Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Lecture – 39

So, let us do it perturbatively and we are ready for it, okay. This is complex set of calculation. So, I am not, we cannot, well there are some schemes to do non-perturbative, but they are complicated. So, we will do perturbative calculation how to compute R less in terms of R naught. R naught was my original variable. Now, under coarse graining what happens to R naught and that will be R less.

I need to derive R less and C less, okay. So, step one coarse graining. This is the most difficult problem. I told you about rescaling.

Rescaling is exactly like what I told before. So, coarse graining is what is a difficult aspect. Average here I will average over in Fourier space, I will integrate those modes, integrate when integral is there, no? So, I integrate those modes. It will become clear in this slide, okay, three, four slides, okay. So, coarse graining step one.

So, this is my free energy. So, my, as I said my phi is phi less plus phi greater. So, this one is like phi square, phi 4, all these functions, no? So, you can just imagine that, let us imagine I have phi squared. So, it will be phi less for phi greater squared. So, I will have term which is only function of phi less, I will have formula which is function only of phi greater and I have terms which are combination phi less phi greater, okay.

So, my free energy will be looking like three parts, function of phi less, function of phi greater and mixed terms. So, these are mixed terms. Phi greater, it may be some non-zero term, but that is like, well, now one more thing I will assume that phi greater is a random variable, okay. It could be having random non-zero mean, but it is a random variable. So, phi greater equal to some average value plus fluctuation and the fluctuation is Gaussian.

Remember we had Gaussian fluctuation, so exponential minus phi greater, it has this kind of, it one is x squared and x was with a zero mean. So, we will use that Gaussian property Wick's theorem essentially to simplify this integral as well as some of this integrals. So, these are critical property, system is in equilibrium, so fluctuations follow Gaussian properties, okay. This is a cornerstone of statistical field theory or statistical physics, the fluctuations obey Gaussian distribution, okay. So, this term will be basically can integrate out, this will be some constant, but what matters is this mixed term, okay and that is what we need to, we need to keep phi less and integrate out phi greater.

So, in a group, we need to select some people and average out and keep others, okay. This term phi less, okay, this term which I am writing it here is function of, this is the linear term, right, R naught plus C naught grad phi square, this one and this is a non-linear term which I wrote it before, okay. Now, so this term is written here, this term is we do not need to worry, it is a constant. So, free energy you can add a constant that is not going to be mattering about the physics part. Now, the mixed part, mixed part is a tricky part, okay.

So, here the non-linear term of the mixed part. So, we have, so it is phi 4, no? So, phi 4 has two phis, two phi you keep greater, less less and two phi greater greater, okay or you can keep three phi greater and one phi less. So, I am just, look, I write this as phi less plus phi greater to the power 4, is a Fourier, so I cannot really write 4, I write phi K 1, then phi less K 2 plus phi greater K 2 and you do 4 of them. So, what will the terms involve? So, 4 phi or less which is here, right. In this product there will be 4 phi greater that will be where? That will be here, correct? Yes or no? When I write this product what do the terms look like? All 4 phi less or 4 phi greater or 2 phi greater 2 phi less or 3 phi greater 1 phi less or cubic in phi less and 1 phi greater, okay.

Now, this one if I average what will happen to this, this average and this average? This is cubic in phi. So, what is the average of cubic in with the Wick's theorem? 0. These also should be 0. So, these terms give you 0. What can give you non-zero is this, this term, but I cannot average phi less, I have to keep average only phi greater, okay.

So, that is what your question is that when you average you have to only average phi greater and we should basically get the blue sphere, that is what we want. Now, system should be just the blue sphere, but probably with modified parameters, okay. So, we need to worry about this term, this part, that is the most critical part. Because if this term was not there, then my parameter will be remain unchanged, okay. and phi greater is already averaged out.

We will not have combination of phi less, phi greater. Phi less, phi greater combination which can give you non-zero is coming only here in phi 4 theory. If you look, if this term was not there, there are only two of them. So, what are the combinations? Phi less, phi less, phi less, phi less, phi less plus phi greater, phi greater plus phi less, phi greater, right, right. Now, this is here already, this is here, right.

