglect; so 1 plus x minus 1. So, it is x so x cube by x, so x square. So, I have to integrate x square dx over 0 to x d.

So, then I got the x, x cube by x cube by, x d cube by 3 say xds theta D by t. So, this cube and so this part from here multiplied with this. So, this will go and 3. So, it will 3 Rt. It will be 3 Rt. And this Cv is del E by del T at constant b, we will get 3, we will get 3 R.

Now, at higher temperatures s this I do not need. Well let us see at higher temperatures; so at sorry at lower temperatures.

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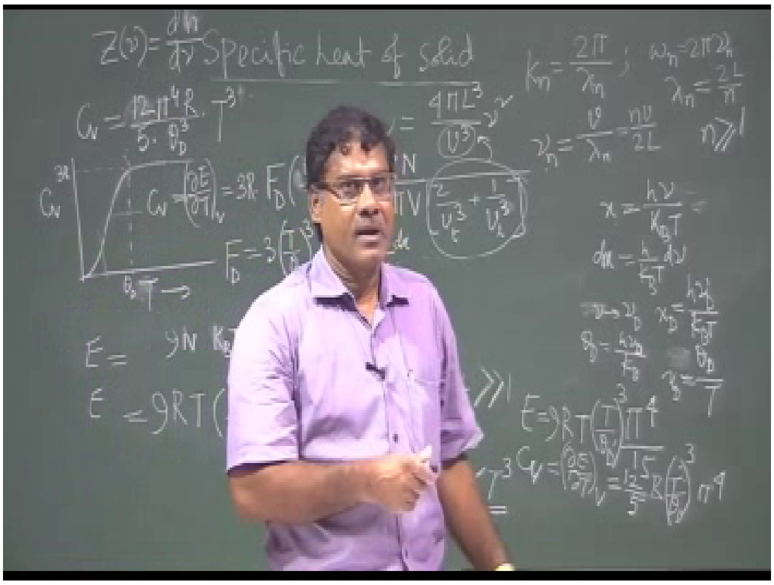

At lower temperatures; that means, T is very, very smaller than theta D. So; that means, theta D by T, these very, very smaller than this. So, this will be very, very greater than 1. This will be very, very greater than 1. So, e to the power x, x is very, very greater than 1 e to the power x. So, it is one can take neglect 1. So this we can write, I think, I delete it here, write it. So, this part indicating part; what I will get? I will get x cube and e to the power x dx. So, this way one can consider, this is another way I think, it is a easier way one can do, that in this case x is very, very x is this. The theta D by T is very, very greater than 1 x is very, very greater than 1; so theta D by T, if T tends to very, very small very, very small temperature. It tends to 0. T tends to 0. So, this will be infinity.

So, this integration we can write 0 to infinity; then x cube e to the power x minus 1 dx. This integration you have readymade solution of this. So, this is basically pi to the power 4 divided by 15, pi to the power 4 divided by 15. So, this way one can, since it is readymade it is available. So, just we took that opportunity. Otherwise as I was trying to find out that it is a very, very greater than 1; so I one can neglect, one just x cube e to the power ex e to the power x dx. So, one can integrate. Also you can find this similar same result.

So, what I am getting? So, basically I am getting here integration value is getting. So, my total energy E now, my total energy in this case E now, this my integration part giving this value and then I have 9 R T 9 RT and then T by theta D cube into this. So, that is the total energy at very low temperature right.

So, now you can find out the Cv equal to del E by del v del T at constant v equal to. So, this basically T to the power 4, see if I differentiate it will give me 4 T cube. So, it will give me 4 T cube. So, 4 into 9. So, that is basically 36, 36 by 15 this 15; 36 by 15. So, T cube R T cube by theta D to the power 4 because no theta D by q.

