

**Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium
Perspectives**

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Lecture – 43

Alright, so let us compute beta that exponent which connects magnetization to $T - T_c$ to the power beta, okay, that is the exponent. So formula for beta is $d - \gamma$ by γ , γ and γ are that critical eigen values, no? So we discussed that before. So this beta is this formula beta, okay. And let us derive it for, let us derive this formula beta to first order, okay, this is the formula. And it should be 0.13, okay.

Okay, so I will do it in the next slide. So d is 3 dimensions, okay, that is what we are looking for. So the formula is derived in a perturbative way, it is perturbation, okay. So $d - \epsilon$ is the definition and ϵ is 1, no? For d equal to 3 ϵ is 1.

So we substitute d equal to $4 - \epsilon$, this goes here. Now γ is $d/2 + 1$, okay. Now this d I substitute from here, $4 - \epsilon$ by $2 + 1$ which is 4, $4/2$ is 2, $2 + 1$ is 3, so $6 - \epsilon$ by 2, okay. So this is my γ . Everything we write in terms of ϵ , okay.

Now γ is $2 - \epsilon$ by 3, okay. So we just substitute, so this is d , okay. And here $6 - \epsilon$ by 2 is 3, so I have to put minus, minus 3 plus ϵ . So this is coming here. And the bottom is $2 - \epsilon$ by 3.

So $4 - 3$ is 1 and the numerator becomes $1 - \epsilon$ by 2, right, so this is that. And bottom is $2 - \epsilon$ by 3, okay. So far it is okay. Now we can do the expansion. So this is clear, no? So this $2 - \epsilon$ by 3 and when it goes up, it becomes 1, so this is $1 - \epsilon$ by 2.

$1 - \epsilon$ by 2, $1 - \epsilon$ becomes plus ϵ half and just keep to first order and we will get that, okay. And for ϵ below 1, this is one-third, okay. So this is how you derive all the exponents. This is perturbative by hand, okay. You do not need, well you can of course, you can put here, you will get pretty similar number, not one-third but something else but close to one-third, okay.

Let us just do it here. d is 3, $3 - \epsilon$ will be $3 - 2 + 1$ which is 2 divided by ϵ is five-third, no? $2 - 1$ by, so five-third. So half divided by five-third which is $3/10$, okay. So this number is 0.3 but this gets 0.333, okay.

Traditionally, people have been doing this in field theory. So expand near fourth and put ϵ equal to 1 or d equal to 3, alright. So next topic is mass and charge renormalization but I will focus on five-fourth theory. Thank you.