Quantum Entanglement: Fundamentals, measures and application

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Week-01

Lec 2: Review of Quantum Mechanics

Hello, welcome to lecture 2 of this course. In the last lecture we had a discussion on the brief history of quantum entanglement and its significance. In fact the last lecture was a very general introduction. As I said that in this course we are not going to focus on the philosophical aspect of quantum entanglement. In the next lecture we are going to discuss the technical aspect of the subject. To do so we need to know some elementary quantum mechanics and that's what we are going to revise in this lecture today.

Also we need to know some mathematical tools. So, before we start with this lecture today we are going to start discussing some mathematical tools and then we will continue that in the next lecture as well. So, let us begin.

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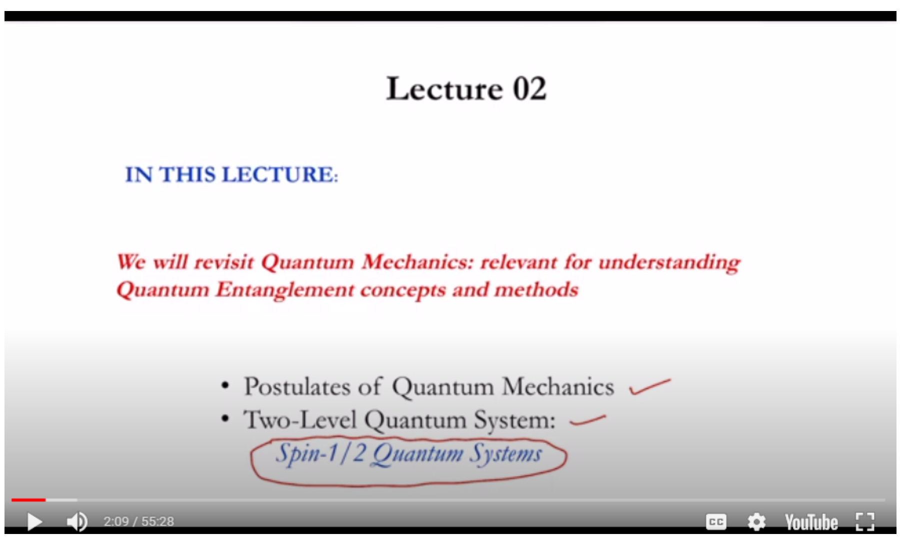

In this lecture we will revisit the fundamentals of quantum mechanics that is relevant and necessary for understanding quantum entanglement and its measures. We will start with discussing the postulates of quantum mechanics followed by discussion on two level quantum system and as a two level quantum system we are going to consider the so called spin-half quantum systems. You will find this discussion very useful later on in this course. We will discuss all the necessary mathematics as we move along and already I assume that you know some elementary quantum mechanics.

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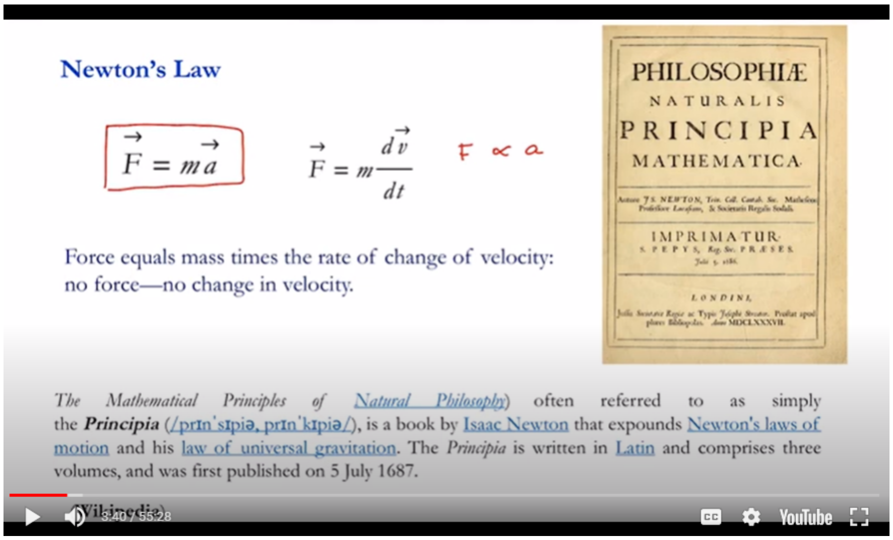

This particular lecture is based on the first chapter of this classic book called Modern Quantum Mechanics. It is written by J. J. Sakurai and Jim Napolitano. If you really want to learn quantum mechanics in a deeper level I recommend this book very highly. Now, let me begin by discussing what we mean by state of a system or a particle in classical physics. As you know that classical physics is primarily based on Sir Isaac Newton's work. Newton was a genius and in fact, his laws of motion were quite counterintuitive. For example, his law F is equal to ma. You see here he said that the force is directly proportional to acceleration which is basically the rate of change of velocity. He did not say that force is proportional to distance or displacement. Right? And experimentally this law is validated in our day to day life. And all the works of Newton were registered in his classic book called The Mathematical Principles of Natural Philosophy in short called Principia.

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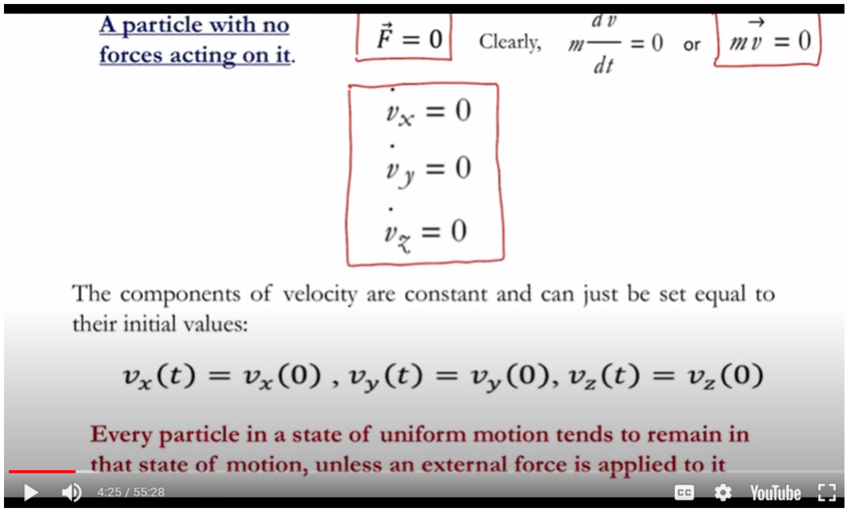

Now, in order to understand what I mean by state of a system or a particle in classical mechanics or classical physics. Let me give a very trivial example where let us consider there is a particle where no external force is applied on it. This is going to result in this equation. Right? And that means that the rate of change of velocity along all the three directions are equal to 0. And this basically means that the component of velocity are constant and can be set equal to their initial values. Right? So, if say if the particle is at rest say Vx of 0 is equal to 0 or this is 0, Vz of 0 is 0 at time t is equal to 0. If the particle is at rest it is going to be at rest or if it is in uniform motion it is going to remain in uniform motion. And this is precisely the so called Newton's first law.

