Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Now, Feynman Propagator that is what we will try to derive. We want to deal with these kind of objects. Multiplication of two operators or in fact four operators. So, we want to construct Green's function. Green's function involves such things or a correlation function will involve like this. Phi x, phi y or I have phi x, phi y, phi z like that you know.

So that kind of, so we need these kinds of operators in future is bit shifted. So, we always want the left operator, this operator to be at a later time. This x naught, x naught is a time coordinate in relativity. So, x naught, x 1, x 2, x 3.

So, these are position and this is time. So, x naught and y naught represent time. So, here this when I put a t this called time ordered. So, the time ordered product. So, when I put this t, I can flip it around.

I can tell you see here I have phi x phi y and phi y phi x both. So, this one involves the top one is when x 0 is greater than y 0 and the bottom one is y 0 is greater than x 0. So, you well basically what do I do when you operate? First state acts on this, then acts on that term, acts here first, then second. I want the second time to be later than first. So, that is called time ordered.

So, that that is what written here. Now, it can also be written. So, this basically the Green's function. So, you write for given well basically I want a number. So, this is the operator, but the Green's function is a function is a is a number.

So, I will be acting on some state or it could be a vacuum state. I hope I am not losing you. So, these two thing become like that. So, theta is a step function. So, y 0 minus x 0.

What does theta means? This one means here y 0 is greater than x 0 only then it is 1. This object is like that. So, this is y 0 minus x 0. So, when y 0 is greater than x 0, then we get 1 otherwise we get 0. So, this is the first part and this is the second part here x 0 is greater than y 0 to be 1 and we get that.

So, these are all compute Green's function and this is a time ordered propagator time ordered or time ordered product. Do not be too afraid of this, but this this is what it is. So, let us really compute. Now, this will probably make it more you will get a better feel for it. In fact, this slide is nice important slide.

So, remember I had written this operators know for phi. So, these operators phi dagger operator and phi operator. So, phi dagger operator remains well I did it probably too fast a dagger p exponential i p x p y and b dagger p b p and exponential minus p y. So, you can just look at your note that is what it is. Now, this operator operates on vacuum.

So, in a operates on vacuum vacuum well these are all c numbers know these are complex numbers. So, the operator will act on vacuum. So, a dagger p will act and b p will act. So, what is b p 0 will give you? It is a destruction operator on vacuum 0. So, this 0 will act on this and what will this give you a dagger p 0? It will give you p it will create a particle with wave number p.

So, in vacuum we create a particle that this is a creation operator. It is a it is a particle and not any particle it is a particle. So, that is what we have in the next line. this is it that this is p and exponential i p y. So, this phase will also be there.

Now, let us look at phi x, but it is acting on left. So, now, this will act. So, I just write down phi x which you I am phi x I am just copying from my previous line it is this. I am changing my dummy variable q to q because I want to differentiate this is a integration variable. So, I just change it.

So, 0 acting on this bra 0. So, what will what will what will happen now? a q will work I give you non 0. So, in fact, the idea is to just transpose it. So, this becomes a dagger q ket 0 and this is q. So, now, bring it back.

So, this is basically this will give non 0 and this will give 0 because if I transpose it then I get b q acting on 0 it will be 0. So, this basically is of only the particles. See my operator has both particle and inter particle, but in this operation I am only working with particles by this by this construction. So, this is basically giving us q and please keep the phase this minus q x.

So, particle particle. So, I create a particle here and here also well this is in the left part. Now, I am going to combine these two. So, this is a state and this is another state bra state ket state and I just multiply. So, what is going to happen? The c numbers and I am going to get p and q and two integrals d q p d q q and this is 2 E p well basically square root 2 E p square root 2 E q exponential minus plus i p minus p y minus i q x exponential i p y minus i q x. What is this? The delta function know delta p minus q.

So, p must be equal to q. So, this is going to act on this is going to be i. So, let us put x minus y. So, minus i q x minus y. So, I just put p equal to q and one integral is gone. So, I get this well I put q equal to p.

So, I put like that p. So, minus i p x minus y. So, this is basically working with particles and remember x is later than y. So, how do I write the diagrams? This is Feynman diagram. So, this particle is created here. Phi dagger is creating a particle know as particle p.

This is the first operates creates a particle then it goes here and it gets destroyed. So, at x it is destroyed. So, this particle has this lifetime in this region only. Please do not think what is this guy doing. So, if you want to think along those lines the idea is there is a electron or there is a particle going like that and it creates a virtual particle.

It could be photon or it could be pion or something something and then another particle comes and it absorbs this particle. So, particle 1 particle 1 particle 2 comes and absorbs this particle 2 this pion and it goes there. So, this is for this virtual particle for this particle it is not connected with that. So, normally Feynman was doing for a photon. So, all these are for construction for photon, but I am not thinking of photon right now I am thinking of a pion of a scalar particle with mass m.