So, then I can write the same way, because this absorb by this differentiation. So, T by theta D cube. So, I think this is the 3. So, this I can write 12 by 5 basically, 36 by 15 means 12 by 5. I can write. So, here Cv, now if you can clearly see Cv is proportional to T cube. Cv is proportional to T cube. That is the great success of Debye theory. That experimental results demands the variation of Cv at low temperature as a T cube Cv varies as a T cube means Cv proportional to T cube so that the experimental result demand and from Debye theory successfully. We could derive this dependence of Cv on T as a proportional to T cube. So, that is the great success of Debye theory, which explain this whole behavior of Cv. So, I think I do not need (Refer Time: 09:02).

So, at lower temperature Cv we got 9 R no 12 by 5. What is the pi? Pi to the 4 will be there are here pi to the power 4 will be there yes. So, pi to the power 4 R or by theta D theta D cube then T cube. Now, this Cv curve we basically it is Cv curve, I think I can delete them. So, Cv curve or I have shown you earlier as a function of temperature this is Cv. So, it is this variation is like this. I think it is not like this it is more sharp here. So, this is the value this is cr value at higher temperature. So, this temperature really Debye temperature theta D and a lower temperature is very, very less than theta dt, when T is very, very less than theta D telling in this region, it is a one tenth of theta D nearly it is considered. So, it follows this. So, upper half here, when theta D is the T is greater than theta D.

So, now what about in between these 2; so one has to basically integrate up this one. So, this is the energy basically this is the energy. So, this is the energy. So, one has to integrate, but I will not integrate. So, that is say your homework. So, it generally after you integrate you will get Cv for this region or in general for whole without considering high temperature low temperature this this limit.

So, in general this comes and it is difficult to integrate just like this analytically. So, generally we take help of. So, we can do numerical calculation. Analytical calculation is difficult, but this form Cv. This form general form Cv in general Cv equal to del E by del T constant v. So, this is all in general things. And that generally it comes 3, I can write like this 3 R, 3 R then 3. So, Cv generally it comes that 3 R. So, these then it come like this.

So, this contains also x T right x cube by kb T. And outside also T is there. So, it is just if we differentiate this one. So, this comes just I am writing the result, I will not do that, but please you can try to do that. See it comes generally, yes general behavior. It generally write this; so 3 R. So, from here 3 R; so rest of the part depends on T. So, this write fermi, that not formi, the Dirac function Debye function sorry Debye function, this is the function of T or I think this one function of theta D by T.

So, this Cv is basically 3 R and the Dirac function sorry Debye function. And Debye function at lower temperature higher temperature limit it becomes 1. And a lower temperature limit you can see it will be in this form. So, what is that form of this Debye function, phi ds? So, that just I will write it; so 3 T by theta D cube. So, it is the 3 T by theta D cube. And then integration will be there 0 to theta D by T right, but this T was theta D by T and then export was there. So, it is the integrative differentiating. So, it is comes nearly x to the power 4 e to the4 x divided by e to the 4 x minus 1 whole squared and then dx.

So, this is the call the Debye function. And that is for Cv, and one has to find out this that is the general form of the of the of the integration. And so it is difficult to integrate it analytically. So, to get this whole curve one has to take numerical value. Numerically one can do it. It is may be computing through computer programming writing programming through taking help of computer. So, one can generate points one can generate points in this range basically. So, for higher temperature lower temperature one can take the limit and find out.

But this will give just if you numerically if you do this, just taking help of computer it is called numerical calculation. Not analytical calculation. While analytical is not possible. So, that is the advantage of taking doing the numerical calculation taking help of computer programming.

So, I think that is the history of this Cv specific heat of solid state. So development of this of this theory starting from dulong petit to Einstein theory, then Debye theory, it is only it is difference. You see the difference of the taking realistic model and including the quantum phenomena do empiric completely it was classical. Then Einstein consider it the quantum oscillator having quantized energy level. And, but he considered this only one frequency in all atoms are oscillating independently. So, that is not realistic assumption.