What about this? Under which theorem? 0, no? One of the phi greater when average I will get 0. So, linear thing does not give you any non-trivial behaviour, okay. You need non-linearity to give non-trivial behaviour, okay. And phi 4, this is the most important term, okay. So, let us write down some, I am just going to go through the algebra.

I hope you will understand the steps. So, here after this partition function Z, okay, I hope you can read from the back. And this is function of r less, c less and u less. This it happens looks like pretty tiny, but I hope you can read it, okay. So, I want a partition function as function of r less, u less and c less.

It is not r naught, c naught and u naught. Now, if I do the exponential of that, so this is the term which is original r naught, c naught, u naught and this is a mixed term, and this is a greater term, okay. So, I made a mistake. So, greater term is kept here, okay. So, I had the greater term, I need to keep the greater term, okay.

So, the linear part of the greater term, because I need for averaging in the previous term, this greater term is kept, phi greater, okay. So, that greater term is here, I keep it here, okay, I apologize. So, this is the greater term. And I integrate out this phi greater, but what is the range of phi greater? It is starting from this sphere of lambda to lambda naught. So, this goes from lambda to lambda naught, the range of wave numbers.

So, this is called K space renormalization. I am doing all this in Fourier space, okay. So, if I take the log of this, I get with a with a minus sign outside, I get this term, this term and log of this with a minus sign. So, this object is here with two terms here, okay. Now, I am going to solve this one.

This is the term, look without this term, R less equal to R naught, no? This is, you just focus on this and this. Without the log term, all the parameters will be unchanged. So, this is the term which is coupling phi less to phi greater, which is going to correct R naught, okay. Now, let us go to this log term.

Log term is most important. So, log term looks like that, S greater phi greater and this is a mixed term, M is from mixed term, so M is mixed. So, what is this integral d phi exponential S greater phi greater? So, this is the Gaussian part and any function f. So, remember we had this for Gaussian variables, this is average f, right. I wrote this property. So, this average f, so we write this is average of this quantity, this is my f, okay, minus log of average of this quantity, exponential this part.

Now, this exponential, in fact, it is pretty complicated, you know, this phi, this S is quadratic already and I do the exponential of quadratic, I will get lots of terms, infinite number of terms, but I am going to do the truncation, there is a perturbation. So, I do a truncation, I write exponential of minus x equal to 1 minus x plus x squared by 2, I do this. So, what will I get? I get this. I just exponential I wrote like that. Now, I take log of 1 minus x minus, what is this object? Expansion of log x.

So, I did two perturbations, I am basically dropping the higher order terms, I am keeping only quadratic in exponential. Now, log I will write as, this is x minus x squared by 2, plus x squared by 2 plus x cube by 3 and like that, okay. So, I keep only quadratic. Now, you write this as, this is 1 and the whole thing is x, this minus, so this x for me, this one is x, with a minus sign out here. You understand 1 minus S average minus S squared by 2.

So, this is my x. So, this is just a pure perturbation, you are dropping lots of terms, but after this operation, I will get this, average of S and this is coming from x, this basically coming from x, this one and x squared, when I do the x square, I do not want to keep the square, because S square will give, become fourth, but I just square this one, so I get this one. This is square this part and we will get that, okay. So, these are three terms we have to escape, two first order in perturbation. This is called order epsilon, is first order, okay. So, this is the word you might see in first order perturbation.

Is it clear why it is called perturbation? I basically truncated the series. Now, look back, we will look at the original equation, this is what the equation looked like, right. And log term, I have these three terms, I ignore the rest. So, now each term has this quadratic term and quartic term, four terms, quartic four. So, my left hand side, this one is here, this object is here and this object is correction, okay.

