Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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It turns out experiments say that this is not correct, beta is not half, beta is less than half. So, basically experiments are finding that it is not working and of course, lot of people worked on it. You will see some names in my future slides. So, what went wrong in Landau theory? So, let us try to motivate what is wrong with Landau theory. One thing is assuming that phi is constant. So, our fluctuations are 0.

So, you can think that how the mob behavior come when people start talking to you know. So, then slowly things build up tempo and the fluctuations of so, there are packets of people who are who are trying to form groups and form opinion right and when. So, that is what happens during transition. So, it is constant phi is incorrect assumption.

Is not that suddenly everything was 0 then suddenly everything became up. There are fluctuations which cause this non-zero to zero transition or zero to non-zero transition. And so, this is not valid and large fluctuations spanning large scales. So, this is the critical point which needs to be taken into account and then only will get the correct theory. And this was done by many people and the leading names are Kadernov, Widom, Wilson, Fisher, McDell Wilson won the Nobel prize for this.

So, let us try to see an approximate argument. Approximate argument because I will some of the arguments when I do the full theory then it do you see that it well it is only approximate. But it is easy to see what went wrong in Landau theory without doing big calculation. So, this is my Green's function right without phi 4 term g0 you may call it. m squared is r right we derived this as I said Green's function is the most important thing for field theory.

G0 k then we can compute G0 of r. I am not including any perturbation right now or phi 4 term. So, how do I get G of r? I integrate this. And so I put this 1 over k square plus n square d d of k. This D capital D is the dimension of the space.

Now this integral is doable I mean we did this in the past we have to choose z axis along r and then you do the integral is doable. But often we do not need to do the integral well. So, I did this integral and in fact, I have derived this e to the power minus m r and this r to the power well I did it for 3D then I said it is 1 over r. Remember this was done in the class for k square plus m square if there is a minus sign then it is a oscillating. Is e to the power i k r or i m r, m is a parameter.

So, I integrate over k so k cannot be there e to the power i m r. So, with this there are poles at i m and minus i m. So, we did this integral. So please make sure that you know how to do the integral. For this part we are kind of getting from the dimensional analysis.

We did it for 3 but 2D I said is difficult it involves Bessel function and the 2D integral is not straight forward for d equal to 2. But we can argue this from dimensional analysis for d equal to 2 and d equal to 4, d equal to 2 is tricky because it involves Bessel function. So, what can we do when what can you argue from dimensional analysis? So, what is the dimension of this integral? So, it is Dk by k square plus m square. What is the dimension of d Dk? Is k to the power d divided by k square, m square and k square should have the same dimension. We are adding quantities of the same dimension.

Please remember you cannot add the quantity of two different dimension. An exponential or sine or cos is taken for a argument with zero dimension or dimensionless quantity. So, this quantity must be dimensionless m r is dimensionless. So, m has dimension of 1 by length which is correct here m is dimension of 1 by length m is dimension of k. So, lot of it you can just do it by dimensional analysis.

You do not need to do the integral, but you see here what is the dimension now? So, this is the k to the power d minus 2. I am integrating k. So, I cannot write in terms of k. I have to write in terms of r. So, there will be r to the power 2 minus d which is here.

I bring it down. So, it is r to the power d minus 2. So, this is the form for G of r without any perturbation. I did not include u phi 4. Now, if that is the case, you can see the correlations.

So, u G of r is same as correlation. I told in the class well before that. So, this one is telling you that my G r is this is a power law for d greater than 2 it is decreasing. But this part is exponential. It is shielded potential.

We discussed that this shielded potential and what is so this guy is following something like exponential something like this. So, here the length scale will come from one length scale will come from here. In fact, length scale will come only from here. This does not give you length scale. Power law does not give you a length scale.

Length scale will come from here and what is the length scale? It is 1 by m which is correlation length. So, this is 1 by m or 1 by square root of r. m square is r, r naught. Now I said the Green's function is same as correlation function in equilibrium. So, now we need to see this fields are correlated in this scale.

