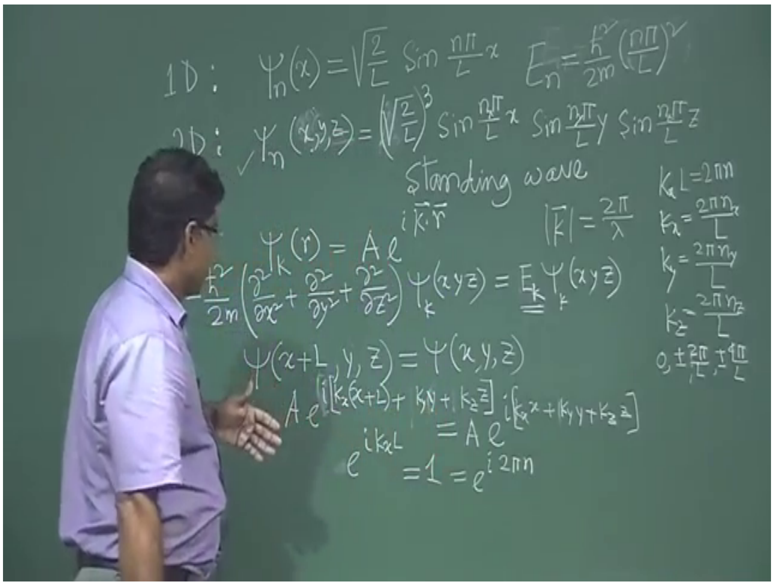
Solid State Physics
Prof. Amal Kumar Das
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture - 37
Electrical Properties of Metal (Contd.)

So, for Fermi free electron gas, it is also called Fermi gas because electrons are Fermial.

(Refer Slide Time: 00:34)



So, for 1 dimension electron gas, we have seen that wave function psi n equal to square root of 2 by L sin n pi by L x and corresponding energy n; n equal to h cross square by 2 m n pi by L square, right slightly below. So, n equal to h cross square by 2 m n pi by L whole square.

So, just for simplicity we have considered one dimensional electron gas, but in reality it is three dimensional, it is three dimensional a metal piece if you take. So, it has three dimension. So, it has x, y, z along x, y, z if its length is L x L y L z again for simplicity if you consider this it is q. So, it is L. So, for that that wave function one can write. So, this for three dimension r one can write or let us write x, y, z; x, y, z. So, this is some constant. So, here square root of 2 by L to the power cube; then sin n pi by L x.

So, one can take L x, L y, L z. So, then square root of 2 by L x into square root of 2 by L y one has to write. So, there sin n pi by L y sin n pi by L z. So, this now it is not n. So, one has to write n x, n y, n z. Now n x, n y, n z, are three quantum number 3 quantum number and this wave function is basically its standing wave, it is a standing wave it is a standing wave. So, using this wave function and corresponding energy one can find, but it is a there is a simple way to deal this three dimensional problem.

So, that s instead of taking this standing wave if one can take this travelling plane wave then it will be simple simpler and easy to calculate. So, this you know this planar wave travelling wave its one can write for three dimension A e to the power i k dot r, right. So, omega t part is not necessary because it is we are interested this independent of time. So, so k is basically wave vector k is wave vector that is 2 pi by lambda now if we take this; this is a wave function this is a wave function for three dimensional case

So, this should satisfy the Schrodinger equation. So, Hamiltonian that is for three dimension minus h cross square by 2 m del 2 by del x square plus del 2 by del y square plus del 2 by del z square then apply on psi k r or x, y, z, right or x, y, z one can write equal to e. So, i am writing this way e k. So, just to differentiate earlier or we can write we can write e k psi k x, y, z or psi r. So, this is the one condition that this wave function has to be the Eigen function where this e k will be the Eigen value.

So, then this will be called this Eigen function Schrodinger equation. So, so here now again the similar this we have to get the appropriate form of e k and appropriate from of wave function for our problem. So, in case of traveling wave it is not standing wave it is. So, so travelling wave means it will in space it will have the periodicity right in terms of lambda; so, in these our case in terms of length L. So, this wave function has to satisfy that condition that periodicity in space.