So, what will it do? You should see the, what is the global picture, this R naught less will be R naught plus whatever comes from this correction. Something will come from correction, that will change R naught and R naught less will be number different than R naught, okay. So, I need to get these corrections and corrections are these three terms, okay. So, let us compute these three terms. Please keep in mind, I need to compute this, expectation value of this S mixed, alright.

S mixed, the non-linear term will contribute, the linear term is already in here. I already told you know, linear term will give you 0 on Gaussian, non-linear term will give you non-zero, which has four terms, two less and two greater. Remember, I had said two less, two greater and that is what is here. So, this is my original term in the expansion. This is for S less with R less, C less, this S less with R naught and this is my correction, okay.

Now, the other term which is S square minus S average square, okay, that term does not contribute here. I am going to show you where it contributes, okay. You have to just wait for a minute. So, let us compute this term. Now, this term was 2 phi less phi k1 phi k2 phi k3 greater phi k3 k4 greater, okay.

And what was the condition on k1, k2, k3? k1 plus k2 plus k3 plus k4 is 0, okay. Now, when I average out these two, what is the condition on averaging out? By the way, these two when I average all of them, phi less is not averaged, phi less remains as is. Post-graining only affects the variables in the yellow shell. So, phi less should not be touched, they remain as is. So, when I average them, what is the property that these two wave numbers are equal and opposite, no? You have to remember all these formulas.

So, I had said that phi k and phi minus k will give you non-zero values. Yes or no? e to the power ikx e to the power ik prime x average, unless k prime equal to minus k, you will get 0. So, the modes of the same amplitude, same magnitude, way numbers but opposite sign, they give non-zero value. So, k3 and k4 must be equal and opposite.

So, these two gives you 0, okay. And this is this Feynman diagram. This is that, this is this term. And what is this? This is called a two-point correlation, no? I hope you have not forgotten. This two-point correlation which is c of k.

This is not small C, okay. This is correlation function which is same as Green's function, okay. All that background work we did now is coming into use. So, this is the Green's function and these two legs are here, k and minus k. This is, so k1 plus k2 will be 0.

So, one of them is k, another one is minus k, okay. So, this two-point Green's function, after I integrate this phi greater. So, remember, I need to go back one slide. Here I have this, this is my original one, phi minus k phi k less k, right? Yes or not? This was the original. This is with R0 C0 U0, right? This is original one. From this term, from this term, I am getting a term which is this loop diagram phi k less phi less minus k.

Yes or no? I showed you. This term, log term is contributing like this. So, this is the term which is changing something here. Yes or no? Right? I mean, I get additional term which is, well, this integral will give a number, it is a number, right? The integral is number and I get a new, new, new term which is number times phi less k and phi less minus k which is here. Even if you cannot do the calculation, you should get the broad idea.

That is what is correcting R0. So, that is what is written here, R0. Well, this term is basically this, this object because this, this is a correlation function, okay. This is, correlation function is what? 1 by k, I used to write k square plus m square, no? Green's function for, for two, for free Green's function was this, right? So, it is here R0 plus C0 k prime square.

I integrate it. There is integral involved. In the previous slide, there is integral. And basically I should get a number for this loop and this is what is coming from after all that algebra. Now, this one is a number, no? Yeah, I get a number. Is it correcting this term C0 k square? What, I should add this number to which term? R0 or C0? This is a number, so number should be added to a number. Had there been a term k square, then I would have added to C0, this stuff.

So, that is why this S mix is not changing C0, it is only changing R0, okay. This is a proof that my R less is R0 plus this number, this number. My R has changed under coarse graining. Is it clear? Okay. So, the interaction term, well, I will do it another way which I do it nobody, well, I did it by differential equation.

I think that will probably be more, easier. So, let us see. So, differential equation I will do it after this, okay. So, let us, so these for R0 correction. But C 0 is unchanged, that is why I had written earlier that C0 less equal to C prime.

Well, right now it is just going to be C, C0, C0 less is C0. C0 is unchanged because this is, there is no term which is proportional to k square in the correction term, okay. So it is only a number I get, okay. Now, what about the other term, the square term? So, other term is S squared minus, okay, this S squared average minus S average squared, right. I had that quadratic, quadratic term.

