Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

Prof. Mahendra K. Verma

Department of Physics

Indian Institute of Technology, Kanpur

Week - 05

Lecture – 32

So, little bit one important theorem in fact you can say, but I am going to demonstrate, I am not proving it, but I am just showing you the theorem for a special case. So, for a free field, I am not putting interaction, for a free field I will show you the Green's function and correlation functions are the same, for the free field without interaction. Alright. So, for a free field my terms were m squared phi squared Hamiltonian plus grad phi squared, this is c in front, but I make c equal to 1. If I do the Fourier transform what will I get? m squared mod phi k squared and grad will give us k squared. So, k squared plus m squared will come often, there will be coefficient in front, but I am making the coefficient to be 1 right now.

That is my effective action, this is effective action. Now, please remember what is the Green's function for this kind of operators? 1 by k squared plus m squared, this is g of k. I am setting frequency to 0 or d by dt is 0, why d by dt is 0? Because we are discussing equilibrium field theory. So, there is no time involved in this, it is a in equilibrium, so time is basically frozen.

So, what is now here, how do I compute phi k squared average? This is a 2 point correlation in Fourier space right. So, suppose someone ask me what is phi k squared average? So, it will be I use this formula. So, this phi f phi will be phi k mod square not average phi k squared, I just put it there. Now, it involves this d phi will be, so I just I would like to write what is d phi? So, d phi k d phi minus k, then similarly other wave numbers. So, d phi k prime d phi minus k prime, so all way numbers included is a product.

So, you can write this as a product d phi k squared for every k, because this is a basically d phi k and d phi 1. So, I am going to make one variable d phi k is this one, but some will product for all k's. Now, so I put this, now here you see that these objects integral d phi k d phi minus k primes. So, my function f is not involving k primes, so I can pull these guys out outside the integrals. And the exponential will involve exponential minus k prime squared plus m squared and phi k prime squared.

So, all of them will come out. So, the dot dot dot and dot dot dot will cancel right. The k prime variables are all independent variables and they will come out and they will be on top and bottom, f i is not there for k primes. So, they will cancel and we left with this. In the top will be phi k squared exponential coming from this S naught term and bottom will be without this k squared, t phi e to the power minus S naught phi which is this exponential part is sitting here.

This is what it is. Now, what is this ratio? If I do the integral, how do I do the integral? In fact, it is quite easy by the trick which I showed you in the last time. So, I have to pull out phi k squared. So, I can just take the derivative with little to this variable, they will bring that out. So, you can do the algebra by whatever way you means you like.

This is 1 over k squared plus m squared. So, from here to there is this, you can just demonstrate it very easily that is that. This is very important derivation. In fact, by example, it is not really full derivation that my two point correlation function is same as Green's function. So, Green's function and correlation function will be used interchangeably in statistical field theory in equilibrium.

This is for two point correlations, but we can also write down well Green's function for high point n point correlation is kind of is not well defined for me, I mean if I from mathematician, but we use this interchangeably correlation function and Green's function be equal is used interchangeably in statistical field theory. So, you need to keep this in mind, this is very important point to keep is in equilibrium. In real space, the Green's function will be g naught x minus y, x and y are two points. Now, that is same as correlation function phi x and phi y. This can also be proven, but I will not prove it here.

If I can do the Fourier transform of the previous slide, then you will get this. Now, I think we are running slow. So, this is a correlation function. Now, how do I compute correlation function from this one? So, now, this is important part. So, we have this full partition function this one, right? And I want the expectation value of this.

So, I put phi k 1 phi k 2, I just put it here. So, this by definition, this part is by definition. I want to compute expectation value of any function or operator, then I just do it like that. I hope this part is clear, no? So, I can do this sum is a well this one, I expand S interaction, this part is a series, Taylor series, this series and this series is in fact, I wrote in the previous slide without phi k's. So, phi k was not there.

So, in fact, this sitting here, this part was derived in the previous slide, but this one will involve I mean that numerator will involve this product as well. And we need this correlations in future, this kind of correlations. I am going to show you bit later. Now, this is a infinite series, but we cannot compute infinite series. In fact, each term is very difficult to compute or well difficult to compute and this call first order, second order, third order terms in a field theory.

So, I think I have it in the next slide. Now, I think let me just mention this, this is you need to understand this part what is meant by first order. So, what is first order here? In this one, I write this 1 plus well you just this infinite series you expand it. So, the first term is 1, second term will be sine interaction phi that is the next term. Then next is square bifactorial 2 and normally we stop at the first order.

Now, not normally well I would not say normally, but in the beginner you start with the first order. Second order term will involve more complicated in the integrals and third order is quite complicated. Now, let us see how this interaction first order term looks like. The above will be similar 1 plus this phi is an s interaction that one. Well, by the way I put a 0 here you see what is 0 here? 0 means with the integration with field with the free field.

I am using this is a S naught is a basic Hamiltonian or basic action free field. So, we are always in computing with related to the free field. You can think of that is a if you do the some series expansion f of x is a 1 plus f prime x x minus x0 well sorry f x0 like x minus x0 like that. So, we are expanding around the free field. And why free field? Because free field is assumed to be Gaussian.

Now, I can compute this use the Wick's theorem to compute higher order correlation. That is a big you want Wick's theorem is going to be used to compute all this first order term second order term. You see it in a minute. Is that clear? So, first order what is meant by first order?