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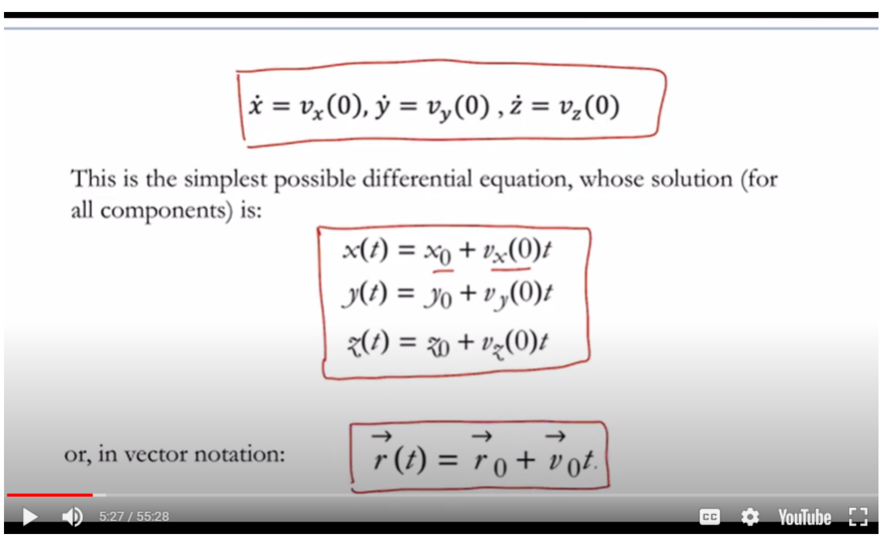

In fact from this equation one can easily obtain this very simple differential equations. And if its solutions are trivial and all of you know that this is going to be the solution for this particular example. What it says is that if you know the initial position and the initial velocity of the particle we can know its future position at a later time. So, in short this solution is written in this form. And this is the way this basically gives us the hint that if we know the initial position and initial velocity or momentum of a particle we will be able to predict its future trajectory. And this is used to define the state of a particle in classical physics. We can in classical physics we can define the state of a particle or a system by assigning its position and velocity at a given time.

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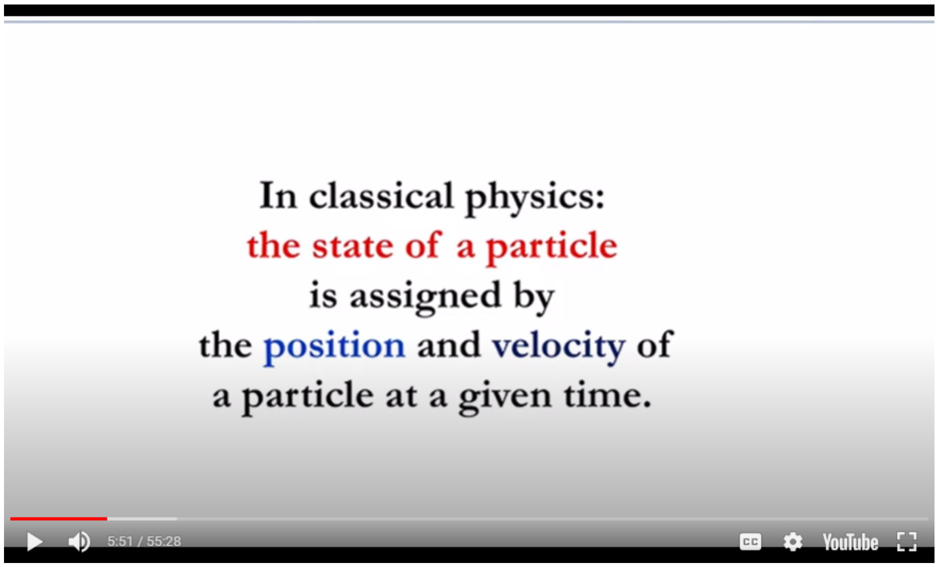

If we can assign position and velocity or momentum of the particle at a given time we will be able to predict its position and momentum at a later time as well. So, this idea of defining or assigning the state of a particle however we cannot extend it further to quantum mechanics.

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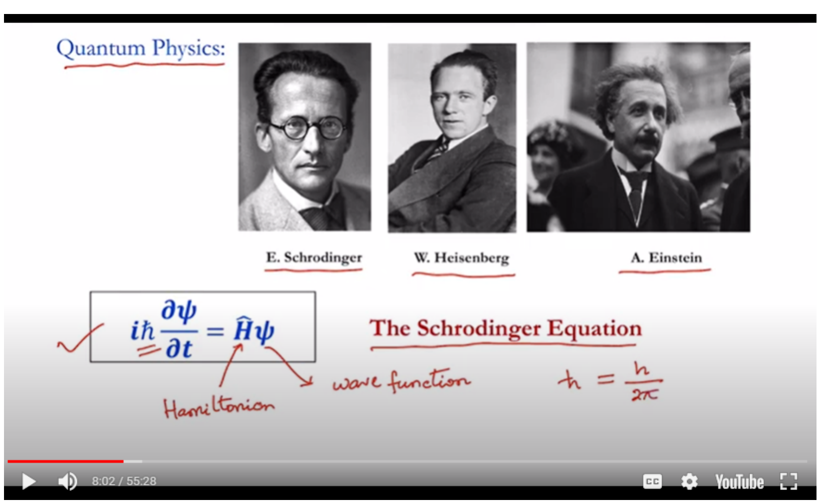