Well, I put a minus, this I have taken from the book. This is all taken from Kopietz. I should give a, okay, one more point. I should, I should go back. This is a factor, this is a factor 6.

Why factor 6? This is a critical factor. This factor 6 because out of this 4, how many ways can we choose 2 greater and, so once you choose 2 greater, then remaining will be 2 less, okay. Because only combination which gives you non-zero value is 2 greater than 2 less because of Wick's theorem. So 4c2 will be 6 and that is why the 6 ways I can do it. And these are basic, this is a property of Wick's theorem, this factor 6.

Now this term, I need to get this term, s squared, okay. Now s squared, so s squared has, so remember I have this 4 phi's, phi k1, phi k2, phi k3, phi k4. So quadratic term, quadratic term, sorry not quadratic, quadratic term, 4 terms involves this and they are integral of dk1, dk2, dk3, dk4, delta of sum of k is 0, okay. Now what we do here is that we take two of these diagrams.

These are Feynman diagrams. So we will involve 8 of this. You could write here phi s1, phi s2, phi s3, phi s4. So the integral will be 4 ks and 4 s, okay. Now I need finally 4 diagrams, sorry 4 legs, quartic term has 4 legs, no? These are 4 legs. But I combine these two and I want to get an object which is only 4, 4 external legs.

What should I do? I should combine them and combine these. And we combine them, I get this phi and this phi averaged which will give me a correlation function. And this phi and this phi 4 and S3 combine, you get another correlation function, okay. So this is the diagram which contributes equal to that. When I do differential equation, it will become more apparent, okay. But this diagram is here and it has two vertex, two u's, one u is here, u by 6 and u0 by 6 is here, two vertex.

That is why you get u0 by, u0 by factorial 4, 6 squared, this two vertex and this 6 is coming because I had to choose two less and two greater. So among them two less here and two greater which are averaged out. Greater ones are being averaged out but you do not make an average like this. This is a, it will give two point correlation, two point Green's function, right.

These are the two legs which we did it already. I want 4 external legs and 4 external legs means this is not you do like this, 4 external legs. Now this is what it is. So we get, okay and so this is the two correlation function, one coming from here, another one coming from here, right. Correlation function is 1 by k squared plus m squared.

Now m is written as R0 here and k is written as C0 k squared. It is the same formula Green's function which we derived before but is coming here. Now we make an approximation that the external legs variables are small. Why? Because I am reaching a coarse grained so it is a big scale. So big scale has small wave numbers, no? Larger wave, larger scale means small wave number.

Way number is 1 by length. So we will have k1 and k2 small. These two guys are going to 0. So this is nothing but k squared, okay. So that is the approximation. You can do the integral no problem but then you have to do many variable integrals.

Well, no problem is you have to do it on computer. It is a difficult integral but this approximation you make that k1, k2 is small. So these two guys will become small and so the one which we are averaging is in a yellow shell which is large k. So the integral we are averaging is coming from here and so and this is a term.

And we do the algebra here. This is 3 by 2 U0 square, okay. And this is quadratic. This multiplied by that. R0 C0 square. And what is it? It is correcting U less. The coupling term, the non-linear coupling term U less, U0 is getting a correction from this interaction.

Well, sorry coarse graining, not coarse graining. Coarse graining is leading to correction to U0 and the final U less is this plus the correction, alright. So under coarse graining I am seeing these parameters are changing. And so you compute effective R and U under coarse graining. So you see R0 and U, R, R0 and U0 have changed from the original values, okay. So under this operation of coarse graining I told you, you know, this, the spins are far apart and this coupling is different now.

We need to do rescaling after this. But before that let me do it another way, this calculation. And I think this is easier what I am going to do now, alright. So we will do computation in differential equation. Okay, so this is the free energy. So given free, given free energy I can compute that, I can derive the differential equation by variation principle, okay.

