Solid State Physics

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Lecture – 56
Thermal Properties of Solid (Contd.)

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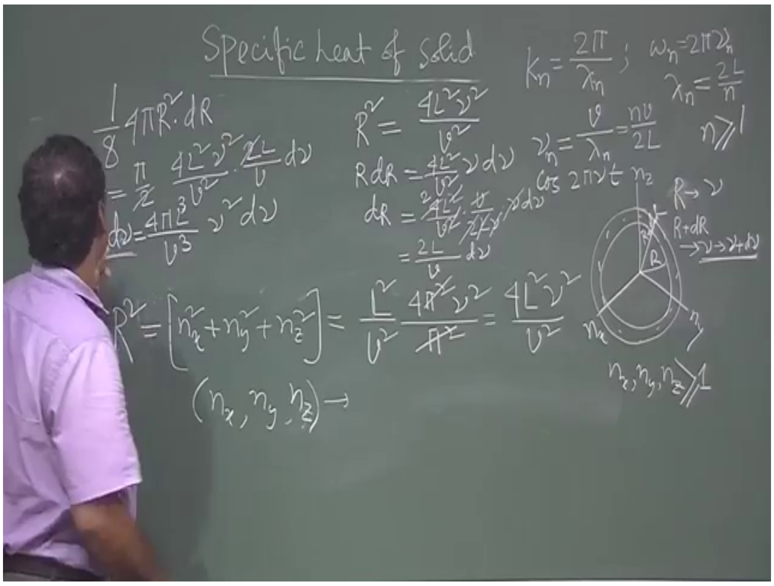

So, I will continue our calculation. So, what we have seen that this density of states, density of modes of vibration is this, in the range of frequency nu 2 nu plus d nu. remember that it is not. So, it is this is not a volume of physical; it is not a physical volume. So, this volume occupy the number of modes, which are having the frequency nu to nu plus d nu. So, what will be the total number of modes in that frequency range. So, that is this, that is, this is the total number of modes in that frequency range.

So, having frequency nu to nu plus d nu. So, here; so, this is the; then we can tell that this is the frequency, this is the number of modes, having frequency nu, and the small frequency difference range d nu. So, if I multiply d nu. So, I will get total number of modes. So, then this part we can tell, this is the density of modes, means number of modes per unit frequency.

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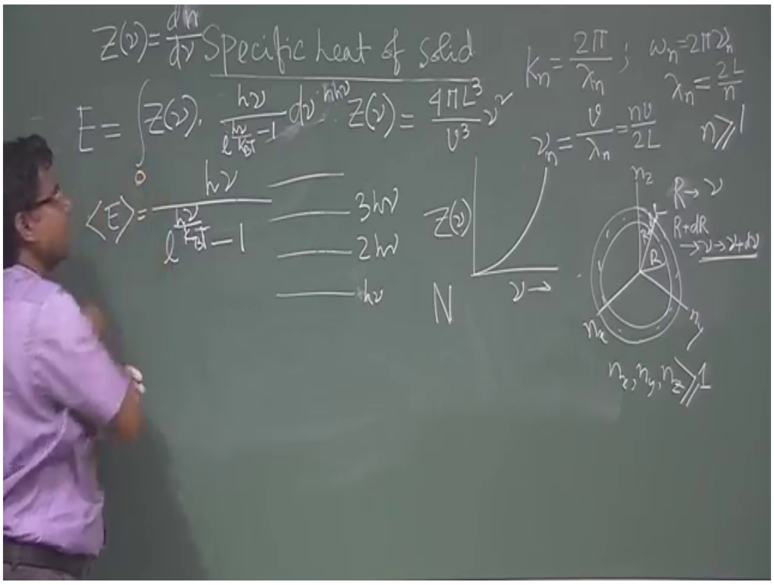

So, this basically density of states is defined z nu, it is basically defined d dn, say dn; this number of modes, or one can give N or whatever dn by d nu. So, that isthat is the per unit frequency how many number of modes.

So, do notdo not be confused with the physical volume. So, then we will think why I have not divided by the volume of 1 mode. So, we got the density of modes at frequency nu. So, I do not need them, I need only. So, I got everything in terms of frequency. So, I got density z nu, density of modes at frequency nu equal to 4 pi L cube by v cube velocity, so nu square. So, here you can see this density of modes varies as a function of, square of the frequency, right.