Because in quantum mechanics as you know because of the uncertainty principle it is not possible to know the position of a particle and its velocity or momentum along a that particular direction along a particular direction with absolute certainty. So, this way of defining the state in classical physics cannot be incorporated into the quantum physics. In quantum physics we do that by the so called wave function and as you know that quantum mechanics is the result of walks of many many scientists and physicists. For example Erwin Schrodinger, Werner Heisenberg, Albert Einstein and there are many many scientists who contributed significantly to quantum mechanics in the development of quantum mechanics. And one equation that is very famous equation all of you know is the so called Schrodinger equation. It tells us how a quantum particle how the state of a quantum particle evolves in time and this is the equation here this is the Schrodinger equation. And in this equation the so called psi here this psi is known as the wave function and it is postulated that the wave function contain all the information about the quantum system. And here in this equation H is the so called Hamiltonian and h cross here this h cross is basically the reduced Planck's constant it is h divided by 2 pi.

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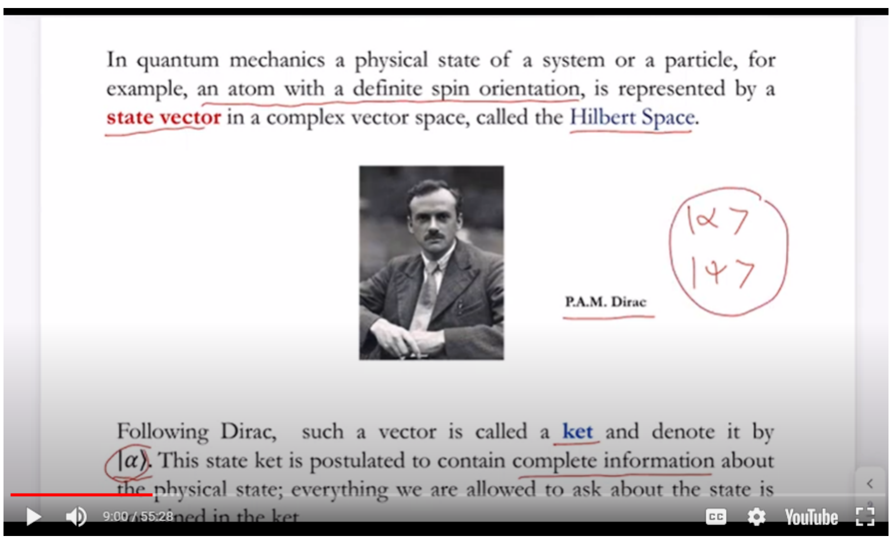

We are going to define the state of a particle by the so called state vector in a complex vector space which is also called Hilbert space. And it is postulated that all the information the complete information of the system for example an atom with a definite spin polarization or orientation is represented by the state vector. And this state vector is called as per the Dirac the British physicist Dirac this state vector is called ket vector and it is denoted by say by the symbol ket alpha. Say ket alpha or you can denote it by say ket psi right. And this state ket is postulated to contain the complete information about the physical state. And everything we are allowed to ask about this state is contained in this state vector ket alpha or ket psi.

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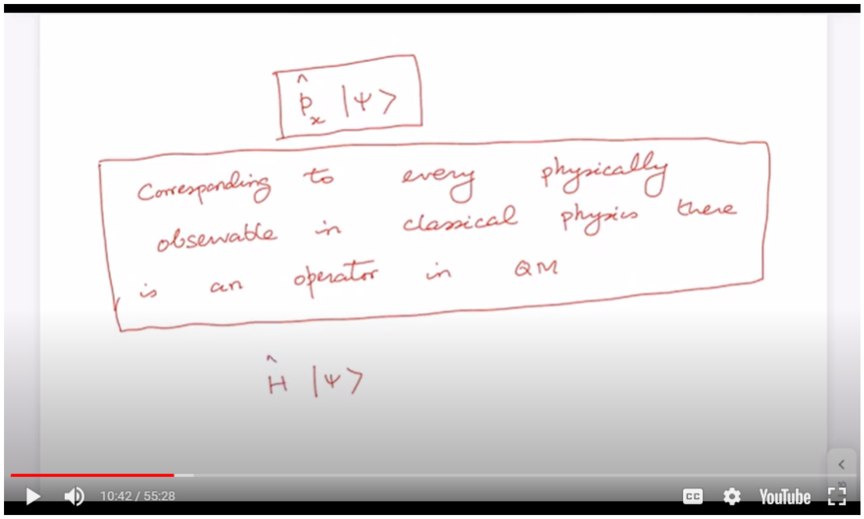

In fact if we want to know about some information about the system or the particle let's say it is represented by this ket psi. And if you want to know about the momentum then you have to operate on the state vector by the so called momentum operator. Say if you want to know the x component of the momentum then you have to operate on the state vector psi by this momentum operator. In fact in quantum mechanics corresponding to every you know the physical observable in classical mechanics there is a corresponding operator.

This is an important statement let me write it corresponding to every physically observable in classical physics, in classical physics there is a there is an operator ok there is an operator in quantum mechanics. This is a very important statement as you can see in my example here in this example if I want to know the information about the momentum then I have to operate on the state vector by the momentum operator. If I want to know about the energy of the system then I have to operate on it by the Hamiltonian operator and so on.

