Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Lecture – 48

Even gauge interaction there are lot of Nobel prizes, but this is recent Nobel prize right in Higgs. So, Higgs is what we write Higgs particle, but there are many people who simultaneously propose this theory. So, the names are Brout, Englert, Englert got the Nobel prize along with Higgs these two guys Guralnik, Hagen, Kibble, Anderson. So, this is condensed matter physics and even Russians like this Polnichov and Midder supposedly they had the idea. So, let us look at what is. So, this is the same Lagrangian which I wrote in the past interaction term and this is the potential and this is free part.

Now, symmetry breaking we need to do symmetry breaking what I said Goldstone mode first Goldstone mode. So, let us look for symmetry breaking. So, I make a change of variable psi s rho e to the i theta. So, we done in the past right magnitude and theta plug it in. So, this parts are straight forward mu square rho square and this is lambda rho 4 right that is straight forward ok.

So, here these operators become pretty nice. So, if I take the derivative then I get del mu rho e to the i theta and this is as is and del mu theta comes here ok. So, we can rewrite this as a del mu rho I have ok. So, we make a new change of variable this part is called i q C mu I make a new variable C mu all right. So, this is same as here and this thing is called whole thing is called i q C mu.

So, you will take the q out of it. So, C mu is basically proportional to A mu plus component coming from theta del mu theta. So, if I plug that in. So, I will get something here will be minus sign. So, if I do all of it there is a cancellations ok I did it myself, but I will not do the algebra here.

So, after this transformation this part the left part is basically these two terms ok. So, one is coming from this guy plus minus will cancel this guy with that guy and this squared square will give us well basically I have to use this C mu and C mu will come. So, the C squared is C mu C mu. So, there is something interesting is staring at us. So, you got something right now which is proportional to C squared ok.

So, that means, the photon may get some mass ok. So, that is you can see something is peeping in here. Now, before going to that let us do it this algebra properly. So, this is a free field, but by the way this f mu nu I write in terms of c mu's. So, we write earlier f mu nu is define as del mu a nu minus del nu a mu, but it turns out this operator will cancel.

So, we can instead of A mu we can write a C nu and this C nu. So, I use C mu. So, I use a new variable C, while C is a new potential 4 potential you may call. Now, let us focus only on the rho part. So, I will not I will right now I will ignore this and I ignore this.

Let us think of only the matter field. Please remember psi is the matter field and f is a electromagnetic field and the interaction is there is via this gauge interaction ok. So, I am just going to write in terms of differential equation I feel much comfortable with differential equation. So, if I keep these three terms I can write down the differential equation right del phi equal to del L del phi mu del mu ok. So, the derivative parts will come here we done this in the past ok.

This is nothing but time derivative square minus grad square and these two guys will give us rho and rho cube right. If they take derivative of rho 4 I get rho cube. Now, I look for a constant solution which is independent of time independent of space. So, that will give me a solution rho square equal to 1 by 2 mu square by 2 lambda. So, that is my new solution.

So, what has happened? So, you have to recall our earlier discussion. So, when mu square was negative I would get paraboloid like this well. In fact, rho is a scalar ok. So, rho will be like this, but when mu becomes positive then I get the solution this solution which is coming from this solution fine, but remember there is a theta field as well. So, this Lagrangian is independent of theta.

So, that is symmetry breaking ok. Now, after this symmetry breaking my solution is one of the angles and particular radius. So, my solution is my psi is square root mu square by 2 lambda and an angle. So, let us choose angle to be 0 ok. So, these are radius and this angle ok.

So, I am choosing along this direction is that clear. So, my symmetry is broken ok. So, what so earlier how many fields did I have? I first we should count how many fields do we have because this is a complex scalar. So, there are two fields right two massive scalar fields psi and psi star agree and these are electromagnetic fields. So, there are two massless photons right.

So, two massless photons here and two massive fields here, but is the reason of this what has happened of symmetry breaking? Do we have two massive fields right now two massive scalar fields? No because this well we will just show you that what is the mass of this field right now along this direction, but these direction along azimuthal direction along theta direction the mass is 0. So, we know at least one of them is 0 mass right. So, one mass is 0 other mass is nonzero. How do I find the other mass? So, this is the set I need to so basically I need to do a perturbation around this. So, let us do the perturbation.

