Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

Prof. Mahendra K. Verma

Department of Physics

Indian Institute of Technology, Kanpur

Week - 01

Lecture – 04

One more important thing we will need in this course is Fourier transform ok. So, lot of descriptions are easier in fact in Fourier transforms with wave number space ok. So, basically waves are there you know when photon as a wave and then it is easier to describe that in terms of Fourier modes like creation operator, destruction operator. Therefore, well in both real space and Fourier space. Fourier space is create a wave of wavelength lambda ok. So, that is creation operator. So, we need Fourier transforms a lot.

So, it is straightforward, but please remember that this is for infinite box ok. We did lot of turbulence stuff where box is finite 2 pi box or box of size L. In field theory is typically we use infinite box ok. It is a we assume the system size to be really large and it works well and we have no need to worry about box size L ok.

And typically d dimensions is typically 3, but sometimes we will use 4 dimensions as well ok. So, our dimension is d, but d could be 2, 3, 1, 4 or in fact sometimes is also fractional dimensions ok we will find that. Now, one assumption is that f x is a square integrable function ok. What does it mean square integrable? So, mod f x square dx integral is less than infinity or is finite. So, this is called square integrable ok.

So, Fourier transform is meaningful or it gives a finite number if we have your f x is square integrable. This is by I am not sure Abel this formalism of Fourier transform was done by mathematicians and that is what it is ok. So, the definition for infinite box we follow this definition f x is a real space function f k is a Fourier Fourier transform. Now, this is what we use the typically I am going to use that f hat for Fourier space. Sometimes of course, it is going to become operators, but that is later right now we are going to use f x to f to be function real function.

Well f k is a complex function f it has a real part and imaginary part f x can also be complex because we are using Schrodinger equation also f x can be complex that means real plus imaginary. I do not think of it as an operator right now. So, my definition is the integral d k by 2 pi d. So, this is what you will find this normalization 2 pi 2 d and exponential i k x i this is my notation I well sometimes some people write is minus sign there, but well people choose both ways, but I like to prefer plus i k x for inverse transform. So, I am going I am using f k to compute f x.

So, this is called inverse transform and I use a plus sign. So, that is what we will always do that notation. Now, given f x I can compute f k that will be a minus sign. Now, this derivation I think Schrodinger well the quantum mechanics also this is standard derivation, but we need these definitions. So, this you should memorize it and this 2 pi 2 power d is in f x well basically d k by standard.

So, when you write d k you put 2 pi 2 power d. So, this go together and d x has no 1 by 2 power d. This you might have seen in your quantum courses some people write in quantum course d x by root 2 pi in 1 d and d k by root 2 pi. 2 pi is distributed among 2 in 1 d and high dimension is d by 2 in both, but in field theory it is typically this is the one d k by 2 power d and that is it this is the definition and you need to know this all the time. So, this is a very small slide and I just want to introduce the definition. So, that is no there is no ambiguity.

So, these are different than what we do it for finite box in turbulent simulations that was a different part. Now comes the Green's function which is slightly complicated. So, Green's function as I said is a critical thing. Now you will see some interesting stuff in the slides itself and I am doing it right now before getting into field theory because once we have this big well I mean I am deriving something, I am just going to say that it has been done in the class before and it is important that we know what Green's function is and this again has been done in your courses before, but let us get this idea very very quickly. Green's function is basically the most important concept of field theory. Thank you.