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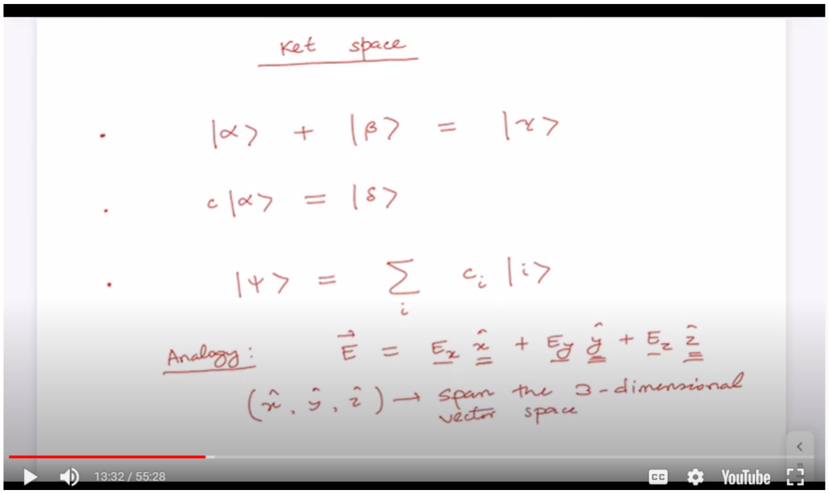

Let us now discuss more about ket vectors. We have already got to know that ket state which is a state vector in the complex vector space contains all the information about the physical state of a particle. Let me mention some useful properties of ket vectors. For example say one important property is that if there are two kets say ket alpha and ket beta they can be added and addition of these two ket is going to result in another ket say gamma. This is one property. Another one is that if you multiply a ket by a complex number say c it is going to result in another ket say it is going to result in ket say ket delta.

One of the most useful property is that if there is an arbitrary ket say ket psi it can be written as a superposition of basis ket say these basis kets are ket i and this is ci, ci is a complex number so this is I am going to break it down I am going to explain it. You see here ci is a complex number in fact you can understand it if you can draw an analogy let me draw an analogy from your usual vector space. In the usual vector space you know that if there is a vector say electric field it can be written as Ex, X cap, Ey, Y cap and Ez, Z cap. So here X cap, Y cap and Z cap are the so called unit vectors along the usual these three directions X, Y and Z respectively. And here Ex, Ey, Ez are real numbers and we say that X cap, Y cap and Z cap span the three dimensional vector space.

So X cap, Y cap and Z cap these three unit vectors are the basis vectors they are said to span the three dimensional because users require three unit vectors or the basis vectors to represent an arbitrary vector. So we say that X cap, Y cap and Z cap span the three dimensional vector space.

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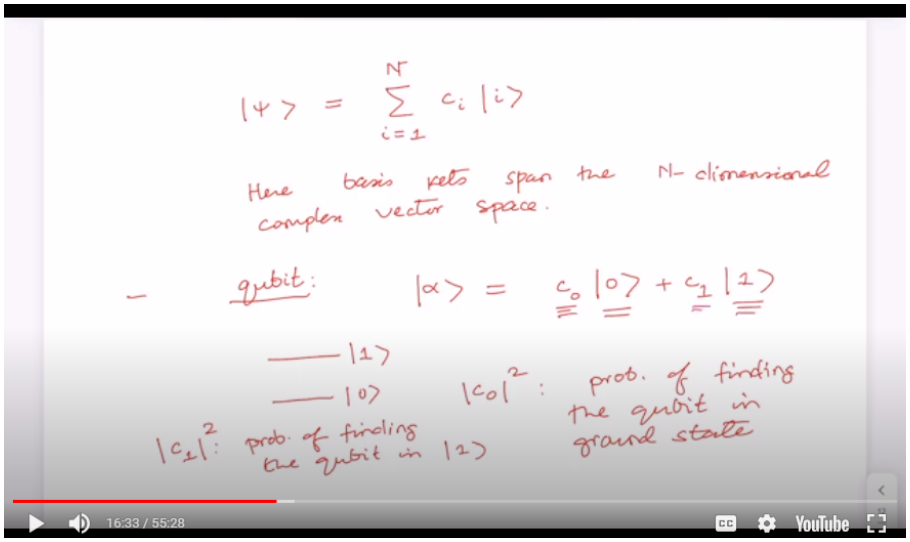

In the same way what we can say is that if there is an arbitrary ket psi is there if I can write it as a superposition of basis ket say ket i where i runs from say i is equal to 1 to n. In that case we say that this basis vector span the n dimensional complex vector space. So here the basis vectors or basis kets rather let me write basis kets the basis kets span the n dimensional complex vector space.

Alright. Just to give you an example so you know you may have heard about the so called qubit. qubit is a two state system if you if I talk about a qubit we are going to talk a lot about qubit in this course. A qubit we can it's any arbitrary states say ket alpha if I represent that arbitrary state of a qubit by ket alpha I can express it in terms of the basis ket say ket 0 and ket 1 in this form. So if I say ket alpha is equal to say c0 ket 0 plus c1 ket 1 where c0 and c1 are complex numbers and ket 0 and ket 1 right these are the basis vectors and this we say that these basis vectors are spanning the two dimensional ket space. And here this ket 0 may represents the ground state while ket 1 represents the excited state of a two level system.

Suppose this is the ground state so this ground state is represented by ket 0 and the excited state is represented by ket 1. Again you may recognize that this coefficient c0, c0 basically gives you the probability of finding the system or the qubit probability of finding the qubit in ground state. Ok qubit in ground state and similarly ket c1 ok c1 it's a complex number so c1 mod square gives you the probability of finding the qubit finding the qubit in state 1 in the excited state. I think you already recall that what the significance of this coefficient and I am sure you have already seen it.

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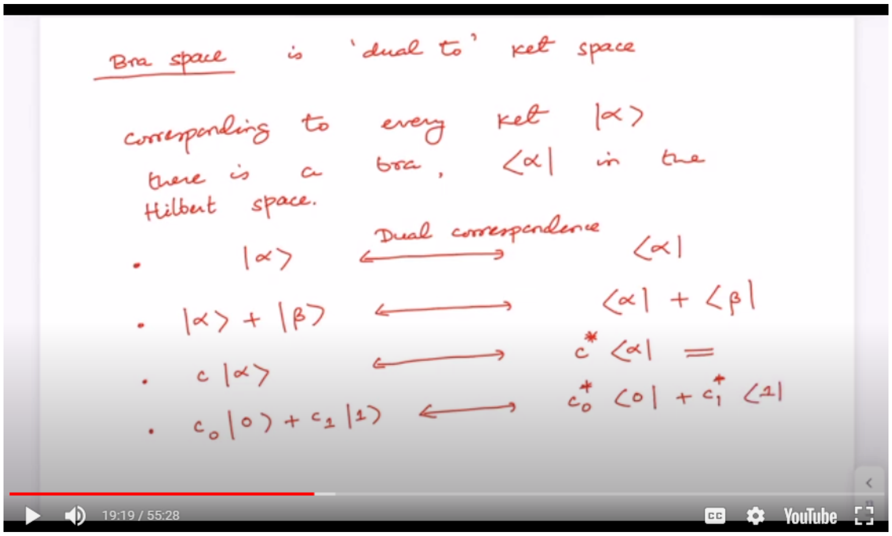

Now let me talk about the so called bra space. Bra space is a complex vector space and it is dual to the so called ket space. It's a dual and I am going to explain it what I mean by dual. It's dual to ket space ok. Actually corresponding to every ket say alpha there is a bra denoted by say by this symbol ok in the Hilbert space. You will soon see the significance of this bra space.

