Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Lecture – 63

He asked this question how can we fix this problem and it turns out people worked on it. I will not tell you the derivation for KPZ equation. So, it is more realistic this equation which I am going to describe now is more realistic which is derived in this book and this equation gives you a reasonable answers ok. In fact, better answers for many experiments and this is a very famous paper which has huge citations I mean lot of people following. So, 1986 you can see it is a very old work 40 year old work roughly. So, the equation which we are going to use now I am not deriving it, it is not really of course on surface growths.

So, so we have a new term this is a non-linear term lambda by 2 lambda is a constant grad H squared ok grad is a gradient ok. So, basically if you look at the proof so this is a surface then the surface roughness basically this is a velocity v of t. So, it is derived in a more realistic manner ok. So, and there is a non-linear term comes.

Now, it turns out how many parameters we got there is a new parameter new is viscosity which as before there is a non-linear term lambda coefficient ok, there is a pre factor and there is a same noise we will use. So, there is a factor D noise correlation which is D also comes into play. Now, this equation analysis because of non-linearity is not easy and we will use some field theory tools ok. So, one thing to keep in mind that lambda is not renormalized ok. So, I am going to so let us do the scaling argument ok.

Let us just do first simple scaling argument and then we will see some hint ok. So, I did this before so basically this term and this term and this terms were there before, but this new term has come. So, I am just going to copy that from my earlier notes dh by dt equal to nu b to the power z minus 2 no sorry this alpha minus z plus lambda by 2. So, grad will give me grad is 1 by length no. So, B minus 2 and h will give me 2 alpha correct.

So, that is the term from h squared is 2 alpha and that and this term will give me b to the power minus d plus z by 2 same as before except I got a new term here ok. So, if I keep nu lambda same not change it then what do I get nu b to the power. So, sorry this is 2 minus z not alpha minus z 2 minus z no. So, that grad will give me minus 2 sorry minus 2 that will give minus 2 and H is alpha what I am doing minus 2 plus alpha. So, alpha alpha cancels I get z minus 2 this we did before z minus 2 and the new term is lambda by 2 b 1 alpha cancel with this alpha and I get alpha plus z minus 2 and then the last term is B minus z and this is so this becomes z by 2 minus alpha minus d by 2 eta ok.

So, from this relation what do I get alpha plus z equal to 2 correct. Now, what will I get from here z equal to 2, but then alpha is 0 right this is a problem alpha 0 means roughness does not change with L ok. So, what is the problem? The problem is assuming that nu is a constant ok nu can change with B ok that is renormalization when I change my system size when nu also will change and this is not nu I mean we have seen it for turbulence the viscosity is different different lengths ok. So, the nu being constant is a gross assumption for non-linear problem ok. So, I have to renormalize nu ok.

So, this nu is some function of B and that that will give us correct result which I am going to derive it in fact. Now, this turns out that d which is correlation of eta also is function of B ok d also gets renormalized, but interestingly lambda is not renormalized ok lambda is not renormalized. Why is lambda not renormalized? In fact, it is a Galilean invariance which I proved for Navier Stokes it works exactly for this problem too ok. So, you see the motivation that we need to renormalize this coefficients to get proper answer ok. So, basically we need to renormalize this these factors.

So, this is my equation KPZ equation ok I think I am losing I hope everybody is following me ok. So, in fact let me prove it how well I am not going to prove the in full detail, but I will show you how lambda remains unchanged for RG. There are two methods, but one simple method is by Galilean invariance. So, it turns out if I write grad h equal to velocity u velocity vector ok. So, I can take gradient of this equation then I get a new equation and that equation is this equation nu grad square u plus lambda u dot grad u plus gradient of eta, but gradient is also a noise some new noise you can call it eta if you like.

So, this equation anyone knows what this equation is? Navier Stokes equation without pressure gradient right and this equation should also respect Galilean invariance right I mean the way we did Navier Stokes. So, this is Navier Stokes equation with pressure being 0. So, it should respect Galilean invariance. So, the lambda should not change that is a simple argument ok everybody is listening. So, lambda fortunately is not renormalized does not change with RG.