So, higher the frequency, so this density will varies as a square of that frequency right. So, at higher frequency, density will be very high. So, it is basically if you draw this, so z nu, it will vary, like its parabolic curve nu square, so it will vary like this, right, so its the nu. So, what we got I have a solid crystal right, it is treated as a continuum medium instead of discrete atoms arranging in this material. So, that was the assumption of this device.

And now there; so, this system will vibrate as a whole, and we will get mode of vibration, different mode of vibration. So, each mode will have the frequency, different mode will have different frequency. So, first mode, second mode of vibration, third mode, n th mode of vibration, so their frequency are higher and higher and higher. So, now, what I want to do. So, this is my system, this system, solid system, I have expressed in terms of mode of vibration. I know the density of modes as a function of nu frequency, right.

So, I want to calculate the total energy of the system. So, each mode will have energy, because it has frequency nu, each mode will have energy, different mode will have deferent energy. Now each mode if its frequency is nu, so its energy will be h nu right, its energy will be h nu, but this mode, this energy may be the lowest energy or it may be the ground state of the, at ground state it is, it may happen, but when you will increase the temperatures, then what will happen. So, this mode, I have consider just one mode, I have many modes.

So, one mode of frequency nu, it is at say ground state having the energy h nu, because this nu is known to me. So, its energy is known to me h nu, but all the time it will not have this energy h nu; although its frequency is nu. So, it can, at when it will be thermally excited, when this thermal energy will given to the system. So, these modes will be excited in higher energy. So, each mode have possibility to go at higher energy at excited energy levels, excited states, having the higher energy level. So, it will occupy higher energy states, having the same frequency. So, its energy will change.

So, there are possibility of a particular mode to have different energy as a function of temperature, depending on temperature. So, what we have to do. We have to know the average energy. We have to know the average energy of each frequency, for each mode. So, now, what will be the energy distribution, having the frequency nu so, but it can have many energy levels. So, initially say its ground state it is here, so its frequency as energy is h nu.

So, what will be the higher energy states for that mode, excited state, for that mode, so that one has to be. Now again in this case; d by assume that considered that solid is made of the atoms, atoms are arranged in the system, as a whole it is vibrating and giving different modes. So, if this solid our cube having length L of its side, if it contains total number of n atoms, total number of N atoms, and then basically all atoms are vibrating together.

Now, we considered that this total number of modes. So, as we have seen earlier that n number of atoms are there, in case of Einstein. So, if it is oscillating, like harmonic oscillator, then its frequency is same, frequency of all oscillator are same, but they have energy distribution n h nu so, for each nu, having the energy distribution following, having the different energy states. So, 2 h nu 3 h nu. So, this is the n h nu.

So, for a particular frequency, it can have these type of excited states, excited or a high energy states. So, here he assumed that, since our crystal solid is made of n number of atoms, and they are oscillating. So, they can take frequency there. So, they are taking this similar frequency, and mode is taking in frequency. So, oscillator also takes a frequency. So, for each mode having a particular frequency. So, it can have energy states, energy levels like harmonic oscillator, so that he assumed, that he considered. Although initially consider controllers with them, now he is considering this, there are atoms, atomic arrangement in the crystal.

So, as if this mode are coming from the harmonic oscillation of the atoms; n number of atoms; so, that way he make it equivalent, each mode is equivalent to a, each mode is equivalent to a harmonic oscillator. So, harmonic oscillator have this n h nu kind of energy levels. So, this each mode will have this, this kind of energy level . So, now, you remember that. Now at a particular temperature, this mode, this mode can stay in any energy level in excited energy state. So, there is a probability to have here or here or here, here. So, there will be energy distribution, right. So, what was the energy distribution; like that is Planck's formula. So, for this particular mode, it has, it can stay in any of this energy states ok.

So, but probability is different to have in different energy levels. So, that distribution is basically well known, the distribution, energy distribution among these different states, different energy levels. So, this is basically h nu by e to the power h nu by KBT minus 1 right. So, this is basically Planck formula, and that isthat is the average energy of a harmonic oscillator that energy description, we got in Einstein theory also right. So, here this, it is the basically one mode is given to Einstein's one harmonic oscillator right. There he considered all harmonic oscillators are oscillating with same frequency. So, that isthat is why he multiplied with this, this is the average energy of each oscillators.

