Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Lecture – 11: Part 2

We will start classical field theory. Quantum field theory is more complicated because of unsaturated relation, the lacration operator, destruction operator, but classical fields we do not need any of it, ok. So these are field variable and we can define Lagrangian equations and so on. So that is what we will start with. So we have field phi xt, so it is function of position and time like a string, you know this string is there, this string. So for this x for every position at given time I have amplitude, ok.

So I can treat that as a field variable, ok. Pressure field in this room, any position there is a pressure, ok. So we have this field. Now so for a string there are derivations, but I think we will skip the derivation, but we can define kinetic energy as a, so every string is, this is a mass element, you know this is the mass here and that mass is changing with position, with time, its position is changing with time.

So it has kinetic energy, half mv squared. So half m is basically rho times dx is mass and v squared that is velocity. Velocity is only along z direction and these formulas are useful only for small amplitudes. For large amplitudes and nonlinearity comes into play and formulas are different, ok. So for small amplitudes these are good formulas.

You can define potential energy. So potential is, ok, so you can, it is kind of elastic energy, ok. So if I, it goes up then it is elastic, well it tries to come back. So that is like, in fact more like oscillator. So it is a d phi by dx, ok, so the derivation I will not do it, but we can in fact again find it in internet or in books.

So d phi by dx squared dx kappa is like a spring constant type thing. So it is a elasticity coefficient, so kappa by 2. Now so you can define action. Similar to what we do for particles, so action will be which is function of phi of xt t minus v, this kinetic energy minus potential energy and so now this is a dt, right, I mean this of course start from t1 to t2. Now this t and v are function of, these functions which are already integrals, t and v are integrals, so I get dt and dx and these are the functions, ok.

Now so we have in particle dynamics we had only integral dt. Now we have dt dx in 1D but in 3D it is going to be dx dy dz. So in 3D we will have 4 dimensional integral, dt and 3 space dimension and this L, this kind of this L, Math-Tell or Curley-L is called Lagrangian density, ok, is called Lagrangian density, ok. And this is integral t minus v dx, this is Lagrangian, this is like before, ok. But I have integral dx because it is a field, for particle we do not need dx integral because it is a delta function object, right, mass.

But here I have this, this is Lagrangian but without dx it is a Lagrangian density, ok. So Lagrangian density is this, half rho dt dx squared minus half kappa dx d phi squared, dt d phi, ok, so this is the Lagrangian density. Now in fact we can derive equation of motion by, we have equation of motion for particle is mx double dot is minus v prime. So similar equation can be derived by Euler Lagrange equation or we can derive Euler Lagrange equation by minimizing action. Now we need to put all those formalism.

So extreme condition, so I defined action with dt dx that, ok. Now I want action to be minimum for a, so for the field trajectory for which action is minimum is the classical solution. The way we have classical solution for particle which minimizes action, here also we will get a field configuration as function of time, ok. So for string it does oscillation, you know, like goes back and forth, dot dot dot, so that solution is a, for that solution my action is minimum, ok. So extreme condition for action will give us the equation.

So delta s delta phi is 0 and, ok, so extreme condition, derivation is very similar to what we did for particles, ok. Now for particle we have d by dt partial L partial x dot, no, that was what we got. Now instead of x dot we have fields, ok. So derivation is very very similar to what we did before. This x dot will be replaced by delta t phi, so this is delta t phi, right.

So that was position time derivative, now I have field time derivative which is with relative time. So this part you can see that it is looking similar to particle time derivative, but it was only dx by dt that time, right, x was only function of time, but this x is replaced by phi, now phi is function of x and t, so I will have d phi dt and d phi dx, two of them. So this is second one with the derivative x. So Euler Lagrange equation for fields is this, derivation is exactly same as for particle, but of course we are constraining that the fields are constrained, they do not change at t equal to 0 and t final. So remember for particles we say dt Lagrange t1 to t2, the position of the particle at t1 and t2 are fixed, right, we do not change the field at that point.

Similarly also the constraint here, the fields are not changed at time t1 and t2, ok. So if you put that constraint like what we did for particles we will get this. It is more algebra but on exactly same lines. In fact, you can see the motivation why these are coming, ok. So these Euler Lagrange equation for fields, Euler Lagrange equation for fields, ok, we need it, so I think you need to memorize it, you can see the form which is this is like dL by dx, so instead of partial L partial x it is partial L partial Phi, then we have d by dx partial L partial of Phi x, these are derivative with derivative x and then we have partial L partial with Phi derivative in time and then d by dt, ok.

Now let us apply to our string, this is my Lagrangian density. So what about partial L partial Phi, this one? 0, no, it is not an explicit function of Phi, 0. What about this one? Partial L partial dx Phi. So take this Lagrangian and take derivative with relative to partial x Phi. So which part is partial x Phi? This is partial x Phi.

If I take the derivative what will I get? 2 will cancel with half, I get minus Kappa dx Phi, dx Phi is a function and I am just taking derivative with respect to dx Phi. Now what about partial L partial L dt Phi is going to act on this and again same thing rho dt Phi, substitute it here, so the first term is 0, second term will be minus minus equal to plus Kappa, double derivative d by dx is coming on that. So d by dx is Phi. Then the second one is d by dt is coming, so this will be minus rho d square Phi dt square is 0, ok. So and that is our, well that is exactly what I wrote.

So that is the equation of motion, this is a wave equation for a string. It is a linear equation which is valid only for small amplitudes, otherwise no linearity will come and great. So we can derive equations of motion for waves or for classical fields once we know the Lagrangian. So for lot of fields we, well most of the field theory we take Lagrangian, we assume existence of Lagrangian but it does not work for Neiman-Stokes equation. It is dissipative, Lagrangian is not well defined, ok.

We can do some trick but it is not that Lagrangian is there for all fields, all, all field theory. Thank you.