Let me give you some examples. For example as I said the dual to this ket alpha say let me say dual correspondence to ket alpha is bra alpha. The sum of two kets say ket alpha plus ket beta its dual is also going to be another basically the sum of two corresponding bra's so that is bra alpha plus bra beta. It's exactly the same property addition of these two bra is going to result in another bra. And now important difference is here that if you multiply this ket alpha by complex number c the dual would be the corresponding bra of the ket alpha but now it is going to be multiplied by a number c but not c it would be the complex conjugate of c ok.

This is the difference. As I have given you the example of this qubit where we represent the arbitrary state of the qubit as a superposition of ket 0 and ket 1 right. The in the dual correspondence in the bra space would be simply c0 as you can see from this particular property it would be c0 complex bra 0 plus c1 complex bra 1 ok.

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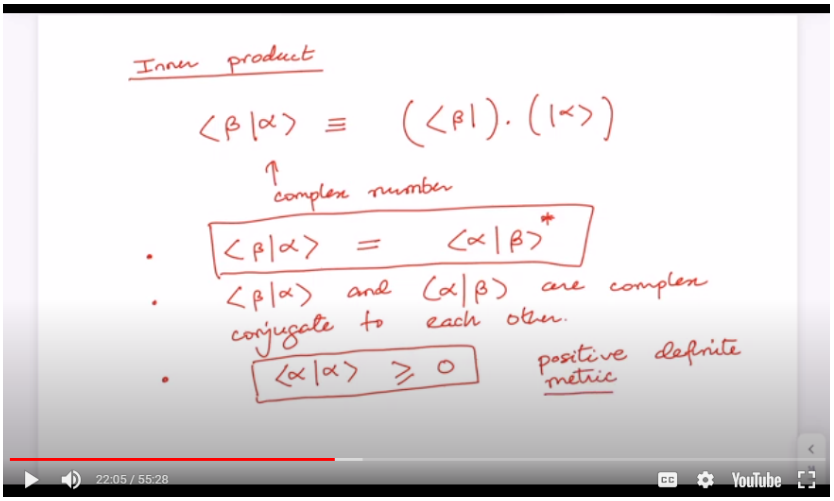

The notion of bra space is actually useful to define the so called inner product and inner product is an important concept as regards the mathematics is concerned in quantum mechanics using dirac notations. It is defined like this suppose you have a ket alpha and a bra beta is there the inner product is defined by this. It is basically the multiplication of the this bra beta bra beta and then this ket alpha right.

So this is how the inner product is defined and in general this product is a it is a complex number. This is a it is a it is a complex number. So this you have to remember it is simply a oh ok one minute it is a complex number. Some important properties of inner products are these two important properties let me mention. So say the inner product of ket alpha and ket beta is equal to this inner product is not equal to bra alpha you know and ket beta rather it is complex conjugate.

So that means that this one beta alpha this inner product is or let me say this one and alpha beta ok this inner product are complex are complex conjugate to each other are complex conjugate to each other. I think this is clear to see from this expression here. Also you should note this another important property is that the inner product of a ket with its corresponding bra is always greater than or equal to zero. Right if it is equal to zero then we say that ket alpha is a null ket and this property inner product if it is inner product of a ket with its corresponding bra is greater than zero or greater than or equal to zero. So this particular property is known as the postulate of positive definite metric positive definite metric ok this is just a terminology you need not have to bother much about it.

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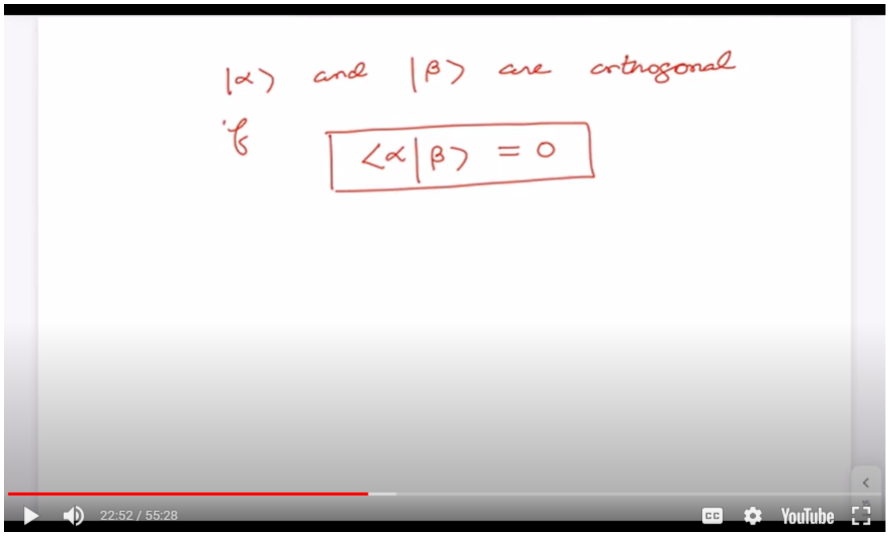

And two kets you know alpha and beta are called orthogonal ket alpha and ket beta are orthogonal orthogonal to each other if this inner product alpha beta is equal to zero. Now this is an important property this orthogonality properties we are going to utilize it again and again in this course. For more information on Dirac bracket notation and its algebra you can pick up any quantum mechanics book particularly the one that I have already mentioned I have mentioned about the Sakurai book.