So, he multiplied with the 3 n number of oscillator, because having the same frequency right. So, here this is the average energy for each mode. In our case this is the average energy of each mode, each mode means for each frequency right. Now our modes, different modes are not having same frequency. So, they have different frequency. So, this is the frequency density, that number density or density of modes are this. So, I have to multiply. So, for this frequency what will be the energy, what will be the energy; e equal to this z nu into average energy is this. So, h nu by e to the power h nu by KBT minus 1. So, this is the average energy for frequency nu.

So, this is the density. So, if I multiplied, so these will be the energy for a mode, having the frequency nu. So, in that mode how many modes are there. These many modes are there each are having this energy, so I multiply it. Now I have to integrate over the different frequency, d nu. Now I have to integrate over the different frequency. Now zero frequency is starting from 0 to generally we take infinity.

But in this case now this d by (Refer Time: 16:28) that this, its frequency cannot be extended up to infinity. There will be some, there will be cutoff of the frequency so, that my total modes should not exceed number of 3 n. Why 3 n. This is the. So, this number of total atoms in this or lattice point in this solid cube we have taken, cube there number of this solids at this. So, each mode. So, I will get n number of mode, n number of atoms are there, so I will get number of modes also will be equal to that, like this if I have n number of atoms in case of Einstein we have seen.

So, n number of atoms. So, I will get 3 n number of modes, because of the 3 degrees of freedom in 3 dimension along the x along the y along the z. So, here also I will get for 3 mode, but each mode have 3 degrees of freedom. So, that isthat is why this will be total mode, number of mode. Means total frequency, number of frequency cannot exceed 3 n. So, I have to integrate this. So, upper limit I have to take, a particular frequency, so that this total. So, that the total modes should not exceed 3 n. So, that I have to find out.

So, basically this is the density of states, density of modes. So, density of mode is this. So, now, d nu.

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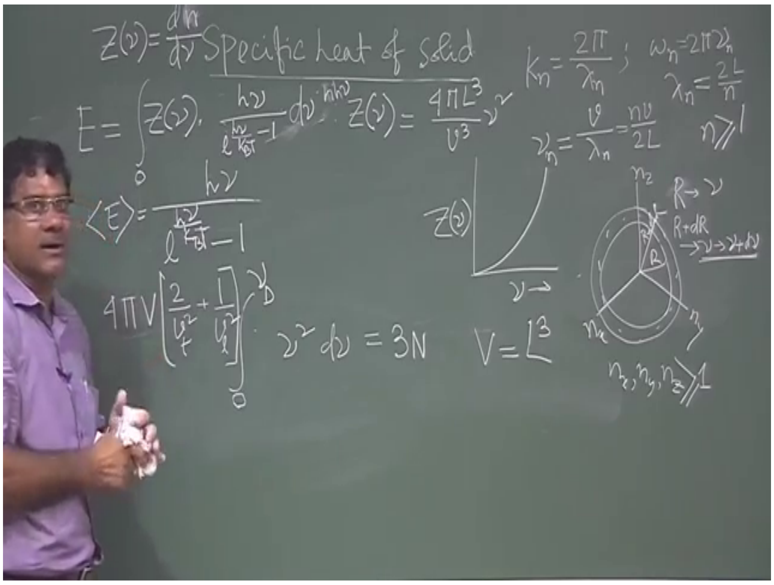

So, if I integrate I over the nu, I will get the total. So, this is the number of modes. So, I will get total number of modes. So, these total number of modes has to be 3 n. So, for that what will be the frequency limit? So, that isthat is called cutoff frequency, divide frequency. So, we write nu d. So, we will find out the value of nu d. So, here you can see 4 pi L square this nu square.

So, I have to integrate over nu square d nu. So, nu square d nu. So, outside it will be 4 pi L cube. So, L cube, we can write V volume of the cube right, and then divide by here one thing we have to do this 4 pi V. So, V is the, V is basically, V is volume which is L cube, it is L cube. So, and then here this in case of string kind of things or. So, here this we have 3 n number of modes, because this is for 3 direction.

