Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

Prof. Mahendra K. Verma

Department of Physics

Indian Institute of Technology, Kanpur

Week - 12

Lecture – 72

So, just the course summary, just 5 minutes or 5, 10, well I guess I will finish in 5 minutes. So, it was a very broad course intentionally, I did want to give a broad picture at least I mean a lot of things I learned myself. So, that is so let me just tell you the what we did summary. So, this interaction of fields, we had dealt with particles. Of course, there are people who do like Ising spin, they work with spins, but then they make a field, but we just worked with fields, Phi-4 field, velocity field, temperature field. So, this was in this course at least it was interaction of fields, classical or quantum, equilibrium or non-equilibrium.

So, I suppose you got basic ideas about all this different aspects, though quantum we did not get too much into detail and I hope some of the projects will give us some idea on to the class when you will sit in the class. So, just to tell you that this is not I mean there everybody has done field theory, it is not that this is first time you heard of fields. So, electromagnetism is a field theory, Maxwell equation is all fields. Of course, you also work with fields with particles, but waves, electromagnetism is field and interactions are waves which can get non-linear which is not I guess you have not done too much of that or probably nothing of that.

So, that is a non-linear field. So, the quantum optics people work with interaction of fields or this electromagnetism wave coming through the atmosphere. So, they get scattered and there is a non-linear effect. So, that is part is a higher order fields or quantum mechanics, you have wave function and you have discussed with fields, but typically that was linear I believe. So, gravity also.

So, if you do non-linearity course that is a field, but in this course we covered these stuff. So, formalism of I did first started with formalism Lagrangian Hamiltonian symmetries conservation laws and I hope you understood this part. So, you can write down Lagrangian of fields. Then we can derive given the Lagrangian no other theorem is very important theorem. Then quantization.

So, we can quantize these fields as soon as you bring in quantum mechanics then these fields are quantized. So, commutation relation comes which is not there for classical fields. So, in classical fields phi k, phi p I mean these are you can just multiply no problem, but in quantum mechanics you cannot just multiply these are operators. So, they become operators and time order products and commutation. So, they are commutation relations for these.

So, quantum calculations have in additional complication because of quantization condition. Perturbation approach I hope you got a gist of how to do perturbation perturbative calculations and I think basically from the middle of the course we have been doing huge number of examples of perturbation. So, Phi-4 theory is a very important theory and I did in detail Wilson theory of phase transition. So, we could compute those exponents which are very important exponents and which has won the Nobel Prize for Wilson for sure and it led to lot of new discoveries. So, renomination group in fact after Wilson it got a new interpretation.

So, I hope you got the gist of that and then we did mass in charge renormalization of Phi-4 theory, but that idea can be easily generalized or it has already been generalized, but you can you can approach that problem of QED mass renormalization. QCD is more complicated, but QCD also I gave you some idea that it is similar idea works for QCD as well. This is called asymptotic freedom right. So, charges so, in QED if you separate the charge they become free or you bring them closer the interaction increases, but what happens in QCD? When we bring them closer it becomes free ok. So, the quarks are when they close by they are free particles, but you would separate them out then they become the interaction increases finally, right.

So, quarks cannot be separated because it requires infinite energy to separate quarks. All that can be derived by this renormalization we can compute the coupling constant how it changes with way number. Then we started non-equilibrium field theory and fluctuation distribution theorem by example and then we went to hydrodynamic turbulence, scalar turbulence I did today, EKPZ, weak turbulence and I did I hope you got the idea that what is the difference between equilibrium and non-equilibrium ok. So, equilibrium has no frequency ok no time dependence, but non-equilibrium has time dependence. Equilibrium has no energy flux, energy flux is 0 in equilibrium, but in non-equilibrium we have energy flux typically.

It need not be constant, but we will have energy transfer across scales and that leads to breaking of detailed balance. So, that is what I did mention in the class, well not mention I think I did in quite a bit of detail ok. So, similarity in contrast. So, like for example this frequency dependence I emphasized on many many times also KPZ RG was very similar to Wilson RG except there was a frequency dependence here. Of course, that made the life more complicated, but if you know Wilson RG very well then you can get the idea of KPZ RG.

In fact, we can do all that stuff is very very similar approach was adopted by the authors and it is now textbook material you can do that. So, the issues which are still part of unsolved problems and there is their activities anisotropy as I said if I apply temperature gradient then the field theory becomes anisotropic or apply gravity then doing field theory is complicated. Many fields like MHD has magnetic field or polymers that makes a problem. Now, there is a new direction called exact renormalization proof, exact R G. We did not do it, but this is another area.

Non perturbative tools in fact this connected I did not do in the class and these are things which I we did not even touch upon quantum gravity and string theory. So, these are challenges if you are interested in field theory and so I just wish you luck. So, apply to new areas and these are open fields. Unfortunately not many people are working on this right now in analytical field theory. In turbulence there are very few people who work on turbulence on field theory now.

Essentially they have gone into applications like soft matter applications or simulations. So, yeah in some sense the opportunity for people who are interested in analytical work that you do not have much competition, but it is also true that it gets hard to publish these papers because it is not fashionable anymore. At least well yeah I would say it is not as much in fashion as before. Thank you.