So that is what we used to do, dT by d phi is delta F by F is variation. And there will, if I took that variation, derivative then I get R0 minus C0 Laplacian squared, this is by parts and this term gives you U0 by 6 phi cube, right. I just take the derivative. So this we done it before, but you can redo it by the same idea. Okay, now we are looking at equilibrium system.

So under equilibrium what happens? This is 0, right, no time dependence. And I will assume that my phi which I am coarse graining is Gaussian. So this one and noisy phi due to heat bath and Gaussian. So it should be basically phi greater, okay. So I will, I should say the phi greater due to heat bath which I am averaging I have assumed phi greater is Gaussian, okay.

So we will basically solve this equation as 0, okay. And I am going to see whether this R0 and C0 will change under coarse graining and U0, the three parameters. So instead of following a partition function and I am going to follow this equation, okay. So we write this in Fourier space, okay. So Laplacian becomes K squared minus Laplacian and this is a non-linear term.

So the equation which I wrote in the previous slide is this. Everybody agrees with that? The cubic term in the, by the way please remember, I think I should write it down, is a cubic equation. So R0 minus C0 Laplacian phi plus U0 by 6 phi cube is 0, okay.

So this term is straightforward. This I hope everybody can do it. And this term is this. Phi cube is cubic, no quartic is cubic for differential equation. It is quartic for the free energy, but it is cubic for differential equation. And this K1 plus K2 plus K3 must be equal to K, right. We did this before. Now we make this partition of phi K less and phi K greater and I am going to average over this phi K greater, okay.

Now please focus on this term, non-linear term. This is non-linear term, no? This is, it is not linear in phi, this is non-linear term. So you just substitute it here, then I will get terms all less, right, I mean cubic, all greater or one less, two greater, but they are two terms, right. So I could have phi K two less, phi greater, K1, so there is two more terms, right. And there is one more combination, two phi less and one phi greater, but two more terms, okay.

So this has how many terms cubic when I do? Eight terms and these are eight terms, right. Two into two into two is eight terms, this is exactly eight terms. So I find differential equation to be more visualization is easy. Now when I do the averaging, coarse graining, so remember I had to do coarse graining over phi greater, assuming phi greater to be Gaussian. So which terms will survive? This one will remain as is.

What about the second term? It is cubic, so it is zero, Wick's theorem. What about this term? Two less and one greater, less you do nothing, less comes out, greater will become zero, again Wick's theorem. All less will become remain less, okay. And so we need to keep this in mind. So it turns out three terms of these are zeros.

This is zero, well this is gone and this is gone, two terms are, well basically these terms are gone. This is original equation. This is what phi less, phi less, phi less. I want to form an equation with only phi less. So remember I have to, I write down equation for phi less here, but the right hand side will involve less greater, less greater and so on.

When I do the averaging, I will get the following. When I do the averaging, I get phi less, phi less plus one more term. Which one? Two greater and one less.

We need to have at least two greater, okay. Two greater or four greater. So I take it to the right hand side. I could have taken, kept in the left and equal to zero, but I take it to the right hand side, u 9 by 6 and one, two greater and one less. And why this factor 3? Because there are three terms, okay. It is easy, we do not need to remember rules.

There are three terms, so that is why factor 3 is there. So this becomes half. Now I need to compute this. Two greater and one less, okay. So one less will come out, no, one less, nothing happens to one less and greater will be, so in fact Feynman diagram will be just looking like this. Two greater and one less and this is a vortex, which is u naught by 2, okay.

Now I need to compute this. So I think you can see the answer right now. This will go out and this will be a correlation function, right. Is not it easy to compare to Green's function and partition function and so on we needed to compute? It is relatively easy. So we take this out, okay and keep only these two and I integrate this.

By the way, here is possible that I can do one more order, okay. I am going to tell you in the next part, okay. I can expand this and I get another higher order term, but we do not go to that order. If I just take phi k 2 out, then I will get this quadratic term, which is correlation function or Green's function. Now what can you say about k 1, k 2, k 3 relation? Remember I had k equal to k 1 plus k 2 plus k 3 vectors.

