Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

Prof. Mahendra K. Verma

Department of Physics

Indian Institute of Technology, Kanpur

Week - 06

Lecture – 38

So, now let us go back to our free energy. So, free energy was function of this continuous variable phi right, phi square grad phi square phi 4. So, we have phi field. Now imagine well please remember that we are working with fields at different scales ok. So, we do coarse graining. So, we are basically looking at larger scales.

In wave number space, so in Fourier space, when I coarse grained it, then which modes are kind of left out or which modes are averaged out. So, think of Fourier modes, this picture is very important. So, these are Fourier space spheres. So, when I coarse grained it, this yellow region will be averaged out.

What do I mean by that? Imagine some signal is made up many many waves right, this is what the Fourier means. Now when I coarse grained it, what will happen to these guys? Small waves if a coarse grained at this level, so this will be going away you know. So, of course, amplitude of this signal, we basically look at these modes only. The lower modes are taken off. Of course, this mode may be interacting with these modes ok.

So, they are non-interaction of these modes with that mode, but that is what we need to take into account. These interactions how are they affecting the parameters, but when you average out these modes are gone. So, it corresponds to in so these are you can imagine 3D modes. So, when I average out this yellow shell is to be taken off and what survives after averaging is file less in coarse graining ok. And after coarse graining we need to make this blue sphere bigger.

We need to make the blue sphere go back to its original size, means I am going to expand the blue sphere to lambda naught. So, Fourier space that is what would it imply. In real space I am shrinking the size, in Fourier space I will be cutting down those modes, the bigger things will be bigger spheres are taken off ok. So, we write down we write down phi field as phi of x is phi less of x plus phi greater of x. And your idea is to retain only phi less x and average of phi greater ok, but it is not that it simply drop them off.

So, my free energy was function of those parameters know I had r naught, p naught, u naught remember that phi 4 theory. Now, when I take these fields out these parameters will be a different number ok that will become in fact we call it r less, c naught less, u naught less. So, after my operation of coarse graining my parameters would have changed if I am going to demonstrate how it has changed ok. So, this is a partition function. Now, partition function after this operation is well we have both this phi less phi greater.

Now, when remember I had this integral of all the modes, but we want to average out phi greater. So, this I want to basically write in terms of d phi less the mode which survives are d phi less. Remember when I wrote the partition function I have some integral all all phi 1, d phi 2, d phi 3 like this. So, what will survive is only d phi less this is a description of coarse grained system, but my S this is a free energy of this this partition function Z. Now, of course, function of phi less, but this S this free energy the parameters would be would have changed some would have changed not all of them some parameter will change this is a general operation and this what Wilson did it carefully ok and this is what I will describe how to do it for phi 4 theory ok.

So, finally, I have this exponential of S this stuff ok. Now, so this is my partition function ok. So, I need to basically compute partition function for this sphere first. You should have only phi less variables no phi greatest variables ok, but it should capture all the physics. So, imagine I had this big spins the interaction I should have those interaction J 1 well not J 1 would be after rescaling, but I first do it by coarse graining J 1 bar will do the rescaling after that ok.

So, let us write down I mean let us get slightly deeper into the procedure. It is a definitely highly mathematical and reference complex, but you have to just pay attention what is being done ok. So, after coarse graining my variables are only phi less right phi greater is washed out my new variables are. So, these my free energy well this terms did not exist in the before in the previous author, but after coarse graining maybe there will be terms like k 4 k 6 earlier I had only grad phi squared no. So, that will correspond to k squared, but under this operation we may have new terms we do not know I mean there could be new term.

So, these are new terms these are possible linear terms. So, these corresponds to grad phi to the power 4 right 4 will give us k 4 this is grad phi to the power 6 and these are all quadratic term well sorry I am not I am making mistake grad square phi squared. So, this so my phi is only two of them phi minus k phi k these are quadratic terms in phi. In the non-linear term so phi 4 term what will phi 4 term give you Fourier space? It is a convolution and this is a convolution phi k 1 phi k 2 phi k 3 phi k 4 and I sum over all the modes. So, this is my non-linear term I could have some more non-linear term I am not writing it here it turns out we can for phi 4 theory these terms will not they will go to 0 I will show you some terms will go to 0 under R G.