So, for any theoretical calculation we take model, we have assume it because taking just real system which is difficult to calculate difficult to do analytic analytical calculation. So, that is why one has to assume some points, one has to consider someone has to simplify the system assuming or thus or some it is called hypothesis or postulates. So, which has no direct proof, but if I consider some assumptions some postulates some hypotheses, and then comes based on that if I develop a theory. And if that theory fit my experimental result, then we tell that this model is correct. So, this model is close to physical model, close to system. So, systems behave like this model.

So, there is a very important and does this theory develop we trying. So, that Debye finally, Debye could be explained nicely, but there are still some deviation of Debye theory. For especially for low wavelength elastic wave of low wave length when wavelength is low then it can see the atoms. So, it will not see the solid as a continuum medium. It will see the discreteness; it will see the atoms are arranged. So for that basically one again has to calculate using that discreteness of the solids. And that can be done nowadays that way only we deal with the system, but I will not go into that.

So, now what I want to tell you, this lattice vibration always we tell nowadays in terms of phonon vibration. Lattice vibration is nothing, but the phonon oscillation of phonons. So, lattice vibration is expressed in terms of phonon. Elastic means lattice vibration means that gives the elastic waves or elastic wave propagates through the solid. And then it vibrates, then solid vibrates as a whole it vibrates vice versa.

Now, this elastic wave, it can be expressed in terms of phonon. So, that is just similar concept of photon. What is photon? That you know this light there is a photoelectric effect right. So, light wave electromagnetic wave.

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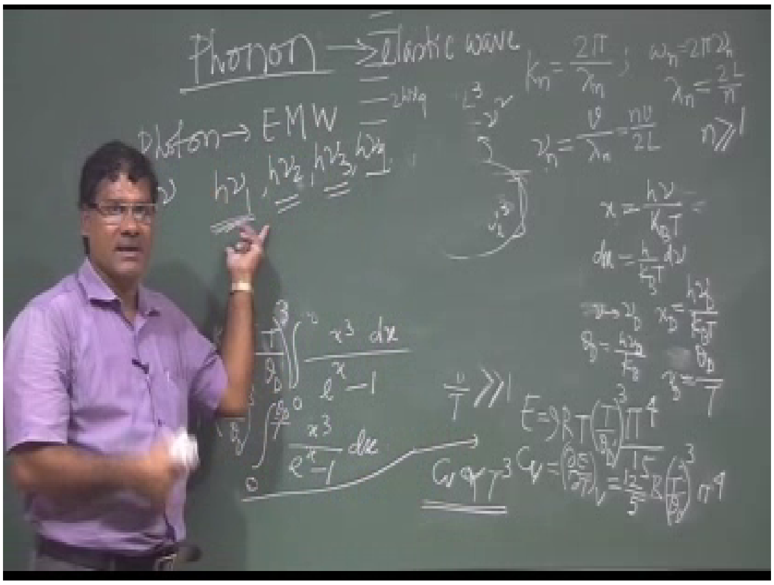

So, I will give some preliminary concept of phonon. It is equivalent to photon; so photon in case of electromagnetic wave. And the similar way this phonon is basically the in case of elastic wave. That is the difference; so basically this photon or phonon that is basically the quantization of the wave. Or quantization of the energy of the wave right; so that is the photon or phonon. In case of quantization of the electromagnetic wave that is photon. And in case of quantization of the elastic wave, then it is phonon.

Basically, energy of a vibration of a lattice vibration; what is that- energy of the lattice vibration. So, lattice vibration is expressed in terms of mode of vibration so each mode of vibration having particular frequency, right? Each mode of vibration is having particular frequency. So, phonon is defined as the quantization of the lattice vibration, is defined as the energy quantization of the energy of the lattice vibration; means quantization of the energy of the elastic wave, because this mode of vibration lattice vibration in different mode. So, mode of vibration is nothing, but the elastic wave. Each mode is a one wave having different frequency and wavelength right.

