Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

Prof. Mahendra K. Verma

Department of Physics

Indian Institute of Technology, Kanpur

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Lecture – 67

So, dg by dl is this. Well, I already know d lambda by dl, d by dl and d nu by dl, so I can derive it. Now look for the fixed point. Fixed point means I set it to 0. So, g is like strength. They call it strength of this stuff.

In Wilson stuff also we have coupling. But the slight Wilson was coupling really. But here it is a g. This g they are expanding.

And now we can, so one, you can see that there is one fixed point is g equal to 0. The g plus g cube, so g can be taken out. Other one will be, we can simplify g square is square root 2 by Kd. So, in fact, there is quite a bit of cancellations and there is some perturbation is also done. The way we did for Wilson, for these epsilon expansion, you can see that I would not exactly get this because these two will cancel with that.

And I think they are cancelling these two. So, there is some cancellation. I have not done this myself. I just take the result. So, the g star, one correction, well, I mean, we can do it, I am not sure.

But this is, the other fixed point is for this equation is square root 2 by Kd. Now if I look at graphically, it looks like this. Now for d equal to 1, for this, we can do it for different d's. Now for d equal to 1, now you have to see which is stable fixed point and which is unstable fixed point. I did it for Wilson, I want to save time because I want to start another topic.

So, how do I check the fixed point which is stable or not? I look at the slope near the fixed point. So, basically we compute the correction, well, if I change L, then do I go away or do I come closer? So, it turns out for d equal to 1, this g 1, this is a unstable fixed point and it goes here to g 2, it goes there for d equal to 1. I am skipping the steps, but that is a stability analysis. We did it for Wilson. So, same thing can be done here.

It is exactly following the Wilson procedure, but it is for dynamical system which has frequency dependence or time dependence, ok. So, for d equal to 1, g 2 star is a fixed point, stable fixed point. So, I should work our dynamics here and for d greater than 2, d equal to 2, this becomes 0 and this is higher term. So, d equal to 2, this analysis kind of feeling, but d greater than 2, it turns out the g 1 star, this fixed point is stable, ok. So, that means the non-linear term is not giving any correction, ok.

If g is 0, then non-linear term is not giving any correction, right and that is like Gaussian fixed point of Wilson theory, ok. So, d greater than 2, the system behaves like a non-linear term does not exist. So, for this equation d by dt h equal to nu grad square h plus eta, ok and that is very easy thing. For this z equal to 2 which has been derived for this equation, ok. In fact, we can do it for from this analysis as well.

So, z equal to 2 and alpha is 0. In it turns out for d greater than 2, this theory predicts that the flux, surface fluctuation is basically not very significant, ok. So, alpha is 0. Now, for d equal to 1, we know that G star is square root 2 by Kd, right, square root 2 by Kd. So, let us substitute it here.

So, fixed point means this must be 0, right, d nu by dl is 0. So, z minus 2 plus Kd g square is 2 by Kd and d equal to 1. So, 2 minus 1 by, so that 1 by 4, right. So, Kd, Kd cancels, this cancels.

So, this is 0. So, z equal to 2 minus half is 3 half, ok. And remember alpha plus z equal to what? And you remember alpha plus z equal to 2, that means alpha is half, ok. So, there is a prediction for d equal to 1 and that matches with the experiment, ok. So, that is a triumph of KPZ equation that these exponents only we are able to get them from theory with long calculation, but this is what we are getting from this theory. And it also tells you dimensional dependence, d greater than 2 we are not getting this coarsening or rather not coarsening, this roughening, alpha is 0.

So, d I already told you about d greater than 2 and summary is, so dynamical art. So, now, this for me I was not trying to do, well I am not giving a big lecture on surface growth, but that was a illustration of how to do renormalization of the flow for dynamical equation or which with frequency dependence equations, ok. So, we can do it following the same procedure as Wilson. This one scheme, but there are other ways to do it, but anyway this is a nice scheme and this RG in field theory can be extended to dynamical equation. So, you do non-equilibrium RG in this stuff.

So, results matches with experiments and dynamical exponents are computed by this procedure. And now this one is, so in remember it was t equal to t prime is t b to power z. So, that is b is like wave number, inverse wave number k minus z and t is like frequency.

So, omega is k to power z. So, this z is connected with omega and k relation.

So, well I am I had to do it properly t prime t and so on. So, basically from this relation I can get this thing relation between frequency and wave number and z is called dynamical exponent, this is dynamical exponent, ok. Stop.