So, if you consider the string, say string or this is your 3 dimensional material. So, I think easy to understand considering strings. So, if you take a string. So, this vibration along this direction. So, this basically tells is longitudinal mode, and vibration along this direction or this direction; that is called transverse mode. So, velocity in case of transverse mode, and in case of longitudinal mode are generally different, they are not equal. So, we have considered 3 n. So, since velocity are different. So, I have to write separately. So, this 4 pi or V cube, so basically here I will write two more transverse for, two mode are there.

So, 2 by v l. No, I think I can take just I am, so this is this. So, this is 1 by v cube. So, I am writing. So, 2 by v square plus 1 by v square right. Since velocity are different for transverse and this longitudinal. So, two are transverse. So, here velocity I am writing v t and here velocity I am writing v l longitudinal. So, since they are difference. So, we can be place one by v square by this. So, then find. So, from here if you differentiate it what you will get. You will get basically nu cube by 3 and nu d.

So, you will get, if you integrate it. So, you will get nu d cube by 3, see integration done. So, I will get this 3 here, it will be 9 N. So, 9 N, and then nu d. So, what I am getting. I am getting nu d, I am getting basically nu d nu d cube equal to 9 N by 4 pi 4 pi V and then this I can write as the, just write 2 by v t square plus 1 by v l square .

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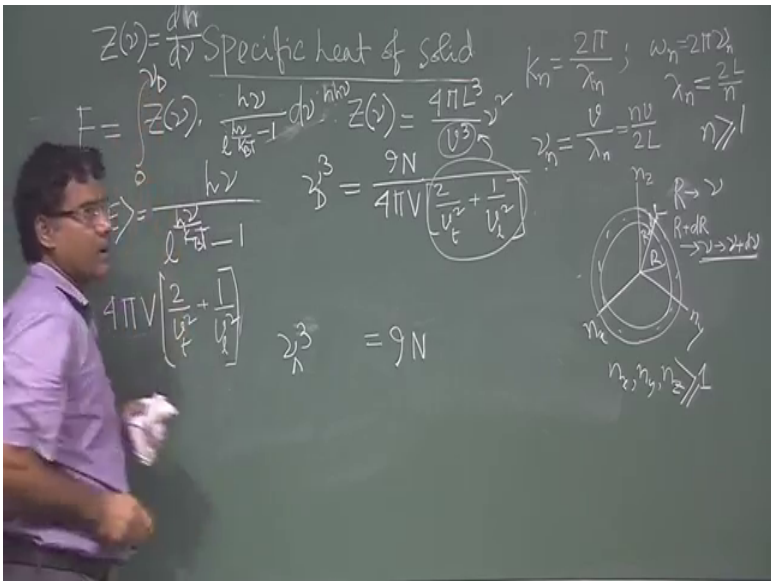

So, here also one should write this when you are considering 3 n. So, one should write this. So, this is basically one can replace this by this one, right. So, here also one can write anyway. So, when we need we will see. So, that is the. So, one can find out the upper limit of the frequency nu d, and nu d is known to us. So, nu d is such, it shows that this we have considered the upper limit of the frequencies are there, it should not cross this total number of modes, should not cross, yet it has to be equal to the 3 n.

So, now, this is the for nu value, this is the energy for this many modes right. Now for d nu. So, that will be the energy, for now, if the nu varies 0 to this. So, this is the energy for a mode of having frequency nu, and now how many modes are there with that frequency;

that is this. Now that will be the energy, this is the energy average energy for that mode, having the frequency nu that will be the energy. Now if I vary nu 0 to nu d. So, that will be the total energy of the system.

