Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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So, let us write down the, so this Fermi Sea is a important concept. So, basically Fermi Sea is like Dirac Sea, but they are not infinite particles, but you just keep filling up, filling up, filling up till it reaches the maximum. So, there are N particles, then I have just keep filling them one by one. So, you might have seen this picture, no, this is the Fermi Sea. So, the highest level, so I am keep filling up, no, this, so you can think of particle in a box. So, the, every box has a energy levels, right, by quantization and keep filling up, then highest one will have is momentum.

So, that is p square by 2 m is energy and this is called chemical potential. So, this is a energy is filled up, up to here, if I mean, I am sure you have seen this in the course. So, I do not want to really repeat it, but just for completeness I am doing it. So, particle below is filled up with Fermi Sea and we write down Fermi Sea as this.

I create particles, I create basically electrons at all these level below Fermi Sea, this is what it means. Vacuum was nothing, I create particles at all these states up to pf and these are in a particle in a box. So, these are free particles. So, you just like a keep filling up in a, in a, in 1D of course is you know that is a level run for 1D we create that. What is, I do not know if I get n square energy.

So, in 3D we have nx square plus ny square plus nz square times h bar square by 2 pi. And so, these are number of particles. So, that is a name that is a chemical potential has a, has a interpretation. How much energy is required to pull a electron? So, the top guy will be pulled up and that is same as E pf square by 2 m. So, yeah, you will pull the top guy only, no? So, that is the interpretation and that is coming from the definition of chemical potential.

So, free electrons basically they said, we write down Hamiltonian. So, as sum of particles, well basically energy of the particles with energy positive. So, this E fermi, the one above that, well positive means above mu. So, this is energy bigger than chemical potential and these are energy below chemical potential. So, E alpha is defined as energy minus mu.

So, this is for the particles. So, the free particles which are conduction band electrons are these guys. The holes are below these are holes and they have negative energy. So, this how we write this is p square by 2 m minus mu. So, this is that one and this guy is this one and these are holes.

Now, we will not get into this, but this is how this whole man-to-body framework is created. And charge, so if they have positive charge, then they have negative charge. They are, basically they are not quite empty particles, but they are interpreted as empty particles, kind of, well they are not empty particles. In relative field theory they will be empty particles, but in, these are called quasi particles, but they are negative charge. But mass can, mass need not be just equal and opposite.

They are certain complications. I myself do not know full details about this, but so these charges are basically coming from here, q n minus q, but mass is not necessarily just equal and opposite. So, few last comments. So, Bosons and Fermions, they are fundamentally different particles. So, simple thing is for Bosons operators commute, for Fermions operators, empty commute.

For Bosons, no negative energy states. So, for Bosons we do not need to create a Fermi Sea. There is a bound which comes from this algebra, nothing required to fill up stuff. But for Fermions there are negative energy states. Fundamental Bosons do not have empty particles, but Fermions have empty particles.

These are part of particle physics and there are lot of discussions on it. But I am just giving you a very simplified picture which helps clarify this stuff. And Bosons fix mechanism which we will discuss later, but the Fermions do not have fix mechanism which will come later. So, important points, I think some, my observation. So, this creation and destruction operators of particles are uncorrelated.

So, look, these Fermions are, the wave function is anti-symmetric, no? So, you may think that there is lot of correlation, right? Is not it? When there is anti-symmetry wave function, that means there is correlation between two electrons. But turns out, yes, there is a correlation because of this, its particle nature that they have to be anti-symmetric or symmetric. But this creation and destruction are totally random operation. They are not happening with certain order. So, what I am interested in is this non-equilibrium nature, the easier some kind of energy transfers in this system or are there some kind of hierarchy system which is built in.

So, you may know that Fermi Dirac statistics, you consider the particles to be non-interacting and random, right? Yes or no? I mean this is how you do construct the Fermi Dirac statistics for free particles. Now, if they are interacting then or if their correlation then that should come in the distribution function. But we assume there to be non, and what is the origin? So, my question was, if there is, they are anti-symmetrized, then we cannot write down them as a, as a random particles which is in structural physics. You get my point? Bose-Menn distribution, we assume the particles to be uncorrelated random in a, in a, at a temperature T. But why do you assume the same thing for Fermions or Bosons? They do seem to have certain correlations, right? I mean they are anti-symmetric or symmetric.

So what is the origin of randomness in Fermions or Bosons? And to me it appears that it is a, when it is created or when these particles are there, there though there is symmetry but it is basically uncorrelated, which is a mystery. Well, this is not mystery, it is a very surprising thing that they are uncorrelated. In fact, the correlations we can compute and they will be zero for field theoretic correlation. So, these particles are in equilibrium. In quantum particles are two in equilibrium like Bose-Einstein condensate, no Bose-Einstein distribution, I am not saying condensate.

Bose-Einstein condensate can have order because of interaction. But the finite temperature or a significant temperature there is no correlation. So, super fluids or superconductors they have interactions and that brings in order and that brings in non-equilibrium properties, multiscale properties. So, we will explore some of this which I am also thinking about them. In fact, that is one objective for me to explore this this phenomena.

So, let us stop here and we can take some questions. Thank you.