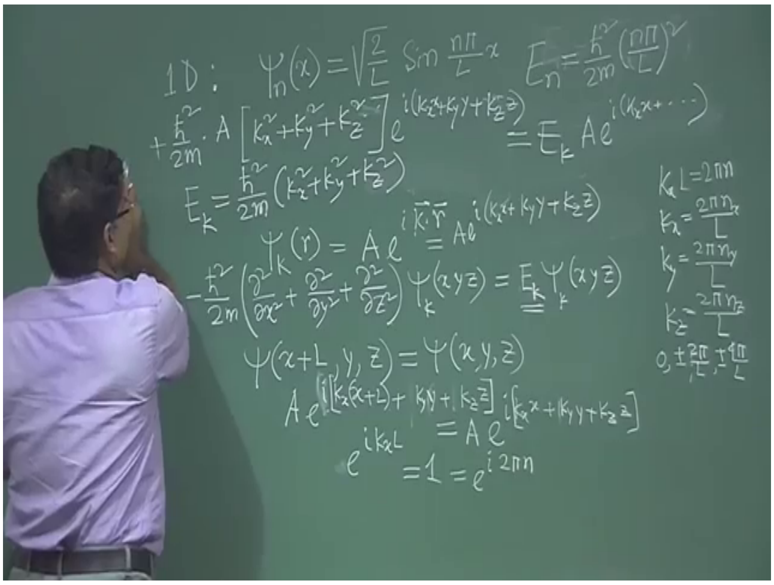
So; that means, basically psi; psi x periodicity as I told this in terms of L y z has to be equal to psi x, y, z, right. So, that periodicity has to satisfy it and similarly for y z, one can one can also check this psi x y plus L z equal to psi x, y, z and for z also one has to one has to check the condition of periodicity. So, this is the; if this is the form. So, you can see that wave function A e to the power i. So, I can write k x; k x x if I write k x x plus other terms will be there y z, right.

So, this x plus L other term one can write plus i k y k y y plus i k z z equal to for this other side A e to the power i k x x plus i. So, let me put in without putting i here you can put i here. So, here k x, k y y plus k z z so; obviously, from here you can tell that e to the power i k x L it has to be it has to be one right. So, so one means e to the power i can write i 2 pi n i 2 pi n it is not n pi check it is not n pi, then it will not be one sometimes it will be minus 1 cos term is there cos cos theta plus i sin theta, so cos pi is minus 1.

So, you have to avoid that one. So, that is why we have to write 2 pi n. So, it is always in terms of 2 pi four pi six pi. So, it will be cos cos 2 pi n will be one. So, from here what we are getting we are getting k x L equal to k x L equal to 2 pi n 2 pi n; so, k x equal to 2 pi n by L. So, similarly one can get k y equal to 2 pi n by L k z equal to 2 pi n by l so put n x n y n z; so k x, k y, k z.

So, they can vary as a this all 3 they can they can take value 0 plus minus one plus minus 2 pi by L plus minus four pi by L right. So, that is the k x, k y, k z, they can vary like this. So, see in this; this is the wave function just arbitrary wave function we have considered. So, now, what about k; so, here we have k x, k y, k z. So, we could find; so from periodicity condition. So, if you found that the L of k x, k y, z will be like this. So, then our Schrodinger equation here this form is.

(Refer Slide Time: 16:54)

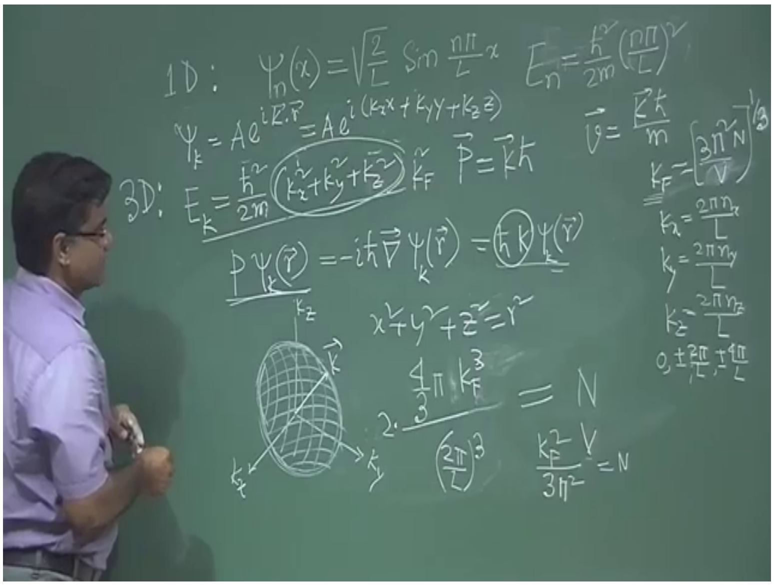


Now, this psi this form is now A e to the power i k x x plus k y y plus k z z where k x, k y, k z can take these values.