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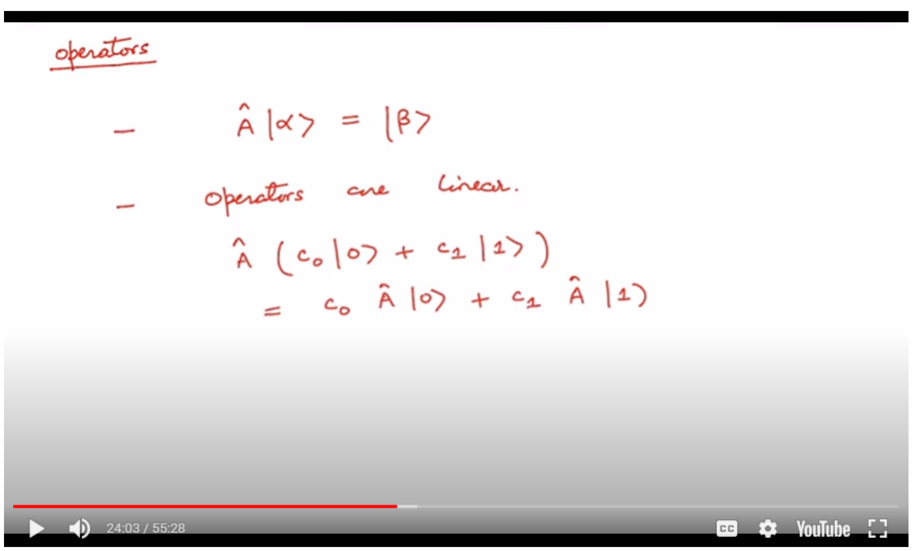

Now we will talk about operators. As I said for every physically observable in classical physics there is an operator in quantum mechanics.

An operator acts on a state vector say a ket say ket alpha from the left hand side ok and this will result in another ket say ket beta in general. And operators are linear operators are linear what I mean by linear to understand it let me consider the qubit state that I define sometime back say arbitrary qubit state is there say C0 ket 0 plus C1 ket 1. If we operate on this by an operator then this will give us say C0 A ket 0 plus C1 A ket 1. So in this sense the operator A is linear.

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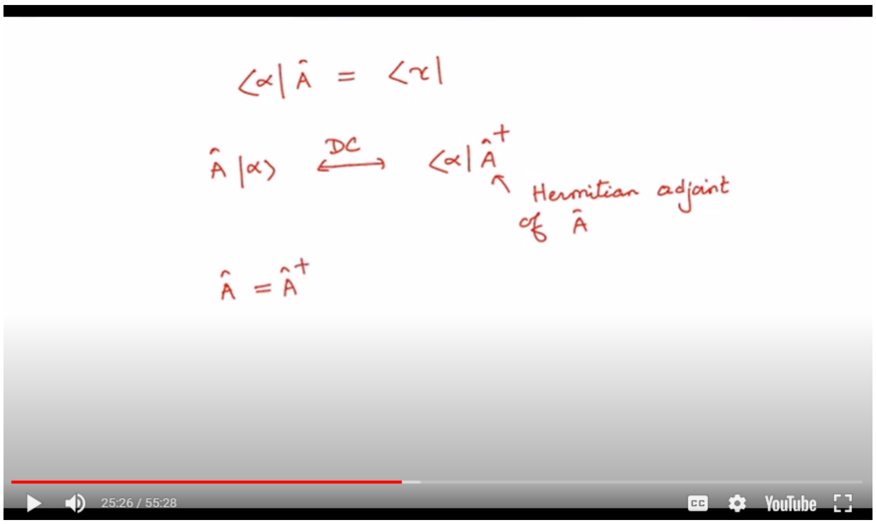

In the case of bra an operator always operates on a bra say bra alpha from the left ok and this is again going to result in another bra say bra gamma.

However one need to be careful about the dual correspondence between the between the ket and the bra operation. For example if I operate take this operation operator A operating on ket alpha its dual correspondence would however be given by say dual correspondence would be given by bra alpha. But here it would not be A but rather it would be A dagger which is basically the A dagger this is the so called Hermitian adjoint. A dagger is the Hermitian adjoint or simply the adjoint of the operator A ok. An operator A is said to be Hermitian if A is equal to A dagger.

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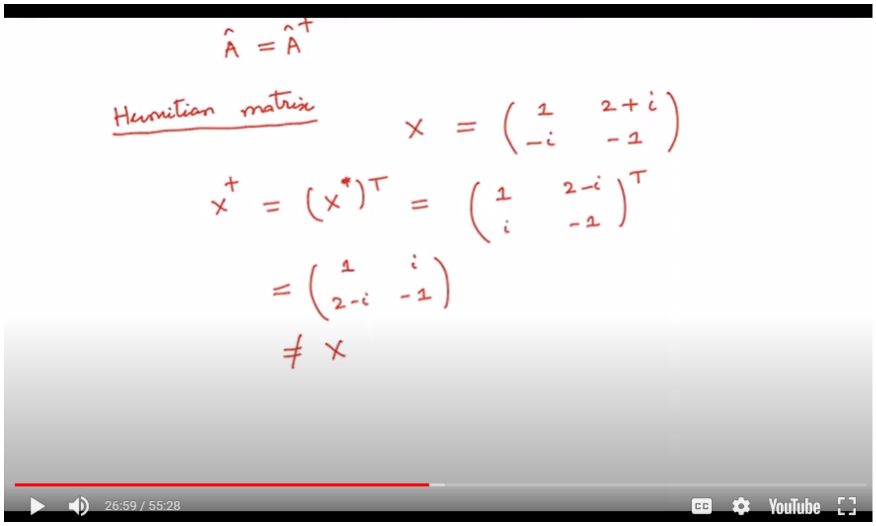

Let me digress a little bit and let us recall about what we mean by Hermitian in the context of a matrix ok. Let us recall about Hermitian matrix because as you may know that an operator can be represented in a matrix form in a appropriate basis vectors and I will talk about it later more about it later. To understand Hermitian matrix let me take the example of a 2 by 2 matrix say say X is equal to a 2 by 2 matrix let me say it is 1 2 plus say I minus I say minus 1. These are the elements in the matrix then X dagger the Hermitian conjugate or adjoint of this matrix X would be basically the complex conjugate of this matrix X and then if we take the transpose right. So in this particular case if I take the complex conjugate I will get 1 2 minus I I and minus 1 and if I take the transpose then I will get 1 2 minus I here I minus 1. As you can see that in this case this X dagger is not equal to X so therefore X is not a Hermitian matrix.

