Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Lecture – 50

Fluctuation dissipation theorem, ok. So, is very important theorem and I will not derive it in this class. I will state the result and I will motivate it, I will just use Langevin equation to connect. So, this relation connects fluctuation and dissipation, ok. So, that is what supposed to be or Green's function and the correlation function, ok. So, I will try to see show you what is dissipation, what is fluctuation in the relation.

So, the statement is that Xt is observable of a equilibrium, it start make system, system is in equilibrium and this Hamiltonian is H naught, ok. So, that is a Hamiltonian of the system, it is function of x. So, this is in equilibrium, but I want to perturb it, I want to make it slightly non-equilibrium. So, perturb it, bring in some fluctuation.

So, my new Hamiltonian is H naught minus f t x, f is a force, a kick basically, you just apply a kick, ok. So, this will create disturbance. Now, you can see that there is a for H naught, I have Green's function, no. So, for my system I define I given H naught I have Green's function, right. I mean in Green's function is 1 by E minus H naught, ok.

So, there are different kinds of Green's function, but one of the Green's function is written like this. So, the Green's function and so, this response system has a response. So, this has damping in it, ok. So, there is a the Green's function has some damping involved in it, but its response is x, x is a response, so x is my wave response. So, x t x t prime that is a correlation function or its Fourier transform will be c of omega correlation, its Fourier transform will be c of omega.

So, there is no wave number here, it is a part, it is a motion. So, I am thinking x is function of time, its Fourier transform will be in omega. So, the statement is that c of omega, this is a correlation function of x. So, this is x omega, x star omega. So, the c omega is connected with the Green's function, ok.

So, this is a fluctuation and there is a dissipation in Green's function and the proportionality constant is this, 2 k B T by omega, where T is a temperature and omega is a frequency, ok.

So, you see the Green's function and correlation functions are not same and the frequency omega has come in. So, this is, by the way this is not far away from equilibrium, the infinite is close to equilibrium. A system was in equilibrium and I just give a small kick. So, this is supposed to be near equilibrium, this formula is valid only near equilibrium.

Far from equilibrium this formula will not work. And this proof you can look at Wikipedia, we will get some idea about the proof, but it is quite complicated, ok. So, I did not want to do it here. So, we will do a demonstration, ok. So, I will try to demonstrate it that, ok.

For a system this relation I satisfy the relation for a given Langevin equation, ok. So, I will basically do a demo. So, we start with the same equation, Langevin, ok. Now, the zeta is of course, random noise, but let us compute the Green's function first. So, to compute Green's function what do I do? I put a source, right, right hand side delta delta t minus t prime.

So, I put a g this right hand side will be delta t minus t prime and the left hand side will be gamma g t minus t prime. There is a definition of Green's function and these are all linear systems. So, linear systems is what people normally work with, it is easy to derive a lot of relations. So, this is exactly solvable. We can solve in Fourier space, means omega space.

So, by the way we are using that Fourier transformer, so f of t is f of omega minus i omega t d omega, ok. So, the minus sign here and f of omega is f t e to the power i omega t dt and this goes from 0 to infinity. t is starting from 0 to infinity, but sometimes we will write minus infinity to infinity, but we say well start from t equal to 0. If you put minus infinity that means minus infinity to 0 is 0, but depends on the authors. Some people write here minus infinity to infinity as well, but time negative, ok, we can avoid it.

So, here when I write this, so I assume that t is greater than t prime, ok. So, response means if I do something I get a response which should be later. So, if I do the Fourier transform this I will get minus this one minus i omega minus omega plus gamma, ok. So, this d by dt will give you minus i omega, no? So, if I just what I am doing it here is this one. I am doing d by dt of this, right.

So, minus i omega will come gamma and g g is replaced by g omega, g t minus t prime is replaced by g omega and delta function becomes 1. So, it is straight forward g omega is this 1 divided by minus omega plus gamma. It is very easy, no? It is a linear system straight forward, but Green's function is look playing a role everywhere, right. I mean it is the most important thing in field theory. Now, let us get to the correlation function.

So, I do the Fourier transform of this. So, same thing this part is that u t will becomes u omega and zeta t becomes zeta omega. So, u omega well I made it a scalar right now, but you can prove dot product as well if you like, ok. So, zeta is supposed to be a vector, no? So, let us assume right now 1D let us just do 1D. So, u omega so I take it below.

So, u omega and u star omega prime will be this guy when I do the complex conjugate this minus I becomes plus here and zeta omega and zeta star omega prime. Now, we do ensemble average and the assumption that this zeta function zeta is stationary that means, its distribution does not change with time and it is homogeneous. That means, zeta t zeta t prime is only function of is function of t minus t prime. You understand the correlation is so if I shift the system by some time I get the same correlation, ok. So, you shift t by alpha t prime by alpha this is unchanged, ok.

So, this is called homogeneous system and also its stationary means things will not change this correlations do not change with time. So, for that system this derivation I will not do this is zeta omega a c c of omega c zeta omega. So, this is a correlation of zeta delta omega minus omega prime, ok. So, this we did it for I am not sure whether I did in this course, but look for homogeneous systems you know sine omega t sine omega prime t if I do average this 0 unless omega equal to omega prime, no? Because sine omega and sine omega prime if they are not same frequency they will cancel if you take t large integrate this. So, the frequency must be the same for it to be non-zero and if it is same then it becomes mod zeta squared.

So, this is what it means this is mod zeta squared and this is delta function, ok. So, the left hand side the correlation function this also will be similar is cos C omega delta omega minus omega prime. So, delta delta you cancel it. So, C omega is this, ok. Now, this part is left without proof that this since it is in a heat bath.

So, the zeta is representing temperature fluctuation noise kicking. So, this is equal to 2 k B T is proportional to temperature is from algebraic concepts 2 k B T. So, my correlation function for Langevin equation is 1 over omega squared plus gamma squared multiplied by 2 k B T. Now, we have two functions both c and g we have this correlation function and Green's function. Can we relate them? Look at the imaginary part of g omega.

So, I multiply by complex conjugate. So, omega plus gamma omega squared plus gamma squared I multiply i omega plus gamma both below and above. So, that take the imaginary part of this what will I get? Omega divided by omega squared plus gamma squared. So, denominator is same as this one, right. So, I divide 1 by omega I get this.

So, replace for this I am going to replace that, ok. So, I get 2 k B T imaginary part of g of omega. So, there is a omega coming here one omega sitting below, ok. So, that is a relation connecting correlation function with the Green's function. So, it is a demo it is not a proof, a proof is more complicated, but this does tell you that the correlation functions are different and the frequency is coming now and yeah you have to be kind of do more work compared to equilibrium. Thank you.