Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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So, this equation I just rewrote that equation, this can be removed. So, if viscosity is 0, then this is a equilibrium solution. So, Navier-Stokes equation also has equilibrium solution. So, it looks odd, no, I mean, it is supposed to be, you said it is non-equilibrium, but what is the equilibrium solution? It turns out, if I turn this off, viscosity is 0 means that term is off, then if I start with also no forcing, no forcing is off, then this is the equation for Euler equation. So, in fact, this is called Euler equation. No external force and no dissipation.

So, in 3D, 2D is more complicated, but in 3D, we start with random white noise initial condition, then T k is 0, turns out. Of course, in average, so, T k will fluctuate, but if I do this, then, so, this is T uk is 0 on the average, let us put this average. So, what does it mean? The energy is fluctuating around 0, but it does not, d by dt of E k is also 0. So, energy does not change with time.

In, the same sense, it is thermodynamic. So, this mode has a constant energy and it is basically in equilibrium, u of k. So, this system with nu equal to 0 in 3D is same as thermodynamic gas, okay. And so, it is a, it is a interesting observation. So, even hydrodynamic equation which is not thermodynamics can also give you equilibrium solution, okay.

So, that is the equilibrium solution, but our focus will be not this, this is not we will focus on, we will focus on non-equilibrium solution. One important point is called detail balance. The uk is 0 means, there is no net energy coming into k mode. Non-equilibrium situation, there will be net energy transfer. So, when I do the Kolmogorov theory, I will show you, there is a net energy transfer across scales.

But in this case, there is something called energy flux which I am going to define soon is 0. So, in thermodynamics, we have molecules which are colliding with each other like that. So, each molecule is basically is kinetic energy is not changing with time and it is not giving or taking energy on the whole from any other guy. It is doing, sometimes it takes,

sometimes it gives, but there is no net gain, okay. So, that is for detailed balance.

So, it is a my bank balance, well everybody has equal bank balance. In fact, there is a model, the economic model of thermalized wealth. So, it is something like communism. So, everybody is equal money. So, we do trade, but well, I will give something to you and you give me something to me.

But nobody grows, becomes rich. Everybody is has roughly same money. So, that is like thermodynamics system. So, this is a in fact, there is a the first paper of this on this idea was written by Meghnad Saha on financial model with equilibrium solution. So, that is a so, please keep in mind the course emphasis is to contrast the equilibrium situation with non equilibrium situation.

Now, I am going to show you the non equilibrium solution after this. So, non equilibrium solution of the Navier-Stokes equation energy transfers. So, you force at large scale that is a this is a key gain gradient. I am discussing 3D, 2D has slightly complex physics, but 3D I force at large scale and dissipate at small scale. So, well I am not proving it right now, but the dissipation that new K squared Ek term, this is significant when k is large, you know, this k square term.

So, k large means small wave number. It turns out the dissipation is active. If I draw this, make a sketch, this is k small, this is increasing k wave number. Then you find that dissipation is strong in this band, but energy is coming in this band. So, imagine that in the system I pump energy at one scale or imagine in a these are this analogy is not great analogy, but it is ok.

You can think of well this is not really a good analogy, but I am giving another example. If I put water in a pipe here and there is a some hole in this pipe. So, if I put water here then when will it go? It follows is this path and it comes out of this path. Some will continue, but if I make a big hole then all of it will just come down right. So, that is what happens in turbulence.

We have energy coming in here it will at small scale and it comes out in large scale, but this is not a correct picture. Correct picture I think let me also say this example I really like. So, you should think of this energy transfer in across scale is in the following way. So, this is the central government Delhi central government which is gives money in from Delhi you know it distributes. Now of course, do not think of other factors like there is a money coming from international source or from money is injecting money or some place taking money.

So, we will assume that money is conserved while while somewhere, but we give energy at large scale which is central government. Then it goes to state level. So, energy is flowing at a different scale, but this is not in physical space this is like is this pipe example is not a great example because this water is flowing in real space. Here money is not really flowing in in real space it is going multi scale to all the state it goes the different directions it goes. And from state it goes to districts from districts it goes to village and here basically well village or in small city or people actually you can say per people this is a smaller scale which is the dissipation scale you consume that or you buy food from that money and you consume it.

So, this is how this is a picture of multi scale transfer. This flux which I am going to describe soon is not the flux like a pointing flux of a little mechanism is this energy transfer multi scale energy transfer. So, we need a forcing and dissipation at. So, it two different scale not there is a same scale and then that creates a cascade the example of cascade I just gave you and. So, this Tu k is not equal to 0 well at some places it is 0. I should say some places 0, but here I am injecting energy.

So, this guy is Tu k is positive here Tu k is negative, but in between Tu k is small, but there are places where Tu k there is energy some people get energy. So, you can think of these are very rich entities not individual entities which are getting money and then they cascade and this is one direction it goes in one direction is not money normally does not flow from people to like that is consumed there, but this is a good approximate theory, but that is what we do. So, this is different than thermodynamics where Tu k is 0 at all scales and we call this detail balance is broken. So, it goes one direction energy flow happens from left to right in this in this picture.