We need this one we need this one and we need this one this prone by Wilson that is this why is a remarkable theory. So, in final theory we should require only a finite number of terms. If you require infinite number of terms then the theory is non-renormalizable. So, all the good theories of physics are supposed to be renormalizable theories well we gravity is a problem that is why is called non-renormalizable, but electrodynamics and nuclear forces can be written in terms of finite number of modes. So, these are so these are property of renormalization theory that you need only finite number of terms under scaling.

So, step 2 is rescaling so that is how does it look like so well let us look at some simple properties which we need. So, these my original so here my variables are these block cross variables. When I do rescaling I basically bring down the scale to this grid. What I am doing I am writing my x prime variable for this box which is x by b here right now b is 4 is that clear to everyone. So, I am compressing it so my what was unit 16 will become 4, 4 will become 1 ok.

So, that is what we got. So, so my integral so by the way this integral must be the same these two integral must be the same under this operation ok. My free energy should give me the same number under these operation ok. So, at this level in the before scaling my variables are phi less, c less, c 4 less and so on, but after rescaling it is r prime, c prime, c 4 prime like that ok and my variable from phi less it has become phi prime ok. Now some interesting observation it turns out which we will do it later the c less and c prime are unchanged ok c less and c prime are unchanged.

So, you can just take my word for it. So, here so I want this integral and this integral must be the same value integral well let us for right now forget about c prime being equal to c less, but when I want this integral to be less same. So, I get c less what will this give us x d dx is a d dimensional integral. So, it will give me x to the power d know if you radius and r to the power d I am just writing this x to the power d radius. Now grad is 1 by length square.

So, I just write as grad square and this is phi less square ok I am just writing this this this this integral here you should think of x is a radius. Now what about this integral this is x prime to the power d right this guy c prime is here grad prime square phi prime square. Now what does x to the power d by grad square multiplied by grad square will give you is radius to the power D minus 2 ok this object will give you R prime to the power D minus 2 what is the ratio of the two R prime by R is b right R prime by R 1 by b ok. So, just substitute it there and assume c less equal to c prime ok it turns out c does not change. So, I am going to show it later just just you have to bear with me then.

So, I am writing phi prime by phi less. So, you take this to the left hand side phi prime by phi less square I get x to the power D minus 2 divided by x prime to the D minus 2 what will that give you x by x prime is b b to the power D minus 2 ok. So, these are property for the variables how does variable scale or how does variable change under scaling is that clear to everyone ok. Now you have to follow the same logic now what about r now by the we have the how does the field change this c and c prime being equal gives you the scaling of the fields by this is very important a field scaling. Now we can use this to find how r less and r prime are related what will I do let us just do it quickly r less x to the power d phi less square equal to r x to x prime to the power D phi prime to the power square ok.

Yes. So, r less equal to r x prime by x to the power d what will that be 1 by b to the power d ok and what is phi prime by phi less to the power d it is going to be b to the power d minus 2 right I just replace it here. So, b to the power d cancel so r b minus 2. So, r less must be equal to is the next slide ok yeah here. So, I am just doing this algebra here r less well I had put r less equal to r prime b minus 2 this is how we prove it. So, under scaling this parameter r will transform like that ok you can apply the same logic c prime I will not discuss it c prime will go b minus 2 c say c 4 less now that is what when b is going bigger and bigger.

So, what happens to c 4 prime decreases right. So, it keeps decreasing. So, that is why it goes to 0. So, that is why we do not need the grad square squared ok you can show the c 6 will also go to 0 in the same manner, but r prime is increasing you see r prime is increasing. So, r is important variable and we can also apply exactly same logic and you can show how does u prime and u less are related u prime is u less to the power of multiplied with b to the power 4 minus d ok.

So, what does it mean for d greater than 4 this will become small right put d equal to 5 is 1 by b. So, for d greater than 4 b u prime goes to 0. So, that is why you might have heard for dimension greater than 4 the non-linear term is not very important in phase transition ok the origin is this scaling formula. We can also talk about u 6 u 6 will go like that. So, it turns out for d greater than 3 u 6 will go to 0.

So, beyond 3 this is not relevant and h we can follow the same logic h prime goes like that. It turns out h will be important for all dimension ok. We can see that d equal to 1 also important d equal to 2. So, this field variable external field variable h is important. So, that is why we need to worry about r prime and h prime ok.

This only under scaling, but coarse graining what happens is not covered here. Coarse graining we need to study how this one effect only. So, coarse graining effect has to be studied and that is a more complicated calculation. Thank you.