So, rho minus rho naught. So, this is rho naught for me it is good of this rho naught and the perturbation is chi by rho square root 2. I am following this gifted amateur book. So, that is chi by rho is the perturbation you may substitute it there. So, actually you can do this algebra. In fact, you do not need to go to Lagrangian.

So, I just substitute rho from here which is rho equal to rho naught plus chi by root 2. Now, this will be rho cube. So, which will be rho naught plus chi by root 2 cube right minus 2 lambda mu square rho naught plus chi by root 2 and this will be d square chi by root 2. So, what do I get here? I will do the simplest possible thing. So, expand these two leading order in chi.

Leading order means drop the higher order like chi square and chi cube. So, this is minus 2 lambda rho naught cube 3 a square d right a plus b cube is 3 a square b 2 lambda 3 is 6 rho naught chi by root 2 square this and this will be mu square rho plus mu square chi by root 2. So, what happens to this these two together? What is the sum of these two? 0 no because that is fixed point. What how do I get the fixed point? In fact, you can just substitute here if you are confused. Just substitute rho naught here these two cancel and here what and rho naught square you substitute it from there.

So, which is mu square by 2 lambda. So, 2 lambda cancels. So, basically I get proportional. So, here is a root 2 this root 2 is I ugly, but coefficient is here mu square by 2 this is a minus mu square by 2 plus mu square 2 this is a mu square by 2 chi by root 2 plus right this is a plus here this is a plus from here and minus here yeah.

So, we get this. So, these my new mass mu by root 2 is my mass. So, I have two scalar fields now one is massless other one is massive and mass is mu by root 2 okay? everybody happy with this. So, this I think this what I get there is a algebra problem ok. So, instead of 2 root 2 is 2 root 2 chi by root 2. So, I made some mistakes is 6 and this is 6 here.

So, this is a minus 3 you know the minus 3 if I got minus 3 and plus 1 here the minus 2. So, the mass is mu root 2 take m square chi mu root 2. So, we got a mass now we have one Goldstone mode which is 0 mass and one is this mass now what happens to the photon. So, I right now I have not touched the photon field. So, let us rewrite this Lagrangian.

So, here this is this part is sitting there rho square q square C square now here I should substitute rho as rho naught plus chi by root 2 ok, but chi is small to lead in leading order I can write as rho naught square q square C square and dot dot dot is the higher orders this is my Lagrangian. So, this is minus mu square chi square this is a mu scalar particle ok which has mass mu well mu 2 root 2 mu that is half come here by this transformation half has come down ok the here. So, this is a new guy which has picked up mass. So, rho naught I substitute now from my formula. So, rho naught is what 6 mu squared by lambda ok.

So, that is what we get here. So, this one I can write as half M square C square when m is this ok. So, the photon which was mass less has become massive and not one all the three components. So, the two photons it turns out that other Goldstone mode which is supposed to be there is not there anymore ok. In in particle experiments nobody has seen that Goldstone mode we now see in Higgs and this three charged particles or three sorry three photons which are massive not charged ok three photons which are massive. So, this is half M square C square three of them ok.

So, photon has become massive ok. So, this is called Higgs mechanism ok. So, what we broke the continuous symmetry. So, we got one mass less than one massive, but when the effect. So, basically what is told is this photon basically eats up that Goldstone mode and becomes massive ok that is a Coleman supposedly interpret popularization he says this photon eats up this Goldstone mode and becomes heavy. So, Efiel has gained mass by eating the Goldstone mode ok.

So, physical implications. So, we start with two massive particles and two mass less photons and what we got is one massive scalar particle and three massive vector particles ok. Now this is for scalar field which is interesting I mean I have given you all the basically details, but for QED of course, you need to do much more work not QED you have to do electroweak E and M plus weak nuclear. So, Lagrangian is reasonably complicated and if I work out Higgs mechanism for that then I get this three vector particles are the three particles which were discovered W plus W minus and Z zero this is ok. This was discovered in particle accelerators before 2000, but this massive scalar particle by the way this particle. So, which is a Higgs particle in all this mu squared by 2 lambda or 6 lambda.

So, that mu squared by lambda with a pre factor that is a Higgs particle ok. So, this scalar particle which was mass less the continuous symmetry breaking it picked up a mass well it was not mass less sorry I think back. So, it picked up this new mass it was there was a mu squared is there. So, it is massive, but it picked up this new mass and there is a Goldstone mode which was 0 mass, but then this photon it have that Goldstone mode and became fat and massive and that is what we got this three massive vector particles.