Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Lecture – 41

So, just a brief on relevant and irrelevant variables, these are very important in renovation group. So this is called renovation operator, you can think of that the matrix is an operator, right now it is linear, but it could be non-linear as well. So, if I start from some parameters, then I will go to the new parameter under this operation right, we showed that R0 goes to R prime, okay. So, R(g), g is a parameter, apply R it goes to g prime, what happens to the fixed point? If I start from fixed point, I remain there under R(g), so this property of g star, g star is a fixed point. Now, I think I will not get into detailed discussion on this, but let us just quickly look at this stuff. So if I, at g star I will remain there, but if I start from g, then I go to g prime, okay, that is what I am going to.

So it is bit of algebra, so I make g and g prime close to g star, so I can do linear perturbation. So, g prime minus g star is nothing but R(g) minus g star and R is a linear operator. So, linear operator then we can write in a matrix form, so it has eigenvectors, it could be right now a 2D matrix, but it could be any dimension matrix. So R lambda l is lambda l, so lambda l is a eigenvalue and this object is eigenvector.

Okay, so if I apply this R, so basically delta g prime, delta g I can expand in terms of these eigenvectors, okay. So I will basically start from here, but I will get how this coefficient Cl is changing under, under R(g). So the Cl, Cl prime is my, what happens after R(g) operation, which is equal to Cl lambda l, okay, so that is what I have written it here and okay Cl prime is Cl lambda l, but this is the beauty of this. So lambda l is written as B to the power Yl, okay, this is my exponent, okay, this is my exponent and Yl is a important coefficient, well eigenvalue already obtained, you know, 2 minus epsilon by 3, but that is in L, not in B. So in fact what we got right now is already Yl, so what I had was, there was not lambda l, but Yl and so Cl is basically going as exponential YlL, okay.

Now we will make, we will make a table, so Yl you have to look for, if I write in terms of l variable, I get Yl as a eigenvalues and if Yl is greater than 0, then it is relevant, that means that parameter is growing with, under R(g), if lambda l, so I am sorry, this is less than 0, if lambda l is less than 0 is irrelevant, in lambda l is equal to 0, then it is marginal. So it can depend sometimes, it cannot depend sometime, depends on the higher order corrections, okay. So for the Gaussian fixed point, let us look at the parameter Yr or Yt, so R is a, that r0 direction, Yr is 2 and Yu is minus epsilon, so these are irrelevant, but Yr is relevant. And for non-Gaussian fixed point, similar story that this is relevant, but this is not relevant, okay. So we had the fixed point, rather we had the R(g) operation, we saw how the parameters are changing with, under R(g), but now we are, we got certain property near the fixed point, okay.

And now I am going to connect it to a stat mech, a phase transition. Was it by the, the non-linear part has been done in classical physics, how to analyze a matrix? Yes, no? Eigenvalues. Eigenvalues. So, dynamical systems like oscillator problem, right. So it is very similar, I mean the same analysis, okay.

So let us now connect it to stat mech. Thank you.