So, constant lambda is not renormalized due to Galilean invariance, but we need to renormalize nu and d these two coefficients ok. Now, let me just quickly do it I am not sure whether all of it will be clear, but let us let us do this steps ok. So, we will write down the Green's function and correlation function for the linear part. So, perturbation I am going to use. So, by this Galilean invariance we show that lambda is not changing under RG ok, but we need to renormalize nu and d ok.

So, that these two thing and it is very similar to what I told about hydrodynamics know this nu and d well we did not for in my calculation I did not renormalize d, but Yoko Dorzhok as well as a functional RG which I was talking about for generating function we normalize nu and d this nu. So, what do I do? We follow this is KPZ the famous paper. So, we write down Green's function for G naught ok. So, let us write G naught. So, in Fourier space do I have Fourier space thing yeah.

So, these are my Fourier space equation is it you understand what it is. So, this term is coming here this term nu K square right that is term there this eta is right here and this grad H square is a non-linear term. So, that becomes a convolution right. So, I lambda by 2 which is missing here and the convolution is product becomes a sum in Fourier space. So, this is H P omega Q no this is omega P omega P and this is Q omega Q omega is a frequency.

So, I have two different frequencies. So, this grad h gives you one P gradient gives you a you do a wave number no gradient of f in Fourier space becomes i k Fourier transform of f of K. So, since I am using p as a variable then that gives me i p from one gradient other gradient will give me i q which is coming from here and is integrated ok. So, this is what we get for these two further non-linear term. Now idea is quite simple actually.

So, we write the Feynman diagram this one is h of p and h of q and this is a vortex ok. So, let us rewrite well basically let us compute it to first order in perturbation. Right now I have written this no I mean this is what I have it here. So, please recall your earlier calculations what I did. So, I write this as a Green's function expand h q as a Green's function times non-linear term.

These my G naught inverse by definition. So, I take these two right hand side and multiply by their full right hand right hand side term yes. So, we write this as a Green's function G of Q 1 c of p is there. Now what we get this well basically focus on the non-linear term I get one h and one h here yes. So, this h of p this should be H of minus P to give you non-zero value right I mean this is we did it before and what will this be by conservation of momentum or the wave number here also the momentum must be conserved.

So, q equal to minus p and what should that be plus k right. So, this should be h of k if you put k then p plus q minus p will go to the left and p plus q is k. So, you see this term is my bubble or self energy diagram that is going to correct this viscosity. This precisely what I did before yes follow me or no how many of you follow me please raise your hand. This is precisely I did for hydro dynamic turbulence.

So, this diagram I just Green's function and these are two u's coming from here. So, G of q no I mean instead of h of k I have write h of q. So, this G inverse will go to the right that from G and this will replace p will replace by R and S. So, we write h of q is Green's function of q. So, I put a 4 vector like that.

So, that is for omega G of q and right hand side will be integral h of r and h of s 4 vector. Now so, you so this comes with this h of r and this have h of s, but h of r will yeah I need to multiply h of p and they will give me non-zero value only when r equal to minus p right because of this noise term h is also fluctuating where eta is fluctuating h is also fluctuating. So, the r must be equal to minus p right that is what we did before I mean this part you should remember and s will be by conservation of by this momentum stuff this is a it should be h of k ok. So, that is what we get. Now, I need to compute this, but compared to so in my fluid turbulence lecture I put h of p this is correlation c of p and I wrote from Kolmogorov theory, but this is not done here we write in terms of noise.

Now, I forgot to say one more thing I think it is ok. So, this is a Green's function and correlation function we are discussed before. So, this is what we will do. So, new normalization this is what I wanted to say. So, I need to separate into less and greater ok.

So, remember we are going to see the effect of this on nu. So, I write minus i omega plus nu k squared h less of I write is a 4 vector. So, k omega k hat is k omega that is a short hand. So, right hand side we involve h less p h less of q plus eta less, but it has other terms too greater greater greater less right. So, we will have term h less p s greater q and interchange and the next term is h greater p s greater q.