So when I do this, what will happen? k 1 plus k 2 is 0. So this is gone. So k equal to k 3. So this becomes k or k 3 equal to k, which is exactly same as what is in the left hand side, right. We have R naught plus C naught k squared U k less equal to, okay. So this part is exactly the same form as in the left hand side.

We get a linear term after coarse graining, okay. Now we need to do something here. So this correlation function will give delta function k 1 to k 2. So this will get out one of the k, one of them will go away.

Now I get k 1, k 3. So this dk 1 will be there, dk 1 and okay, so k 3 will come out in a minute, okay. Okay. So okay, there is one more delta function, you know, there is one delta function k 1 plus k 2 plus k 2 equal to minus k. So the two delta function, one coming from here k 1 plus k 2 and two delta function, so two of them go away. And we only get one delta function, one integral here.

The two delta function, I did not write it here. This delta function is here, okay. Or you have to write k 3 as k minus k 1 minus k 2 minus k, you have to write this as from here k minus k 1 minus k 2. Then you have only two integrals, okay. So this is exactly same as what we got before.

This is my correlation function, which is, which I write as minus u naught by 2. This is that stuff, correction. And what will it correct? This is a number. So it is going to correct r naught, fine? Okay.

Now this is a R naught correction. Now let us look at t 2. T 2 is where I am going to do one more order. Well, it is still first order is called, but I am correcting u naught. It is a bit tricky. So I have two greater and one less. Now this greater in differential equation, I have linear operator phi equal to some right hand side, 0, okay.

Now we write as Green's function G of k phi, sorry, linear operator G of k is 1, right. So I am going to expand this phi of k, phi greater as Green's function G of k and right hand side.

So once we have the Green's function, I said, I can write down the solution in terms of Green's function, right. So how do I solve? So here I can solve this phi greater. So L phi greater is right hand side, well phi greater is right hand side is minus u naught by 2, u naught by 6, sorry, minus u naught by 6, 3 u's, 3 phi's, okay.

Non-linear term I keep in the right hand side. These are linear operator, okay. So what is solution of this? If I have an equation like this, what is the solution? So L is written as Green's functions inverse, right, G of k, k. So phi greater of k is Green's function of k and this integral, this guy, okay. We did this in the past.

We write the solution in terms of Green's function. So I am going to write one of this phi greater in terms of what I wrote here, okay. So one, sorry, so this is less. So one less, this less is kept here, okay. This is the less part.

One greater part I write as Green's function, this Green's function. And the Green's function will give you this Green's function. So I, pay attention. This Green's function and three u's. One u is here, one u is here, another u is here.

Three u's, no? Okay. So three u's I write as a, so one greater is here and I write one greater here. So for this part I will keep two less and one greater. I will not keep two greater and one less, but I will keep two less and one greater. I am not doing averaging right now. Averaging will be done later. So here I will have two less and one greater and this one greater, this greater, this greater is coming here and this greater will combine, okay.

So these are Feynman diagram way to represent it. This greater is going to combine with this greater, okay. And we will get a correlation function from here. And Green's function is same as correlation function. So you get the two correlation functions. Is it clear to everyone? So express one of this, expand this u greater is a Green's function and three legs, three legs, one leg, one leg, one leg.

But two legs are u less, phi less and third leg is phi greater. And then phi greater, this phi greater is going to combine with this greater because I need a non-zero value, otherwise I will get zero. So I need two greaters. So one greater from the new one and one greater remaining from the old one, combine them and you get correlation function here.

So this integral will be three L1s. I write L1, L2, L3 coming from here and these original dk1, dk2, dk3. This is a delta function which I am not writing here. And so one Green's function, the Green's function is here. Two Ls, three Ls, one L1, one L3, these are two less-less guys.