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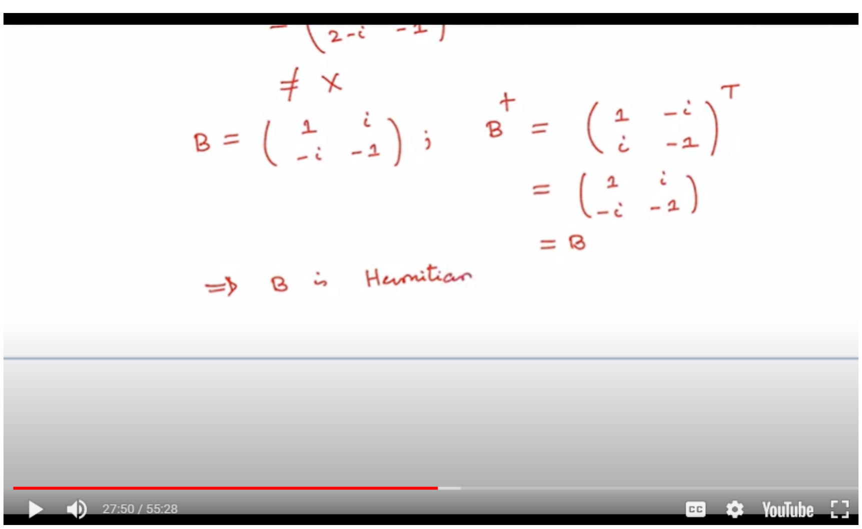

However if we take say say a matrix B is equal to say 1 I minus I minus 1 so in this case the B dagger the Hermitian adjoint would be simply if you first you take the complex conjugate ok. This is what you will get and then if you take the transpose then this is going to result in 1 minus I I minus 1 and this is exactly the original matrix B so in this case this matrix so this implies that B is a Hermitian. So I think just it's for recalling purposes I am mentioning it.

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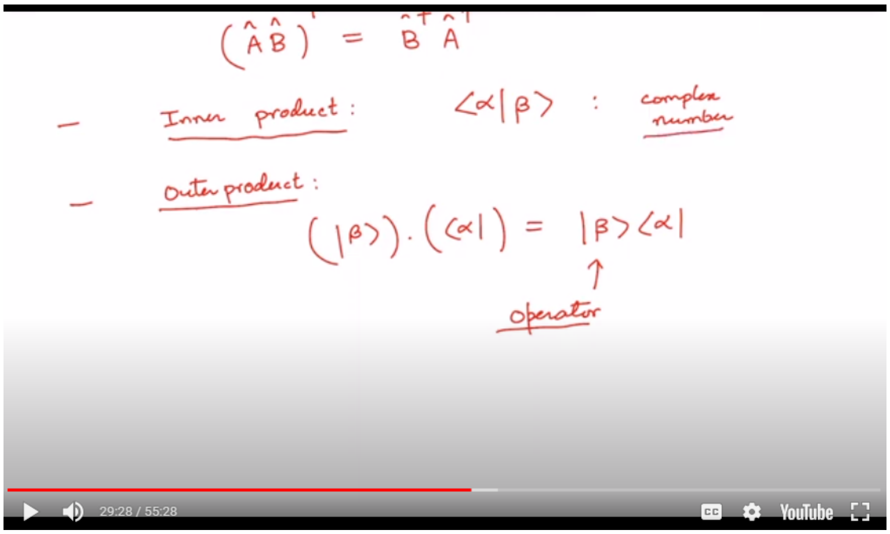

Another thing you should note is that the Hermitian adjoint of a you know the product of two operators say A B product of two operators if I take the Hermitian adjoint of this product it is going to be equal to product of the Hermitian conjugate of B dagger A dagger.

Please just note down the note the order in which they are multiplied. Now sometime back we defined the so called inner product. So sometime back we defined inner product between two kets. In fact it was defined like this so this was what the so called inner product.

So there is something called outer product as well. An outer product outer product is outer product is defined in this form it is basically product between a ket beta and a bra say alpha. And this would be simply like this. Now this resulting product resulting operation is an operator. It's an operator. On the other hand in the case of the inner product it's just a number it's in general it is a complex number right so this is a huge difference between inner product and outer product.

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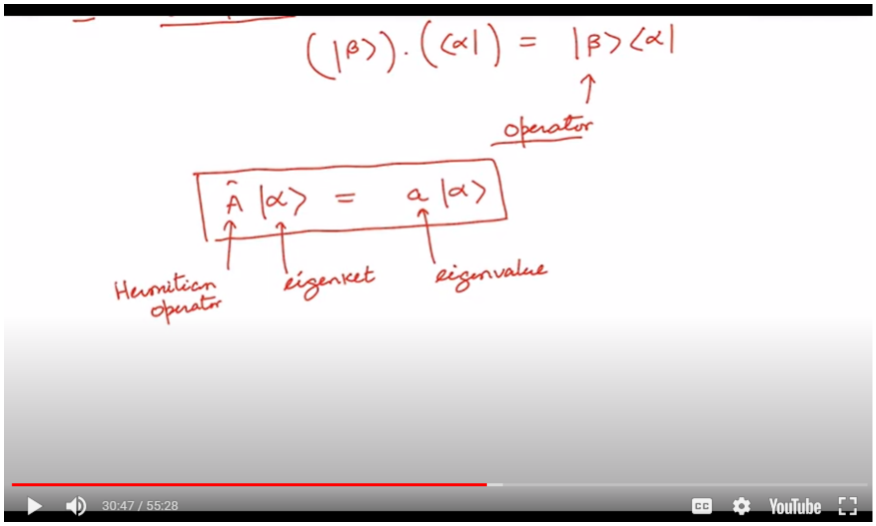

Now coming back to the operation of an operator on a ket many times we get suppose we have this ket A ket alpha. And if we operate on this by the operator A then what may we get is that we may get the same ket and multiplied by real number say A. Whenever we have such kind of a situation and this is basically known as the eigenvalue equation. In this case this ket alpha is called an eigen ket because of its peculiarities.

It's called eigen ket and this operator A is called an Hermitian operator. It is called a Hermitian operator. And this A is called the eigenvalue. Hermitian operators are important because if that is the case whenever you have this eigenvalue equation all the physically observable quantities in classical mechanics are represented by Hermitian operators in quantum mechanics. Okay we'll talk more about it now.

