Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

Prof. Mahendra K. Verma

Department of Physics

Indian Institute of Technology, Kanpur

Week - 10

Lecture – 55

So, nu star what happened to nu star? So, this is a picture somewhere down the line. Nu star is bit around 0.5 for our for all these around 0.5 this is a range I am going to show you one slide, but the question is yeah this is a nu star ok. So, nu star is for this x axis is b and y axis is nu star and Kolmogorov constant too, but we will discuss that Kolmogorov constant computation later. So, nu star is ranging from around 0.5 it decreases to around 0.4. It is function of b, but it is around 0.4, 0.5 and interestingly these also the number which you obtain in Navier Stokes equation.

So, shell model is giving us a very good handle on computing nu star. This is from theory ok the formula which I just wrote I just substituted there and take square root I will get nu star, but how do I verify using numerical simulation? The solid line is our theory and the numerical simulation we did I am going to show you how I can get these numbers for different b and they are quite close to the theoretical one. So, how do I do it? So, these were how to compute nu n from simulation. So, remember this is an assumption, but it is a good assumption.

So, what is that in physics we make models there is nothing called perfect theory in physics. Nu theory is proven in physics mathematics we prove theorems, but in physics we never prove theorems. Say that if I do some computation and that matches with the experiment on numerical simulation then my theory is good, but tomorrow's experiment may go against my theory by some regime it may not work then my theory has to be re-modified or maybe abundant and new theory should be created. So, it turns out this model is good it works to good approximation for small t minus t prime ok. So, let us see whether this this thing works in simulation.

What is C bar? It is u n at time t u n star t prime ok t and t prime are shifted this is this quantity. So, I can compute this in numerical simulation I get a time series of u n I get time series of and I just shift it and compute this correlation function ok. So, in fact, I divide this by u n t square. So, this will give me what this exponential part. So, I take this unequal time correlation divided by equal time correlation I get a time series and I compare it with exponential ok.

So, this is what we do in simulation and this was done by Sadab in our lab. So, x axis is we put nu n k n square tau and y axis is the normalized correlation this one C bar by C and for this is non-dimensional quantity. So, for this band is exponential some constant times k n to the power two-third tau is a good fit ok. In simulation this is what we get I am not going to exact like things fit on the line this microscopic phenomena we do not get like on dot I mean this is not GR verification ok this is what we get pretty approximate curves ok. Now, but interestingly this is going as k n to the power two-third nu n k n squared is k n two-third right because nu n is k minus four-third and this is so great I mean this function of k n is working.

Now, coefficient is 0.57 now what should I compare this 0.57 with this 0.57. So, you can look at it here ok nu n k n squared this k n squared is k n two-third is here and this whole object must be same as 0.57 right I compare. So, nu star k root K Kolmogorov epsilon one-third is 0.57 0.57 ok. Now, epsilon one estimation was 2 ok K Kolmogorov of course, I need it because I cannot so we took k Kolmogorov from theory I think it is around 1.7 K root then that will give me nu star and that is how we compute nu star from simulation and that nu star is matching reasonably well of course, not on the line, but reasonably well with the simulation with the analytical prediction simulation results are close to the analytical prediction and that is how we compute nu star ok. So, this is what we get here and this is for different B we can compute it we did this for different B shell model is pretty cheap. So, we can do it for many many runs now these are non-equilibrium solution, but we also can work equilibrium solution partial model. So, remember RHS was un plus 1 un plus 2 right.

So, RHS was un plus 1 greater un plus 2 greater there was a star in one of them ok. So, for equilibrium solution un plus 1 un plus 2 are totally random quite noise. So, this is 0. So, there is no correction to nu bar where right hand side was 0 where right hand side is 0 for this. So, for equilibrium solution there is no correction to nu bar.

So, nu bar is same at all scales for all un's for Euler equation and that is what we get here this is 0. So, we basically get the right hand side is 0. So, there is no renormalization for Euler equation ok. So, equilibrium solutions are somewhat trivial that way and no correction. So, summary for RG is the interval scenario nu n is function of k which is k n minus four third and nu nu star is roughly 0.5.

A Kolmogorov solution works here. Well, I did not say that Kolmogorov solution is a unique solution, but I said it is one of the solutions. So, this was self consistent well let me guess it works or not and look it works that is what we say this is a self consistent theory and equilibrium case basically is no renormalization ok. So, I think I did a lot of things, but I hope is clear most of it any questions? No this idea is not to find the solution of un, idea is to find correction to nu bar.

