Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

Prof. Mahendra K. Verma

Department of Physics

Indian Institute of Technology, Kanpur

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Lecture – 45

e square by 12 pi square, e is the electric charge in CGS units. So in field particle physics people normally use CGS units. So it is not in coulomb but it is in ESU. And e square by 4 pi is alpha which is called fine structure constant. And fine structure constant is computed is called 1 by 137 when your momentum is close to 0. It will become clear so in a minute.

So I can solve this equation exactly like what we did before. So this lambda is okay I should not, lambda is e square no? Okay for electric thing lambda e square by 4 pi I think I have right now I forgot but the e square, is a coupling constant. Is the interaction term no? For two charged particles what is the interaction term? e square no? So this e square is come here. So exactly same logic.

I get this. Actually I may have a typo here. I think it is e cube. I do not remember right now. So this equation I am slightly not sure because I did copy paste here.

But anyway the running coupling constant we derive exactly same manner what, well I mean the derivation of this is complicated. This requires huge work, Nobel Prize for Feynman and Singer and Tomodaga. But this huge work but once you have the beta function I can compute how the coupling constant will change with momentum. And so that is the momentum. That is the running coupling constant.

It is called running coupling constant because coupling constant changes with momentum. Now this is the experimental data. So why this axis okay. So x axis this Q is momentum okay. So this is normally in particle physics this Q is called momentum okay.

Now it is two particles are coming so it is a relative momentum but we do not need to get too much into that. So this is like p okay for what I wrote p. p square to M square. I think I am making too many mistakes. M square okay.

So this is the momentum. Now when I increase momentum my coupling constant is increasing. This alpha is the coupling constant is increasing okay. And these are experimental data. Now if you go to very very small momentum then you would hit this line.

This suppose it will suppose hit that line okay. And this line is 1 by 137 okay. Of course I think people have done a lot of work to get these three digits okay. Now I will not discuss that. I do not know but this is how you derive the running coupling constant for QED.

So see that when you increase the momentum my coupling constant is increasing. What does it mean? When I increase the momentum that means I am going closer and closer to the electron no. To probe the electron closer I need larger momentum. When I go closer then I see more charges okay. Now what is it why should I see more charges? The reason why I see more charges is if I have electron there are this vacuum is not free space.

This electron will create lot of electric field and that will create electron positron pairs. So they create e plus and e minus. Non-electron positron pairs also create quark, anti-quark pairs all sorts of pairs. And which will come closer? e plus will come closer, e minus will come closer. Negative charge will attract the positive charge.

So e plus will come closer and e minus will go far away. So it turns out effectively it is keeping some e plus closer, positive charge is closer. So it has it is shielded when I go closer and closer. So the shielding of charges when I go of electron because of the virtual particles. This is called vacuum polarization okay.

So this is vacuum itself is polarized like material polarization. Vacuum is not vacuum like free space. It has all these properties. So free electron there is no nothing called single electron. Electron will create lot of virtual particle, real particle.

I should not say virtual particle. Real particles which are pairs, particle, antiparticle pairs and this always shielding because of this. This new particle will shield electron. So if I am too far away then I see less charge. If I go closer then there will be less shielding right.

I mean if you go meet the bare electron that is infinite charge, huge charge okay. So this is called I mean you do not go close to the electron but this is what is a formula for the coupling constant okay. So this is how we derive this charge renormalization okay. And why is it this is called running coupling constant okay. This is running, it is changing with wave momentum.

Now this for QED, what happens to QCD, quantum chromodynamics? That is even harder to compute this beta function. This beta function is even harder okay because it has coax and gluons okay. That is you have to work with coax and gluons and it is definitely not easy. But we can write down I mean this was written by people. This is the equation for the running coupling constant which is very similar except a sign change.

If for QD it was minus, for QCD it is plus. So what happens is a plus sign. So for large momentum what will happen? This is M squared. What happens to large momentum? It will decrease in fact become 0 for large momentum. So you see that is what is happening to this coupling constant is going decreasing with momentum and if you go to huge momentum then it will become 0.

