Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Lecture – 56

So, we did RG for viscosity in the last class right for the shell model, but there is one missing piece. So, what about the coupling constant? Coupling constant is a pre factor for the non-linear term right that is what we did for phi-4 theory. So, for phi-4 theory we had lambda phi-4 right in the Lagrangian and then m squared phi squared right and then of course, there is a gradient which becomes k squared gradient dot phi o squared right. So, we renormalize m squared this was not renormalized, but lambda was also renormalized. Now we want to know whether lambda for what happens to lambda for the shell model right this is the interesting question. It turns out lambda for the shell model let me let me just first show you what is this lambda.

So, this renormalizing the coupling constant for shell model. So, these are the non-linear term everybody agrees with that. So, there is a factor in front lambda like lambda phi-4 similarly we have lambda into non-linear term. Now, it is possible that lambda will also change with under RG when I rescale there is no guarantee that lambda should not change, but it turns out there is a symmetry argument.

In fact, there is a Galilean symmetry for the Navier Stokes equation and based on the symmetry arguments you can claim that lambda should not change under RG. It is not easy to well it is not well according to me it is not possible to show for the shell model directly, but shell model is supposed to represent Navier Stokes equation in turbulent regime. So, we will say well for Navier Stokes equation lambda is unchanged. So, for the shell model also lambda will be unchanged that is the argument I am going to make. So, we want to know whether lambda what happens to lambda.

So, as I said the proof is that Galilean invariance which I am going to describe in a minute implies that lambda is unchanged lambda. In fact, lambda should be equal to 1 including RG. So, let us look at the proof which is pretty simple. So, these are Navier Stokes equation right. So, this is the partial du by dt and this is a non-linear term you do not get it, but I put a factor in front lambda which is 1 at the onset, but we see whether it becomes non different than 1 under RG and this is grad p and this is a viscous term.

Now what is the Galilean invariance? So the equation well basically the idea is that the laws of physics do not change when I go to a moving frame or stay in the lab frame. Laws of physics are the same. So, this is a lab frame which you denote by x, x is a real space coordinate and time is time coordinate, but when I go to a moving frame then my coordinate system is so of course you say x, y, z, t the 4 coordinates 3 space and 1 time, but I am right now expressing only x. So, in the moving frame the coordinates are x prime t prime ok, but Galilean invariance you know that t equal to t prime know the time does not change. Lagrangian invariance of course t goes to gamma t prime minus x v, but we are doing the Galilean invariance which is good for non-linearistic flows.

So, what is the relation between the two? So, this is the moving frame is moving with velocity u naught along x direction we will assume that ok. So, that is my mean speed. Now we want the equation to remain unchanged. So, equation should be same when I replace u by u prime grad by grad prime t by t prime is that clear. So, we know this transformation I mean.

So, I will not go one by one for example, x equal to x prime plus u naught t right t prime you might have seen this velocity u equal to u prime plus u naught which is x right. So, velocity in the lab frame is more. So, relative velocity will get added up. So, these are very standard time derivative and space derivative are slightly involved, but that is also doable we just do standard partial derivative formula then you will get this transformation. So, as I said x is that t remains unchanged, but space derivative the partial derivative like t goes like that ok.

This you can easily show d by dt of any variable phi is d by dt prime. So, we write d by dt prime phi dt prime by dt plus d phi by dx prime dx prime by dt right. So, that is how we will get this this stuff ok. So, dx prime by dt will give you u naught clear I mean that is how we just use the standard mathematics formulas to derive this relations ok. And the velocity transforms that way and pressure is unchanged ok.

Pressure is coming from the divergence of velocity and divergence non-linear term is unchanged ok. So, we substitute them in terms of u you put u prime for t you put all that. So, these are what we get ok. I am avoiding the algebra ok, it is boring for you and boring for me as well. So, we just replace u by u prime plus u naught t.

So, this is the this is part. So, dt prime so I just just plug it in ok. So, here velocity field is replaced by u naught x hat plus u prime and then we have divergence and of course, I have u naught added, but u naught is a constant. So, Laplacian prime is gone ok. So, what are we getting here? So, we are basically this is the equation I get for the prime variable everything is in prime ok.

Of course, u naught is a parameter ok. Now, does it look like original equation? This is a set we should look like the original equation that will be the invariance right that when you say laws of physics is unchanged when I use u variable or u prime variable, but they do not look the same right I mean I should get same equation. So, I expect this to be lambda u prime dot grad prime u prime ok. So, this is looking good, but this term is a new term and this is a new term. Pressure is same as before viscous term is same as before this is same as before with u change by u prime, but these two terms are a problem right these two terms.

So, one way is to cancel the two term one is coming with a. So, one term is lambda u naught. So, this x hat dot grad will give you d by dx right d by dx prime u prime this is this term. Now, what about this term? So, these are exactly looking same form except lambda is sitting there. So, what will cancel it? We have set lambda equal to 1.

If I set lambda equal to 1 this will cancel with that. So, this shows that if I want to impose Galilean invariance then lambda must be equal to 1 ok. Now, this is set for all scales, but I derive Navier-Stokes equation at a coarse grain level that should also be symmetric under Galilean invariance. That means, this derivation is to be applicable at all coarse grain levels. So, that means, lambda must be equal to 1 at all levels ok.

So, lambda is not renormalized ok under coarse graining. So, d lambda by d L, L was that B equal to e to power L right that is the thing is 0. So, the beta function is in fact is 0 for this ok. It is interesting proof, but this well I mean this is straightforward. Now, of course, I did not prove for the shell model, but shell model is a set is a basically a model for the Navier-Stokes equation.

So, we expect the non-linear term for the shell model to should be respecting this property. So, lambda is unchanged even for the shell model. So, Galilean invariance is a very important symmetry which immediately tells you that lambda is unchanged and actually. So, basically shell model is not in real space. So, I cannot apply this argument ok, but as I said similar physics is NS equation.

So, lambda is not renormalized for shell model 2. So, please keep in mind that new has changed and this new goes as k minus four third is function of k and this exactly what we get for the Taylor diffusion. So, viscosity in fact increases as length with length. So, bigger the Eddy is bigger than diffusion. And so, this is a coming from theory.

So, interestingly we are able to derive this using RG theory.