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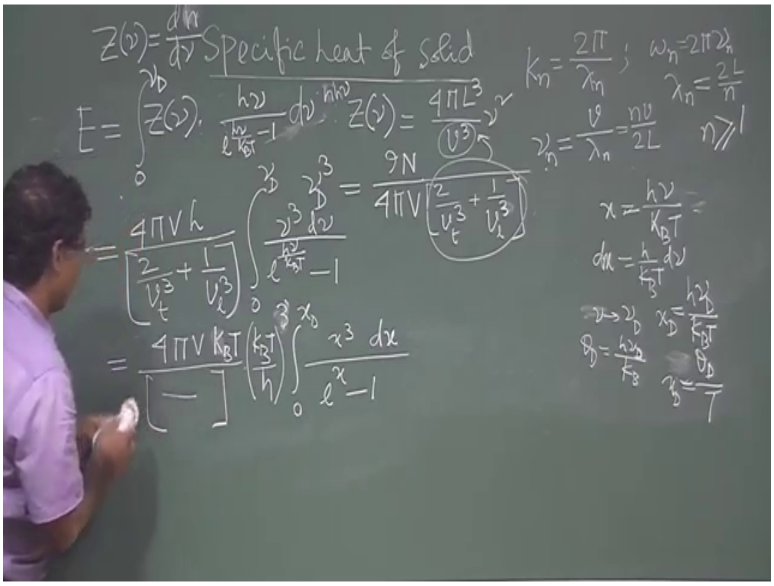

So, that I have to calculate. So, you replace e equal to, you replace this. Here I did again mistake it is v cube. So, this is then v square, this will be v cube, this will be v cube.

So, here you put this value what is z nu. So, this 4 pi V 4 pi v L cube is V divided by V cube is this 2 by V tan cube plus 1 V L cube. So, then what I have, here I have. So, then h also. So, this I am putting here. So, if I put that one here. So, this I am taking outside of this frequency, because its the independent of frequency, h is independent of frequency fine. Then frequency part I have to write inside, frequency part I have to write inside. So, that is basically, here I am getting, here nu squared is there. So, nu is here. So, it will be nu cube divided by nu cube, from here only nu squared is coming. So, nu, nu cube e to the power h nu by KBT minus 1 d nu. So, here I can write d nu.

So, these, for integrating this one I have to consider that. So, so let us take x equal to h nu by KBT. So, dx equal to h by KBT d nu right. So, when nu tends to 0, x tends to 0, when nu tends to nu d, when nu tends to nu d, x tends to h nu by KBT. So, from here. So, for x nu tends to nu d. So, I will get xD equal to h nu d by KBT. This xD here, if I define that h nu d by kb is theta d, d by temperature theta d, xD equal to theta d by t, where theta d is h nu d by kb.

So, this this now comfortable to us. So, here what I can do. Here your nu cube. So, nu is KBT by hx KBT by hx. So, this I can write 4 pi four pi v h by this term, then here I can write 0 to xD. Now nu I will replace by x. So, nu cube means I am getting kb by kb kbt by hx kbt by hx. So, K B T by h into x into x into x. So, x I will keep inside. So, nu cube. So, xs cube. So, this will be cube, fine then divided by e to the power x minus 1, and for d nu I will get kbt by h dx KBT by h dx.

So, KBT by h. So, it will be 4 into dx. Now here h is there. So, what I can do. So, 1 KBT by h, I could take out. So, keep it 3. So, KBT by h. So, h-h will go. So, here basically h it. So, here KBT. Now you see this part 4 pi v into this, if you take this side equal to 9 N by nu d cube, right. So, 4 pi V by this term equal to 9 N by nu d cube. So, I can simply replace this.

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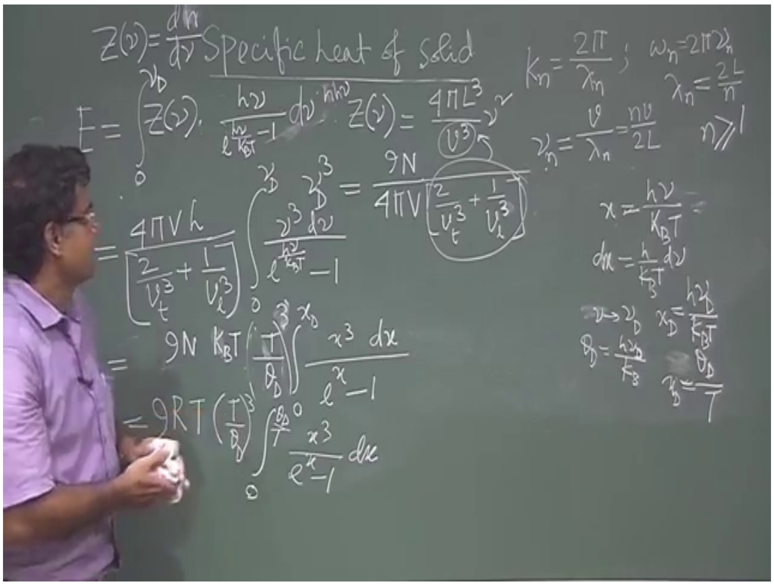

I can simply replace this by 9 N by nu d cube.

