Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

Prof. Mahendra K. Verma

Department of Physics

Indian Institute of Technology, Kanpur

Week - 02

Lecture – 07

So, Fourier transform, we will look at the formulas again. This will be important for our all the lectures in fact. So, it is defined, we are in field theory we will use infinite box. Unlike finite time series or in turbulence we have finite box, but we will use box to infinite in d dimensions. So, d could be 1, 2, 3 or 4 as well or even fraction. So, for Fourier series, Fourier transform one condition is that f x must be square integrable, that is a condition.

So, that means the integral of mod f x squared f x can be complex, like wave function in quantum mechanics is complex. So, mod f x squared integral is finite. So, this is a condition. So, please make sure that this condition is valid.

Sometimes we let go like Fourier transform of a unit, you know constant, you say delta function that is not square integrable, but still in physics we let go of few things. This is sometimes we violate it, but strictly speaking mathematicians would say that we should have this condition. Now, the definition is we are going to use for this course f x is a function in real space, x is a real space coordinate x, y, z and k is a wave number k x, k y, k z. So, I represent the Fourier transform of f x as f hat k, we put a hat. Right now this means just a complex number, but later on we will this hat will also be used for operators.

But right now we just think that that is a complex number. So, we can go from Fourier space f hat k to real space, real space the function can be complex in real space by multiplying this by exponentially i k x, a k dot x is a dot product and I integrate with this d k divided by 2 pi to the power d. This 2 pi to the power d is because of normalization and so this is the integral. So, now you will always find this kind of combination in quantum field theory. In 1 d it is going to be just d k by 2 pi.

So, you will link I mean you basically blindly put d k by 2 power d. In Fourier so this is called inverse transform, you go from Fourier space to real space inverse transform. Now, I can go from real space f x to Fourier space. So, I just have to do integral d x e power exponential minus i k x, please note the minus sign. So, that is the notation I will follow for f hat k to f x plus sign and f x to f hat k minus sign, that is the notation we will follow in this course.

There is no 2 pi division here. Now, this normalization is if I in fact f hat k this definition if I do d k by 2 power 2 pi to the power d this should give me if I in fact integrate this know I should get 1 no sorry if I do this I should get back f x. So, you can do it you can check it yourself that this is consistent we can invert it. So, you play around with this. This is standard stuff in quantum mechanics you might have done it or mathematical physics this standard definition. We follow this definition in this course.

Now, this is for real space dimension to Fourier space. Now, sometimes we will also need to do the Fourier transform or time series is function of time f of t. So, time and frequency. So, these are also Fourier transform. So, you have time series we can do the Fourier transform that will give you frequency.

So, if our time series f t I integrate from 0 to t. So, time series starts from t equal to 0. Sometimes people start with minus infinity to infinity dt but then we assume that f t is 0. So, if you see minus infinity in to infinity for t this t then my function is 0 in this region and then it may start like that f of t. So, for this part the sign is flipped.

So, this is a plus sign here and minus sign here. So, when I go from Fourier f hat omega to f t I put a minus sign. If you recall for my real space stuff I had put I k dot x for going from Fourier to real this is a plus sign here but here I put a minus sign. The reason is quite obvious that imagine a wave going in the x direction like this. So, what is this wave written as you may only write you might know that e to power i k x minus omega t.

So, k is positive the wave propagating along x direction k is positive know this is a positive momentum. So, k is positive and omega is also positive that is why x and t both increase with time well x is increase with time and time of course increases. So, the wave going in the forward in the right direction is given by that. So, this is a plus sign for i k x this one but minus sign for omega t. So, please remember this is a minus sign here and now inverse transform f t to f hat omega I have to put a plus sign.

So, this is when we just deal with time series there are times we just deal with time series but often we deal with both space and time wave function is function of space as well as time. Then I have to transform from f x t f is function of position and time to f hat f hat k and omega. Now this function is e to power i k dot x minus i omega t. So, they said for the waves. So, imagine that we have some function which is function of x and t and Fourier transform it is a function of love waves moving in time.

So, this is the amplitude f hat k. So, for a wave of wavelength k and also frequency is also I mean you also transform rather decompose in frequency space. So, we will see some examples in a while. So, this is both space and time. So, I just combined last two slides and we get here.

Now we can get so this from going from Fourier with k and omega to real and time but we can do the inverse I can get from f x t I mean my function which is in real space is a function of time I can go to f hat k omega and the sign is of course we need to flip it this becomes minus and this becomes plus. So, we need these definitions. So, I thought I will put it here. So, this is about it we can I can take some questions if you have any. So, this is fine.