So what does it mean? Coupling constant being 0? It has interpretation. It is again won Nobel Prize. Low force, the quarks are free when they are close by okay. So the quarks do not I mean put the two quarks together they are happily sitting side by side. When you take them away then it becomes harder.

There is a force when you take them away. In fact you cannot separate them, make a free quarks because the force is increasing with distance and it is apparent here. That is what it meant. If I decrease the momentum that means I separate, a coarse grain of became larger momentum then I am my force is increasing. My coupling constant is increasing.

So it is impossible to get a free quark. So this is coming from of course the beta function computation is hard but once I have the beta function I can compute this training coupling constant. For phi-4 theory is easy. I showed you how to do it.

We will do that okay. So this is for the charge for the coupling constant. Now I should also look at the renormalization of M squared. That is another parameter. So let us look at M squared renormalization, running mass okay.

So this is my equation. So here I get delta m squared. This is the formula I showed you okay. Now this integral the next step is easy. This one right, it is exactly same this omega t is surface area. But this one we want to do slightly sophisticated.

We can write down as m squared is small so I write as m squared outside 1 plus k squared by m squared okay. I am going to do some approximation anyway but so this one has two terms okay. I think so I will skip the algebra okay. I will just go to the this integral can be easily done but it has two terms one coming from 1 over m squared other coming from k squared okay. So this will have d minus 1 dk no no sorry no no other way round other round k squared is big.

So I should take k squared out not other way round. So I should take k squared is large. So I want to do expansion then I need to do that 1 over k squared 1 plus m squared by k squared okay. So 1 is 1 over so k d minus 1 by k squared and I do the expansion this is 1 over k squared 1 minus m squared by k squared right.

This goes up until expansion. So other term is 1 over k 4 k d minus 1. So two terms and what gives you important contribution is this term. This time is small okay. So this term is ignored in fact. So I will not discuss this in detail but this term is ignored and the term which is d minus 1 minus 4 which is d minus 5 right.

When I integrate I get k d minus 5 dk right d minus 1 minus 4 is k minus 5 integrate I get lambda d minus 4 by d minus 4 right. I mean that is the integral. So this is lambda d minus 4 will become 1 because these near 4 and I get d minus 4 m squared is coming from here and this is the term coming from lambda to the power lambda naught minus lambda right. I mean this is their lower limit and upper limit. So this coming from that lambda to the power lambda naught to the power 4 minus d or d minus 4 and lambda to the power d minus 4 okay.

So this is the my correction okay. So this bit of algebra but I think we will skip some steps. We will just keep this term. Now B again expand is like e to power l.

B is e to power l. So this is log. So you can write this as e to power l 4 minus d 4 minus d is epsilon okay. So this gives you that log B okay. Basically it gives you L like before 1 minus this what is what will that be 1 minus 1 plus epsilon l right. So 1 1 cancels so this is epsilon l. Epsilon will cancel with that and get l and l is log B.

So this is log B. So m squared correction is proportional to log B. So I can derive the equation, differential equation. So that is m squared well now again we need to stop a minute. m squared decreases we go to higher smaller momentum.

There is a minus sign sitting here okay. So now I need to go to higher momentum. So I need to change minus to plus. So I just this sign I change it if I go from lower wave numbers to higher wave number.

So this is become plus. Now this can be solved. In fact this is a bit simpler. I mean this this equation I need to solve. I will skip the steps and I write down the final answer.

It is this. M squared is M naught squared plus this. So this momentum mass is increasing when I go to higher momentum okay. So this is what we get. So we can see that both charges and mass for phi-4 theory increases with momentum okay. Now I made a remark for Q D without deriving the beta function but okay I mean so this is how we do it. So summary, see phi-4 theory charge and mass get renormalized due to interactions.

Both charge and mass increase with momentum or else we go to small scales. Q D has similar behavior okay and Q C D in fact is other way around. Okay I did mention it. Coupling constant becomes small when you go to lower higher momentum and that is it. Stop.