Now, here it is cube, this is cube. So, this nu d cube I can put inside. So, this is basically 9. So, N kb. If it is for 1 gram mode. So, I can write R, 9 R into T and this I can write as I defined KB by what is this h nu d by KB is theta d h nu d by KB is theta d. So, this KB you take here KB. So, this is theta d. So, this I can write T by theta D cube. So, then what I am getting t by theta D cube, and then this part is. So, 0 to xD, xD is nothing, but theta D by T.

So, this I can write theta D by T, this limit theta D by T, and then x is basically theta D by T, x is theta D by T. So, this I can write theta D by T, this you keep it. So, this is xs cube by e to the power x minus 1 dx. Now we see this is the total energy of the system. So, now, we have to integrate this one.

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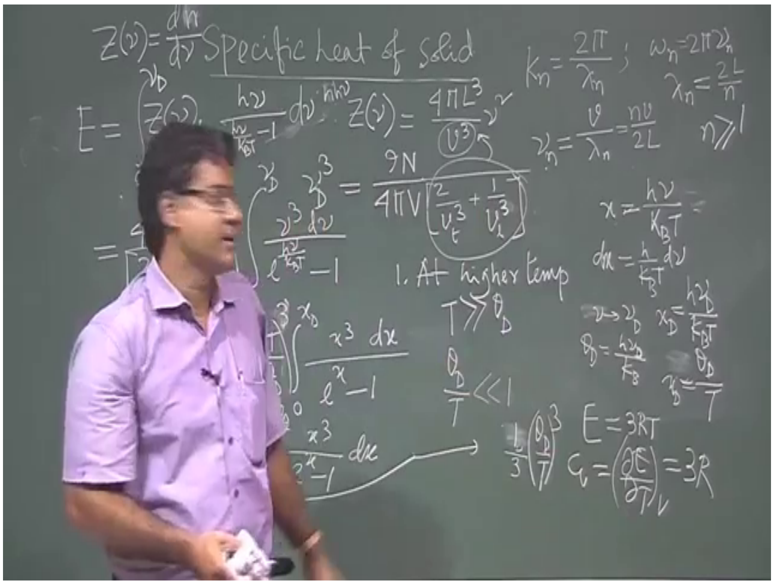

Now, let us consider this first case, this at higher temperature. So, as I mentioned that there is no meaning of higher temperature, lower without any reference.

So, here at higher temperature, means temperature is very greater than theta d. So, then one can find out that theta D by T; that is nothing, but x xD theta D by T; that is xt. So, theta D by T is very. So, T is very higher than theta D right. So, this will be very less than 1 and this is nothing, but this x right, its x. So, basically x is going towards the x, is very small right

So, because xD ultimately, here this xD will come right after integration. So, that is why this term, this x, it is a. So, this, its smaller term, under this condition its very small to compared to 1. So, this you can e to the power x minus 2. So, its equivalent to x 1 can write this is x. So, this is x, and this is xs cube right. So, it will be x square dx. So, x square dx means xs cube by 3. So, xD cube by 3. So, this will give me this integration. This integration part will give me for lower temperature, for lower temperature it will give me xD by xD by 3. So, xD. What is xD, theta D by 3. So, xD is nothing, but 1 by 3 theta D by T cube right

Now, here we see T by theta D cube, here theta D by T cube. So, this will go, and this 3 is there. So, it will be 3. So, I will get basically 3 RT. So, at low temperature total energy I will get E equal to for low temperature, I will get 3 RT. So, CV equal to del E by del T at constant V right. So, it is equal to 3 R, it is equal to 3 R. So, at higher temperature, its correctly we are getting 3 R. Now at lower temperature. So, I think I will stop here, and then I will continue in next class.

Thank you for your attention.