This equation is same as before this becomes 0, this is non-zero term and that is going to correct nu ok and that is what I am going to compute ok. So, let us save time. So, I am going to just show you why we get this complicated expression ok. So, let me draw it here. So, I told you that this two h greater p s greater q gives me Green's function.

So, we are doing from the bare function or from the free function G naught of q and we got two 2 h here h greater p h greater minus p and this h of k less. Now, h of p I again expand it ok. So, this one G of q is sitting here this one G of q is sitting here this G of q already got ok. Actually well I made a mistake. So, let me just do change the variable ok.

So, I am just going to write this as p and q above this formula is written that way. So, this is p and this is q and minus q ok. This integral of course, here one term in Green's function is above ok. The two Green's two terms one is this other one is that like this and that, but I am focusing on this one. If you know one of them you can solve the other easily ok.

So, this G naught of p is here p 4 vector. Now, how do I write h of q? So, I write to first order. So, remember it was G naught inverse q h of q 4 vector 4 vector equal to forget the non-linear term I am doing linear only so eta of q fine. If you drop the non-linear term that is what I will get. So, what is h of q? h of q is equal to g naught of q eta of q 4 vectors correct.

So, this is this term G naught of q eta of q got it. So, this is this one. What about h of minus q? It is this term ok. So, these my correction this one. So, I got it here and the vertex term one is lambda and one is lambda p dot q which I wrote in the previous one.

You also lambda basically get something like p dot q ok. So, that is lambda square and this is p squared q squared ok. There are some more stuff, but basically this is a sketch. Now, where do I integrate this? I integrate over a shell.

This is K greater shell ok. So, K n plus 1 to K n and Wilson think it is K by lambda to K right. Remember that so lambda and lambda by b that is what I use for Wilson, but idea is to sum over a shell ok. Is that clear? So, I need to integrate this. Now Green's function I already know the formula it is 1 minus omega q plus this p ok omega p plus nu p squared. So, I this algebra is reasonably complex, but it can be done and all that gives you some number integral ok.

So, the integral this what we get. So, this is the term which is coming a correction nu tilde ok. So, trust me that well I did not I have not done this calculation fully, but this is from where I have received. So, but the form looks similar in lambda square d why by the why how d is coming? d is coming because of this 2 etas right eta q eta minus q that is D no eta square is D. So, we get this D lambda square D and why this nu cube? So, we said basically in this there are lot of approximations are made. We set omega to 0, low frequency modes, large time scale and the nu is coming.

The how many g's are there? 3 G's 1 G, 2 G, 3 G ok. So, 3 G's gives you nu cube that is nu cube there. This integral will do it over a shell and this integral will be function of lambda lambda by b ok and we can do this. But we do one more step. What is step? I have done coarse graining right now, but following Wilson we do rescaling. So, when I coarse grained it my boxes become big right.

I have coarse average or small boxes. So, my boxes have instead of small boxes I got this big boxes. So, you shrink it again ok. So, in fact we want to follow Wilson scheme. So, you shrink it again and that is a scaling step, rescaling step. A rescaling pre factor is exactly what I derived by scaling argument.

It is b z minus 2. I push the system back to a smaller small size. So, this one gives you and nu less. So, this is nu less ok. So, this nu less is here. So, this part b is e to power L our standard thing which we did e to power L and you can expand for L small.

So, it is going as 1 plus. So, b to power z minus 2 will be this one. So, it will be L times delta L times z minus 2. So, that is what I got here. Now, we substitute nu here and b will come here ok.

This b will come here ok. So, if you do all the algebra and then I get a differential equation. So, this dL will come below. So, change in nu which is you can see from here this is original nu and this is a corrected nu where the total nu. So, the difference will give us d nu by dL and this dL is sitting here and that is a factor ok. So, I think I will stop here because I think I am probably putting too much material ok. So, let us stop here. Thank you.