Tapestry of Field theory: Classical & Quantum, Equilibrium & Nonequilibrium Perspectives

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Week - 02

Lecture – 10

So, I will do little bit of Green's function more because we will learn how to use dimensions. In fact, you can verify our results at least by dimensionality. So, I think this is important trick we need it later as well. So, we will spend just 5-10 minutes on that. So, Poisson equation I wrote I derived the Green's function or it is like a potential for a point source, that is the Green's function. So, this is Green's function definition.

Now, I want to look at dimension of G. What is the dimension of G? I am not going to derive the result because we already done it, but remember in 1D it was constant, in 3D it is 1 over R. So, how do I reconcile it from dimension analysis? So, in fact, it is quite interesting to see from here. So, Laplacian has dimension of 1 over L squared, L is the length, MLT, mass length time, G is here.

Now the delta function, now this is for it would be 1D, 2D, 3D. In 1D what is the delta will be 1 by L. So, in 1D. So, Green's function has dimension L squared by L which is L. Now what was the answer for Green's function for 1D? It was this, this was Green's function x by 2 mod.

So, indeed G x x prime was x minus x prime. So, this is the linear potential for 1D, here the potential is linear and the electric field is constant. So, it is consistent. In 2D what do you expect? Laplacian will be 1 over L squared, Laplacian is always the same, is 1 over L squared, but right hand side is delta x minus x prime vector. So, in 2D is 1 over L squared, so is 1 over L squared.

What should be the Green's function for this? Well, we do not get constant, is log r or log the difference. So, G x x prime is log of x minus x prime, whether please note that it is divided by some distance, in the distance is size of the source, you know. So, there is some dimensionalization is done, but log has 0, no dimension, log has no dimension, no, I mean it is in fact L to the 0. So, this is indeed constant. In 3D, left hand side remains the same, G by L squared, right hand side is 1 by L cube.

So, G is 1 by L and answer is r, 1 by r, 1 by 4 pi r. So, these are all consistent, you can verify from dimension that this is what we get. So, this for Poisson equation, I think this is just straightforward, but we should know how to do this stuff. Now, wave equation, this slight difference, right hand side has delta t minus t prime. Now, left hand side, indeed I have to divide by c squared, no, I mean that I did not write it but so, left hand side dimension remains the same, 1 over L squared.

Basically, Laplacian and d by dt f by c squared is same dimension, G, right hand side dimension, let us look at 1D. So, delta will be this is 1 by L and this is also we have to say t by c, that is we have to make it space dimension, everything we want to make. So, if you look at textbooks, where it is done more carefully, it is again space dimension is 1 by L squared. This also has a dimension delta t by c, this is basically we need to convert it to x, because here I want same dimension. So, that means G is constant.

That is why we had this front which was going but in fact, it was constant till minus t to t. Now, you have to recall the derivation which we did some before. So, this was half and in fact, the answer is constant. In 2D, G by L squared, this is going to be 1 by L cube. So, this one coming from here, so G will be linear, sorry, 1 by L.

I did not derive it, we will skip. 2D normally involves some Bessel function. So, if you are interested, you can look at typically this is done in the electrodynamics course or electrodynamics textbooks, good books like Jackson would have this stuff. 3D, G by L squared, this is 1 by L4. So, G is 1 over L squared.

And so, what was the answer for 3D? The answer was delta R minus t by c divided by R, this is what I derived. So, this delta function has dimension 1 by L and R has dimension 1 by L, sorry, 1 by R has dimension 1 by L, so it is 1 by L squared. So, it is consistent, it better be. Dimensionally things should make sense. Often we do not know the real answer, well, we do not know the answer but we can make sensible guess by dimensional analysis.

Now, I will not work it out for diffusion equation, I will leave it as a homework that please check the dimension of Green's function for diffusion equation and Schrodinger equation. Schrodinger equation and diffusion equation have same dimensions other than I which is square root of minus 1, there are no dimension but you please verify for this one. So, that is what I wanted to communicate regarding dimensions of Green's function and we will use these ideas later. So, we are going to use similar ideas in future. Any questions on this? Straight forward, no, but so I think I guess you will have no question. Thank you.