But this is a technology. So, the technology can apply in other places too. We are just thinking of tools how to how to do it. So, this is for particle. Well, this is 1, but now here x is greater than y and I have phi x phi dagger y. What if I flip it? Well, by this called advanced Green's function is going forward in time.

So, this particle is going forward in time. So, you see this time axis and time is going ahead time is going like that. So, this time is later and this is before. That is quite less forward in time and I am going to show you a little later. So, in fact the Green's function we do all this labor you will find something which we already get forgot for PDE.

But all this work is useful and important. So, this is working with quantum field theory we are discussing particles. So, we cannot really say well what I did for PDE will work well is it happens that we get the same function, but this technology is very important. So, now let us do it the other way round.

So, I will add phi x y acting on 0. So, now this phi x operator is at this one. So, which will give non 0 value? Among the two there is A Q and B dagger Q. B dagger Q because A Q

0 vacuum is going to be give 0. So, B dagger Q, but B dagger is antiparticle. So, it creates a antiparticle with a wave number q.

Clear? And it comes a plus IQ. There is no minus sign here. Now, I apply phi dagger y acting on bra 0. So, which one will be non 0? So, this will be this one will be non 0 E p. So, this antiparticle with wave number P is a destruction of antiparticle.

Now, combine the two phi dagger y phi x 0. So, phi x is first, but it creates a antiparticle. So, this antiparticle is created here at x this time axis. Now, then phi dagger y, whether it is a dagger, but it destroys antiparticle. So, dagger normally you think that it should be creating, but it is destroying antiparticle destroying it.

So, it destroys it here. So, it is created here and is destroyed there and it goes that way. So, antiparticle goes forward in time, but you want to think of particle then the particle is going like that. Particle is going backward and this is advance time compared to this. So, this is the later time.

So, from future it is going back. Of course, they are virtual particles. They are not really something is going and these are only technology. So, these are all based on what the virtual particles are created. Why is it going back in time? I create a time machine, none of it is possible.

So, this is a tools to complete make computation. Virtual particle in some sense is also a tool, is not there, there is a particle there. It is basically mathematical construction Green's function, but final results are meaningful results. These are technologies which we need to compute numbers. So, here I get this object.

So, very similar thing that I get a i p x minus y. In earlier slide it was minus and here it is plus. So, this antiparticle which is going from x to y. So, this is first and this is later for antiparticle. But we think of particle then it is going back.

So, that is a technology which Feynman created. But there are other two people who did simultaneously Swinger and Tomanaga. But Swinger was doing all with Green's function full integral and so on and it did not catch on as much. Swinger is also considered very very smart man and very highly respected physicist. But these diagrams are Feynman's diagrams.

So, that caught on, he became more popular. So, this called G minus for if you think a particle then it is a going back in time is called retarded potential retarded Green's function and these are some names, but you should get the idea what it is. Now, I can do bit more algebra. This is bit tiring I mean I am skipping lot of details, but if all of it is there in that QFT for gifted amateurs. So, this is the time order product.

So, remember it has two parts. So, you need to keep both particle and antiparticle in you need to do that algebra know. So, this why these are two terms. If you keep only one term then you cannot get the whole physics. So, that is what you should think intuitively.

So, these are the two terms which we derived right now. Now, we can also write down theta function like that using complex variables. If you substitute it there then and do algebra which I will skip we get like that. So, this coming from theta function if you just plug this theta function appropriately and do the algebra and do this to come to integrals and so on you get this. Now, this is energy know P naught is energy and P 1, P 2, P 3 are three momentums.

So, you get energy minus E P squared. E for energy for particle with momentum P. Now, E squared. So, E square E P squared is P squared plus m squared this is Einstein formula know energy is. So, what is E squared minus P squared this P well this P vector square P vector square minus m square. What is E squared minus P squared? These are called so this called P mu square or 4 momentum square.

You guys are familiar with this? So, we have time and space vector. So, you can construct P squared and X squared. These two quantities are not invariant under Lorentz transformation. So, if you have one coordinate system which measures P in X, you compute T squared and compute X squared well delta T squared two events delta T squared delta X squared. These are not constant for Galilean transformation of classical physics delta T is constant.

Two events time difference must be constant, but relativity is not, but what is invariant is delta T squared minus delta X squared with appropriate C square multiplication this is constant. Similarly, for relativity is E and P transform under Lorentz transformation. You will get different E different P if I do go to Lorentz transformation that means, you will boost, but the P mu square which is E square minus P square is constant. The Lorentz invariant and these are very important for various reasons and also theoretically. So, this what we write P squared this is the P mu squared on this is not 3 momentum square this is E squared minus P squared minus m square.