One greater guy which is coming from here, greater guy. And one greater which is here, this greater and one less, this is one less. I need three less. Remember the original of in the left-hand side, I have three phi less. I want my new equation to look exactly like old equation except that I am only allowed to change the parameters.

So this is great. Why is it great? Because I have one less, another less, another less, three less. So this is multiplication. So there is a U0, this vortex and this vortex is this vortex. Vortex is this term, this front of the legs. In the nonlinear term, there is a factor in front that is called vortex. So you write as these three less which is, the circle ones are kept here and the integral ones whatever we need to integrate, we write as dk1 and these two greater, these two greater will combine to give you ck2.

k2 plus L2 is 0. And so we got all this stuff. So now if you do the algebra, so we have to follow this condition, this function and k2 plus L2 is 0 coming from here. And what do we get from here? What is the condition from here? This was Gk1. So k1 variable, k1 wave number is leading to three wave numbers.

So what should be condition on the wave number? k1 equal to L1 plus L2 plus L3 coming from here. These are the new babies. So the wave number must be conserved. This is the momentum conservation. So this one and then we have k1 plus k2 plus k3 equal to k.

You put all these conditions, then you get some contraction, delta functions and we get this. This is collapsing into that. So this part is nice before, but you get integral. We have one more condition. We again assume that this external legs are small. So that means L1, k3 and L3 are small and that makes it, this condition comes.

Exactly like what this very, it is similar to the calculation for the, in terms of partition function, but I find it easier. At least I am comfortable with this. And this is what we are getting as a correction. I am sorry, this is collapsed into this PPT conversion and the pre-factor U0 square by 4.

This is not correction because I need to write the equation which is same looking, similar looking as the original equation. So let us now put it together. This was original equation, without the non-linear term, without the right hand side.

So I take the right hand side to the left. So the first term which was U0 by 2, I have taken it to the left. This is the first linear term. This is correcting R0. This is R0 correction and the other term which is coming from this diagram which was this diagram, well I mean basically I have three legs. I made the diagram before now.

So basically I wrote the diagram like this. So it was diagram like that. My diagrams are different than what we do for Green's function. Green's function has four legs. But since I am doing with differential equation I have three legs for the non-linear term.

So this is the term and this we get. The pre-factor is minus U0 square by 4. I have taken it to the left. So what do I get? So now I need to combine them together. So I will, where should I add this term? I should add it here. Where should I add this term? I should add it to the non-linear term.

L1, L3, K3 are dummy variables. So I can rewrite the L1 and L3 as K1 and K2. So these are the two corrections. So exactly same as before. So I am, well I am, my new equation is looking like this except my R less is equal to R0 plus, this is delta R0, this object is delta R0, this one, this object delta R0 and U0, remember this is U0 by 6 here. So I need to worry about that U0 by 6. Already pre-factor is there. So U0 less by 6 equal to U0 by 6 and da da da da and that gives you equation for U0 less is this, which is identical equation is what we got before.

So these are the corrections which we got from coarse graining. Now we still need to do rescaling. So whatever, so I did this derivation before. So I get R less to get R prime. So what we are doing is right now I had the blue sphere inside and yellow sphere outside.

So now you need to make the blue sphere go to the same size as the big sphere. So that is by factor B. So that operation involves this. So for the big sphere, my parameter will be R prime not R less.

So I just need to multiply by R squared for R prime and for U prime I need to multiply by B to power 4 minus D. I derived it before. So this is my R prime. You ignore Zb. So Zb is functional re-automation which we right now we do not need to get there. Zb is 1 for you.

I cut it from that book by Kopitz and U prime is this. So this is the transformation of U0 and R0 under RG which has two steps. One is coarse graining, other one is rescaling. Now it has, you see that it has changed. Now so that is it. So we will start from here and we will see what is the fixed point for this transformation.

So we are looking for a scale invariant system. It turns out under near phase transition, R prime is same as R0 and U prime is, so we will get a fixed point. So we need to get and from that we will derive the properties of phase transition. So that we will do in the next class. Thank you.