Imagine m is small then this there will be this guys are basically will go like that. So, in this region the field is correlated. So, my zeta is a correlation length and this guy will look like zeta to the power 2 minus d. I take it up. So, I am making only hand waving argument but this is dimensionally correct.

Not precise because I am ignoring the phi 4 term but it will give some idea right away. Now what happens for d equal to 1? Put d equal to 1 this correlation is zeta and if zeta is large if m is small, this is large then it is strongly correlated system. So, what happens to d equal to 2? Is constant. So, you can see in 1 d and 2 d there is strong correlation but in 3 d it becomes 1 over zeta. It is like a correlation is weakening.

So, this 1 over zeta if zeta is large. So, correlations are weakened after d equal to 2 beyond 2 equal to 2. So, between 1 and 2 and these only approximate arguments and do not take it too seriously but this telling you that d equal to 1 and 2 order is very difficult to break. And phase transitions are not possible in 1 d and 2 d. Equilibrium phase transitions do not happen in 1 d and 2 d.

And this is a theorem. The theorem is called Mermin-Wagner theorem. The proof is more complicated but you get an idea here that this correlation is strong in 1 d and 2 d. And that is one reason why we are not going to get phase transition in 1 d and 2 d. But in 3 d is possible but still it does not tell you how big is the fluctuation. It is telling you that there is order in 1 and 2 d and it is coming from Green's function.

Now assuming the same form of phi, where phi I can argue is like zeta to the power I take square root. I am doing lot of hand waving. The phi will be 1 minus d by 2 from here. So assuming this phi I estimate the non-linear term. So let us estimate the non-linear term.

This one. Well I am doing in a different way but you can do that way as well. So here, okay, you can do in both ways. So this is going to be x to the d. So let me just follow the same way.

Zeta to the power d, no? d to x. And u naught, let us assume u naught to be constant which is not the case. Both u naught and r naught will change with temperature. I am telling you before hand but we will assume right now that this is not changing. And phi 4, what did I say phi was? Phi was zeta to the power 1 minus d by 2. So I put this phi 4 will be zeta to the power 4 minus 2 d.

So this makes it zeta to the power 4 minus d. So this is what you got. So the non-linear term if I integrate is like zeta to the power 4 minus d. If I put d equal to 3 then I get strong nonlinearities. So non-linear term is pretty big if zeta is large.

So it cannot be ignored. So Landau goes wrong here that he is ignoring non-linear term which is not possible. So Landau has the keys to non-linear term only to get the transition. But he ignores the fluctuation. So this term is like including the fluctuations. So I am waving lot of hands but we will do it rigorously later.

So this is what we will do in later term. So this is what it says that non-linear term, the fluctuations of non-linear term cannot be ignored. This term I need to do the fluctuation. In fact this term also has to be kept. This term is the same order as that.

I am not writing it here. This term is k square, phi square. You can do the computation. This term and this terms are same order. So Landau says grad phi, ignore it.

No, it cannot be ignored. What about H? This factor will come again and again. 4 minus d will come later. The H phi we can also get, so non-linear term is unimportant for d greater than 4. Yeah, so this is important point. So d greater than 4, this will go to, let us say d is 5, then 1 over zeta, this goes to 0 for large zeta.

So this thing again will come in feature in more rigorous way. Now let us look at the, this H phi term. H phi term will be zeta d and H is constant, I will create constant, 1 minus d by 2. Phi is 1 minus d by 2.

So this becomes d, so this 2 here. So we get zeta 1 plus d by 2. So the, this source term H phi is always important. So we have 3 factors, well 4 terms. r naught, c naught, u naught and h, 4 of them. And it turns out for our calculation we need to see which ones to keep, which ones to ignore.

So these are the questions we are going to ask, we will answer in future. So it turns out H has to be always kept. But u naught, keep sometime, do not keep sometime or it does not matter sometimes, it matters sometimes. And R naught will always matter. So these are some answers which we will, we will discover. Thank you.