So, this quantity is, so this is what we get. Now you may recall what is this? Let us go back to our equation. Our equation is dT squared minus Laplacian square plus m square psi is 0. Now, so this is basically the Green's function g inverse Green's function inverse. So, this guy will give us dT squared will give us minus omega square I omega I omega

Laplacian will give us k square plus k square plus m square. So, if I go to relativity then this becomes, so this is only the sign stop minus E squared plus P squared plus m squared.

Now, I change the sign, sign changes with me I can put a Green's function definition as well. So, this is E squared minus P squared minus m square with a with a minus sign. So, this is what I got here E squared minus P squared which is which is here. So, this guy is basically not basically is exactly this. So, this is the Green's function and we derived it for with the partial differential equation.

So, with all this machinery we basically get the same Green's function. So, this is called Feynman Propagator, this is you should know this word this Feynman propagator. Of course, this was quantum field theory, but is interesting that we get same thing with a differential equation. Now, let me just get now this combination of two diagrams this one forward Green's function and this is a advance Green's function retarded Green's function together it gives you that. So, this combination deals with particle, antiparticle, together.

So, this is our propagator or Green's function. Now, Green's functions are like represent interactions. Now, this interaction was very important and this called if I do the Fourier transform of this what will I get? Go to real space this is a momentum k omega space. I can do the inverse invert it. So, you have to go to basically suppress omega and you write this is only P squared minus m squared keep omega 0. So, what is 1 over P squared? Green's function this Green's function in or k squared you remember this no I think this is I am this well g of k 1 over k squared what is the g of r? Is 1 over r is a coulomb interaction 1 over r you can in 3D.

So, the way to see this is well one simple way I told you to do dimensional analysis e to the power i k dot r d cube k that is what it is. So, this is going to give us by dimensionally k on top it is a dimension of k on top. So, k has dimension of 1 by length. So, this is 1 by r. This coulomb particle electromagnetic field this is the coulomb interaction Green's function.

Now, what I put well this is a plus here actually omega squared minus k squared minus m squared. So, k squared plus m squared. So, this one I mentioned in the my lectures is going to be 1 over k squared plus m squared leads to exponential minus m r by r. And remember we have this P squared is coming as e squared minus so, the omega squared minus k squared no minus m squared.

So, e squared minus P squared minus m squared set omega to 0 omega is small. So, you get k squared plus m squared which is this. So, this shielding of the coulomb potential. So, this what is a Green's function in real space or is interaction in real space. So, what we drew this diagram like that like that like this.

So, these are supposed to be nuclei's this could be proton and neutrons. In fact, this was idea by Yukawa he won the Nobel prize for this. So, protons and 2 protons and 2 neutrons in helium nuclei they are not repelling each other. Well, there is a repulsion coulomb in repulsion, but coulomb repulsion is so strong that it could basically break the nucleus. So, what is holding a holding them together? There must be some other force which is holding them together. And Yukawa postulated that there is another force which is short range and that force is going like that.

This whose Green's function is this. From here you can compute the mass right you know the R then you know the mass, R you can set is the size of the nucleus. So, you can well I mean this Yukawa's work. So, you can get mass and he postulated some mass and they in fact they indeed found a particle with that mass. And these are pions I think some some this mesons particles and this is the particle which corresponds to this Green's function. So, this algebra or this field theory is not useless they are very important and this is what is the potential for this.

No, I am sorry not R by m this is m R, m should be on top. And so Yukawa this called Yukawa potential is not only for this also in atoms we have screening of electrons do not see the coulomb interaction, but this is lower interaction because the electrons are being shielded by other electrons. So, this very important interaction. So, I think we will end with one homework I am going to give you. By the way this homework please do it.

So, and please revise your Green's function mid sem is coming close. So, all this was relativistic. So, this is full relativity which I did not did in more detail which will require more lectures, but can we derive non-relativistic limits or Schrodinger equation from these formulism. It turns out it can be done. So, this relativistic wave function you replace this by psi non-relativistic multiplied by exponential minus i e t by h bar h bar. And this energy we say that well kinetic energy is too small means half m is equal to too small compute m c square.

So, you just put m c square here. Now, you can substitute this in the Lagrangian for the Klein Gordon. We have written in the Klein Gordon Lagrangian or you can put this in the Klein Gordon equation. And then you should derive Schrodinger equation. So, if you substitute it m c square term will be go away and you should be able to under appropriate approximation.

You should derive Schrodinger equation for psi n r derive the Lagrangian. So, you should be able to do it. So, we will end here and I can take some questions. Keep the recording on for let us see how what the how is the questions come how are the questions coming. So, questions? It is all clear is it or is it nothing is clear. No, it is a difficult thing, but so read up that book that book is well it is difficult is not I mean I tried to so maybe you can turn off the recording. Thank you.