So, we are well ok good question. So, I am making an assumption that I force at large scale I should have mentioned it, but I did not I force at large scale this is a cascade, but in the inertia range force is not playing any role. So, F is not appearing in the inertia range equation for RG equation and we are looking for the effect of the coarse grain mode un plus 1 un plus 2 on the viscosity under for steady solution of un. So, let me go back to the idea is not to solve this equation that is not the idea, idea is look for correction to. So, we are looking for ok I should go to previous slide yeah my idea is to look for the how the nu bar is getting increased or is increasing because of this term which is averaged out.

So, look in this room I have full Navier Stokes equation, but I do not want to look at small scale of the flow of the room like a large eddy simulation. When I filter them out then what is the enhanced viscosity at a bigger scale at alias scales. So, our our modes were all the way right let us say in fact, when I was doing it I did not go all the way I had up to kn plus 2, N plus 1 kn I had up to kn plus 2, but I would say well what is if I go up to only kn I do not keep these two modes then what is my viscosity at kn that is what I am looking at. So, my objective is not to solve the shell model, but my objective is to look at the effective viscosity. So, that was what I said I compute the correlation un plus 1 un t un t prime ok.

So, so let us yeah I will go to that equation yeah. So, I compute this this object which is un t un t prime. So, this is your time series know. So, I can do this is a complex conjugate in one of them then I divide by un mod un square average. So, this will give me this exponential part Why this is a real solution we got.

This is a semi log semi log y and it is linear in that part is not for all scales it is for. So, this is the RG equation RG un is not work does not work for all times it later for bigger times is a decorrelation this works the Markovian approximation. So, this will work only when for this is order less than equal order 1 this is a what we see my formula this formula is not working going to work for all times right. This one this is Rn this is Rn now this is exactly this.

So, this is exponential constant this is kn two third this guy function of kn two third. No. Even all the way like that. Yes. No it does not well we worked infinity is not supposed to work no by the way it is this is.

I mean. No no. I will get. By the way this is not expected to work in fact, it is wrong to extend it to large t this is the condition this condition less than equal to 1. That is correct. Ok one is somewhere here ok. So, it does not go up to one ok now DNS ok may be something different, but this is trial and error and that is what we get in fact, this is somewhere here 0.7 or something 0.7 that is up to only up to that. But this kn whether this kn these are all initial range N these these guys are initial range our kn is initial range for the time according to me nobody is completed this first shell model what I showed you this one.

So, this shell model this is a correlation a whether if you go for long time then this is the inter correlated rather 0 correlation this guy this correlation will become 0 for large t minus t prime. So, this expected to work only for this band not for all t. This is from this memory time memory time. Yes Markovin approximation. That is right ok anything else is it clear or not clear ok.

So, I think are you guys all tired or we want to do the next one how many for next one I am bit tired, but I can go on for half an hour. So, next is flux how to compute Kolmogorov constant that is the next one which has less slides, but ok maybe you should do next time right I think it is saturation rethink saturation how many for today ok. So, I think we will go to the next one, but so, let us record a little bit one more few things which I missed out I should ok. So, few important point you should remember. So, this is the first real RG calculation for non-relativistic system.

So, you might have noticed some differences I mean if you pay attention and with some well you may not have noticed. So, you can compare Wilson phi-4 RG with shell modal RG. So, few things which are obvious the d by dt there was no time here at all d by dt was 0 here d by dt is there is a dynamic, but of course, we are looking at steady state. So, d by dt is not playing a big role, but it is a steady state what we look at second important point is the Green's function correlation function G of k and C of k there was no time in it remember there was no time in it it was time independent it was a equilibrium system whether equilibrium system time is frozen time does not change if you are in equilibrium then time does not move for you there is no arrow of time for equilibrium system. So, G of k C of k's are independent of time here G of k and C of k are dependent on t t minus t prime and they are different functions one of them was theta t minus t prime exponential other one was C k.

So, they are different functions they are not the same function and they are function of time here the flux was 0 equilibrium system flux is 0 not only in phi-4 theory, but also for Euler equation for non-equivalent system pi k is not 0 detail balance is broken. So, there are so, I gave only one scheme self consistent theory this is one of the simplest demonstration of non-equilibrium renovation group there are more complex ones. So, I will mention them with later. So, there are like Yakhot Orzhov is one of the most famous Yakhot Orzhov for Yakhot Orzhov 1986 then there is a Martin Siggia is called MRS Martin Rose Siggia I think this is 73 something FNS Forster Nielsen Stefan 76. So, there are different schemes to do RG the RG is a.

So, we can do functional randomization using partition function similar things is done here I use basically differential equation. So, I do not think we have time to get into details of it, but I will try to mention this stuff it for 2 minutes for each of them little bit later. So, for point to note is that this non-equilibrium and equilibrium RG's are different, but of course, they are common theme that we do coarse graining and effect of coarse graining on different scales that is a common theme, but the tools and assumptions are different. So, let us stop here and I will do the flux part in the next class. Thank you. Thank you. Thank you.