So, for now from this now this wave function has to satisfy this equation Schrodinger equation, right. So, now, you apply here if you apply this Hamiltonian on this wave function. So, what you will get I will get. So, this is just you put this on here. So, what i will get I will get minus h cross square by 2 m, then it is a will be there and. So, del 2 del x square. So, it will act only on this, right. So, I will get basically k x square k x square I will get k x square, then from here I will get k y square then k z square then this wave function e to the power i k x plus k y k x x plus k y y plus k z z equal to e k A e to the power i k x x plus, etcetera, right.

So, from here what you are getting you are getting e k energy expression e k equal to. So, I did mistake. So, i is there. So, i k x will come out when you will differentiate i k x. So, i k. So, twice i k; so, i square minus 1 i square minus 1; so minus 1 minus 1 minus 1 minus and this minus plus. So, it will be h cross square by 2 m k x square plus k y square plus k z square right. So, that will be the energy. Now, you can write in this form k x equal to 2 pi n x by L, right. So, this is one form this is one form you can also write in terms of 2 pi n x by L, etcetera, right and wave function already I think that is the wave function psi k.

(Refer Slide Time: 21:17)



So, here; so, for three dimension three dimension, right. So, psi k equal to A e to the power i k dot r that you can write e where this I do not need this. So, this is the result we obtained for three dimensional case; so, this. So, this is the wave function for this problem now if we apply operator on this wave function. So, you will get Eigen value. So, on this wave function I applied this energy operator I got this one if I apply momentum operator if I apply momentum operator on this. So, I should get momentum.

So, this where this p momentum operator; So, this operator form is this i h cross del equal to this. So, this will give you k basically k k i k, it will give you i k i k. So, i square minus 1 this minus. So, h square h cross and this will give you k and that will be there that will be there that will be there psi r. So, one should write in principle k. So, as if this you have applied momentum operator on this wave function this wave function right and you are getting this.

So, this is the Eigen value of this operator. So, momentum you are getting momentum you are getting p equal to k h cross k h cross where k is where k is square root of this right k is square root of this and in terms of L, you have this k x, k y, k z value. So, if you want to the want to get the velocity. So, one can write this way also momentum one can write this way also if you want to get velocity. So, velocity is. So, momentum by m; so, velocity will be k h cross by m.

So, if we look at this expression. So, I can tell this is a. So, this is the equation this is the equation of sphere in k space, right because you know sphere equation x square plus y square equal to some r square that is the radius of a circle this equation of radius equation of circle and this in three dimension it will be sphere, right. So, this is in x, y, z coordinate in space this is the sphere. So, this is the similar form right this is the similar form. So, this in k space in k space momentum space Fourier space its. So, reciprocal space in case of a crystal, right it is reciprocal it is reciprocal of this length you see. So, it is the equivalent to reciprocal space k space in k space. So, it will represent sphere it will represent a sphere.

So, basically coordinate system in k space this is the k x this is the k y this is the k z, right. So, this will k x square plus k y square it will be present as sphere it will be present as sphere anyway. So, it will be present as a sphere in k space where this is the k value this is the k vector k vector and it has it has component k x, k y, k z, right. So, so this volume of this sphere will be volume of this sphere will be four by 3 pi k square right k square right and if we look at this value k x, k y, k z, k x, k y, k z. So, here this smallest value is. So, if we take origin this 0 value k x, k y, k z origin 0 value and then next next one is 2 pi by L then four pi by L

So, the smallest one is 2 pi by l. So, as if along k x, k y, k z; so taking smallest dimension 2 pi by L in all 3 direction you will get a elementary volume you will get a elementary volume for a particular set of k x, k y, k z, right and this k x, k y, k z in that volume you will get only one set of k x, k y, k z or a particular wave vector a particular wave vector k and corresponding k x, k y, k z value smallest value will be this. So, that is the smallest volume is 2 pi by L whole cube,.

So, within this sphere in that volume how many elementary volumes are there which these volume will represent basically one energy state one orbital's because k x, k y, k z. Now it is a nothing, but the quantum number it is nothing, but quantum number, but with different just multiply with some factor; so its k x, k y, k z quantum number; so, for a particular set of 3 quantum number k x, k y, k z. So, its minimum volume is this; so, now, in this volume total in this volume total in this volume how many states will be there how many orbitals will be there.

