Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Lecture – 51

So, in the last class we were we started discussion on non equilibrium field theory right. So, I took Langevin equation and we constructed the Green's function correlation function and discuss briefly about fluctuation dissipation. Now, we will go to well that is in fact, it is a toy problem, but we will do more complex problem today. So, it turns out turbulence is a nice example of non equilibrium field ok. We will have velocity field and that will show you non equilibrium features and I am going to tell you exactly which part of this theory of turbulence is non equilibrium ok and we will keep that in mind. So, we can say we can contrast equilibrium versus non equilibrium ok.

So, the one of the most famous theories for Kolmogorov series of turbulence and that is what I will discuss today. I set up the equations, but I will just we will use this as a framework ok. So, some of you well most of you I think know Navier Stokes equation, but some whom are not familiar with this for them this is the equation for the fluid flow fluid flow in this room. So, u is the velocity field it is not velocity of the molecules ok, it is a velocity of the collection of molecules.

So, it is a continuum field that means, I can write u as function of x and t. So, this is a non-linear term u rho rho u is called advective term. So, the left hand side is total derivative d by dt of u and right hand side is the force like this acceleration. So, Newton's law is acceleration is force by mass, but of course, not a point particle or isolated particle it is a continuum. So, you have to think of force local forces.

So, gradient force then external force which could be gravity or it could be electromagnetic force and the viscous force this is a viscous force because of differential in velocity. So, this is the Newton's law and we will look at the properties of this equation for this class or in the in fact, in this course we will assume that flow is incompressible ok. That means, the volume of a segment when it moves it does not change or we can think of density is constant during the evolution. So, there is a quantity which is very important for Reynolds number which is a ratio of the non-linear term the non-linear term right is the quadratic in u with a ratio of this and viscous term. So, it is in fact, u dot grad u divided by nu grad square u and that happens to be u L by nu.

Nu is a kinematic viscosity which has units of centimeter square per second you can see that u L has the same units as nu. So, it is a non-dimensional number and the flow becomes turbulent when Reynolds number is around 5000 or more there is no critical Reynolds number depends on the system, but it should be significantly large non-linear term should be much more than the viscous term ok. So, Fourier space so, we will work in Fourier space. So, the same equation we can write in Fourier space right I mean I discussed the Fourier space many many times. So, the Fourier u k is a Fourier transform of u of x this is a non-linear term I am going to write down the expression for that in a while.

Pressure gradient becomes minus i k p these are force and this is a viscous term right the Laplacian becomes k minus k square. In the non-linear term it is you write for incommensural force you write like this u dot grad u becomes the gradient can be taken out it can come out because the divergence free condition. So, it is this and the non-linear term so, right now I am writing a vector equation is convenient to write. So, non-linear term this becomes a product because the convolution and that gradient gives you i k this i k is coming from the gradient and u j u i is coming from here and of course, p plus q is equal to k convolution has this condition right so, that we discussed that before. So, this is what we get and the pressure we can determine by the incommensural condition.

If I take a divergence of the whole equation that I can get pressure or divergence will be i k dot. So, if I do that I get pressure which it will come like this. So, I am going to skip steps in fact, I am not really I am trying to give a quick overview of turbulence. So, to discuss all of it will take time, but we will go fast, but I hope you get the gist of it. So, these are non-linear term and u and that will treat as a perturbation when you do field theory when you do the renormalization group of this equation then we will treat that as a perturbation I am going to do it soon.

So, you have to just wait for few minutes. So, we define energy spectrum is in fact, it is called modal energy. So, given u of k you can take mod square. So, that is the units of energy know well density is 1. So, this is called modal energy every mode has energy and that is the energy and the total energy will be integral of u k right.

Now, I am going to sum it up. So, this integral is the total energy we can write down equation for the modal energy. So, from the Navier-Stokes equation we can derive it. So, I wrote the equation for d by dt of u k I can take a dot product of this with u star of k then add it to complex conjugate plus complex conjugate. So, that will give me d by dt of mod u k square and that is equation well I divide by 2 then I get this equation.

So, the non-linear terms the one which was u dot grad u in the energy equation there will be another u I am multiplying another u right this u star k. So, it happens to be k dot u q u p dot u k star imagine part of this. So, u the k mode is getting energy from other modes by non-linearity right. So, we will also make a triad like that k p q right p plus q is equal to k. So, here this k mode is getting energy from u p and u q.

So, the u p is giving energy to u k ok. So, that is the energy going to u k. So, there is a formalism which is useful, but we do not need to get into that. So, this guy is giving energy, but what about this guy is like mediated ok, but we do not need to get there for today's lecture. So, this is the equation for the energy transfer by non-linearity this is the energy coming into k from external force.

So, we write this in fact, as f of k f u. So, this energy injection per unit time to wave number k from external force and this is a viscous dissipation. So, there is a loss because of viscosity and that is a viscous dissipation term. So, every term has a name this is called T u k ok. So, this is a non-linear transfer term and this is the injection rate due to force external force and this is a dissipation.

So, D u k is positive the minus sign is outside ok. So, dissipation rate these are all per unit time this d by d t is here so per unit time. Thank you.