So, then I can tell lattice vibration or elastic wave or mode of vibration, they are quantized in lattice right. So, this is the quantization of this lattice vibration in terms of mode of vibration, they are basically quantized. And they are quantized and each quantum of a particular energy quantized. So, that is h nu smallest quantum energy that is h nu there is the phonon; that means, a mode of vibration or an elastic wave is made of phonons. An electromagnetic wave is made of photons. Means one electromagnetic wave may not be one phonon one photon. So, it can have a number of photons. So, few numbers of photons together gives a electromagnetic waves. Similarly, few number of phonons gives a elastic wave, gives a mode of vibration.

So, now as I told you that each mode have a particular frequency and that corresponding to that frequency there is energy h nu right. Now in ground state it is energy may be h nu, because this frequency cannot be changed h nu, but when it is thermally excited due to external excitation, that mode it can go at higher energy level. So, that higher energy level as I told this like harmonic oscillator. It has h nu, 2 h nu, 3 h nu, n h nu.

So when electromagnetic; when this elastic wave or mode of vibration is in nth energy level; so it is energy of that mode in down state. It was h nu in excited state it nth energy level, when it is exactly nth energy level. So, it is energy is n h nu. So, then we tell that this excited mode of vibration or this elastic wave, it consists of n number of consist of n number of phonons.

So, in crystal vibration, lattice vibration, crystal vibration we have seen 3 n number of modes. So, 3 n number of frequencies are there. For each mode if it has ground state energy, it is smallest energy in terms of this quantity h nu 1, h nu 2 h nu 3 h nu 4 right. So, these are the energy of each of different mode having different frequency right. And now each of this mode it is a lowest energy. It has own energy higher energy levels likes harmonic oscillator it has higher energy level.

So, it can stay it can when it is excited. So, it can go in any other higher energy level of it is own energy level. So, these are in terms of, this will be 2 h nu 4, but in this case 2 h nu 1. So, it has own energy higher energy levels. It has own higher energy levels; so this at a down state. So, for each mode have it is, each mode at down state. So, as a whole this is a whole wave elastic wave. We can consider when it is ground state. We can consider it is one phonon. It consists of one phonon when it is at higher energy level nth energy level. So, it contains n number of phonons; these also n number of phonons. These also it higher state n number of phonons.

So, each mode can have any number of phonons. So, number of phonons is tells the it excited state. So, at down state each mode of vibration, lattice vibration equivalent to elastic wave is a phonon. It has 3 n number of types of phonon. Type means this phonon at down state. This 3 n number of modes are there so that we can tell that 3 n number of phonons are there. All of them are having the smallest energy the quantization of the energy; so smallest energy h nu 1, h nu 2, h nu 3, h nu 4 etcetera.

But each wave or mode of vibration, it can consist of few number of phonons when it is in excited state when it is in second energy level; so 2 h nu 1 or 2 h n 2 etcetera. So, it consists of 2 phonons, 3 phonons, 4 phonons, n number of phonons.

So, now as a whole my system, that crystal vibration; now I can tell that this I have this is nothing, but system of phonons. Phonons are having this h nu 1 energy, 2 h nu 1 energy 3 h nu 1 energy right; as each mode can have this different energy when it is excited. And this is another phonon for another mode; so this h nu, 2 h nu, 2 h nu to 3 h nu 2 etcetera.

So the all mode are in down state; that means, the system is having 3 n number of phonons having the lowest energy. So, each wave elastic wave is one phonon, but each phonon have probability to go in any excited state. And that excited state it expressed in terms of some number having the same quantization of the energy either h nu 1 or h nu 2 or h nu 3 whatever. So, this lattice vibration is that is why, I considered it is system of phonons.

So, I think that is much I could tell you just to give you the idea of phonon. So, whenever this scattering from the lattice vibration, that we just tell scattering between electron and phonon; scattering between electron and lattice. So, that we tell scattering of electron and phonon. So, I will stop here.

Thank you for your kind attention.