So, this k square I can replace 2 pi n by L square right 2 n pi by L square and. So, I have to tell something more about this. So, this sphere we have considered here k x, k y, k z. Now if I consider that the electrons are orbital's are we have n number of electrons. So, now, electrons are placed in these orbitals. So, how many orbital's, we need to fill how many orbital's, we need to fill or to accommodate this n number of electrons. So, so if we if I tell that this is the sphere. So, whatever k value whatever k value.

So, here I will go back here to k square. So, whatever the k value at which k value up to that k value if we write that is k f up to that k value if electrons are filled. So, this is the uppermost orbit orbital's that is where electrons are filled then that is called Fermi level and if its momentum is k f now up to this Fermi level all electrons are there all electrons are there. So, then how many levels are there.

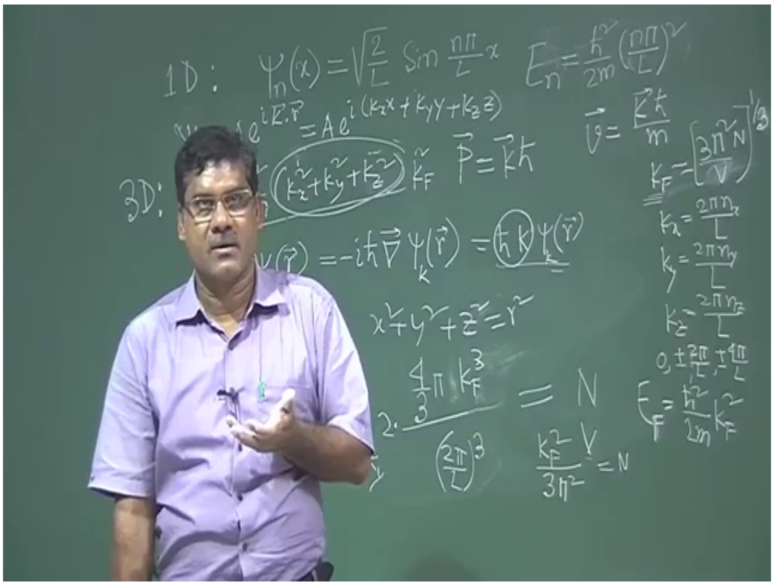
So, from here I can say that is the volume of this sphere and for each orbital this is the volume. So, that will be the this divided by this that will be the total number of orbital's and each orbital will contain 2 electrons one is spin up and another is spin down. So, I will multiply with 2.

So, then this will be equal to N proper number of electrons. So, here this k f we have chosen the uppermost; the orbital's which are filled and that is energy of that orbital's is all the Fermi; Fermi energy and this Fermi wave vector. So, from here you can find out this value of k f you can find out the value of k f. So, just do it. So, it will be 8 pi by 3 and here 8 pi cube. So, you will get you will get you will get here pi square; so this 3. So, I will get k f square divided by divided by 3 pi square, right and then this will go up L cube L cube this will be equal to N.

So, basically your k f that is the value total this is the total number of electrons N and then from here you will get k f, I will get k f equal to 3 pi square 3 pi square by 3 pi square N by L cube k f square. So, this is 1 by 2. So, I think that is the value of k f 3 pi square N by V. So, L cube one can write V volume L cube one can write V volume one can write V volume. So, here I can write V volume. So, your 3 pi square N, but this is I did mistake it seems k f cube that is what this volume is not k f square k f cube. So, that is the reason, I was not able to get the correct result k f. So, it should be 3 it should be 3 right 3 pi square N by V 3 pi square N by V 1 third s.

So, here you can see this k f it depends on the density know N by v. So, total number of electrons in volume V; so, this is the density number density right. So, this k f is a proportion to the cube root of number density cube root of number density. So, if k f is known. So, what will be the; what will be the e f; e f this is the k f. So, these if I take this one as a k f for Fermi and k f; so k f square.

(Refer Slide Time: 37:59)



So, your energy; your energy e k your energy e k e naught k e f for Fermi one; so, that will be h cross square h cross square plus 2 m and this is k f square k f square and k f square is this. So, if you put this one again it will be two-third, right.

So, again this energy is again energy will depend on only on density right number density of orbital’s or energy levels. So, I will stop here we will continue in next class.

Thank you for your kind attention.