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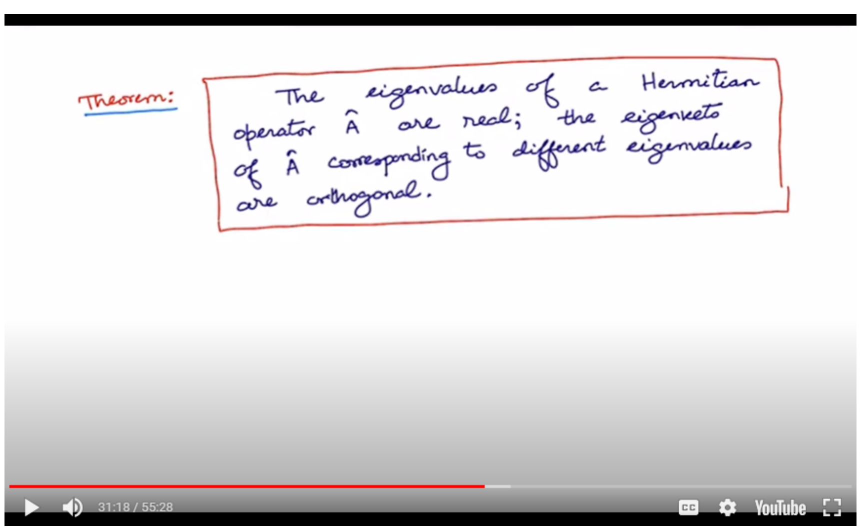

There is a very important theorem in quantum mechanics that says that the eigenvalues of a Hermitian operator A are real. And the eigenkets of the operator A corresponding to different eigenvalues are orthogonal. This is a very important theorem and I think it is worth to prove it and let me show you how to prove it.

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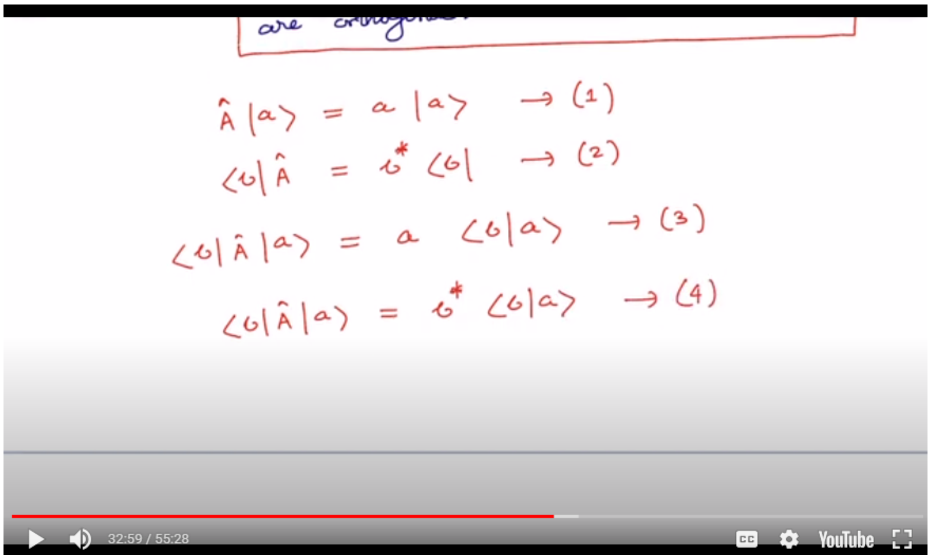

You know that this eigenvalue equation. So A A ket A is equal to A A where ket A is the eigen ket and A is the number eigenvalue. And let's say this is equation number 1. And because this operator A is Hermitian we can also have that if the operator A operates on a bra B and it operates from the left it is going to result in say this equation say B complex. It's a number complex number B right. So this is what we have and say this is my equation number 2. Now let me multiply equation 1 by the ket by bra B from the left.

So let me do this from equation 1. If I take this operation I multiply both sides then this is what I'll get. I multiply it by bra B and this is what I'll get. Let me say this is equation number 3. And multiply both sides of equation 2 by ket A on the right. So if I multiply equation number 2 from the right by this ket A this is what I'll get. It is easy to follow. Let's say this is my equation number 4.

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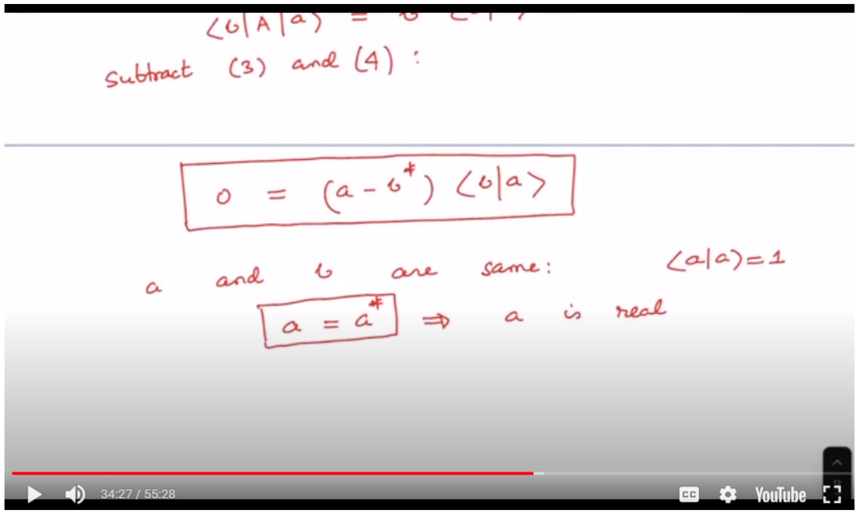

Now let me subtract equation 3 and 4. Subtract 3 and 4. And then you can easily get 3 and 4 if you see. Then you will get 0 is equal to A minus B star this inner product right. This is what you'll get. This is an important equation now and from here this we can easily see what the theorem says. Now if say A and B are same. Say A and Eigen value these numbers A and B are same. Then clearly what you will get? You will get A is equal to A star.

Right? And this implies that A is real. A is real. Because you will get A this inner product will be equal to 1. So that's what the first part is proved. The Eigen values of Hermitian operator A are real.

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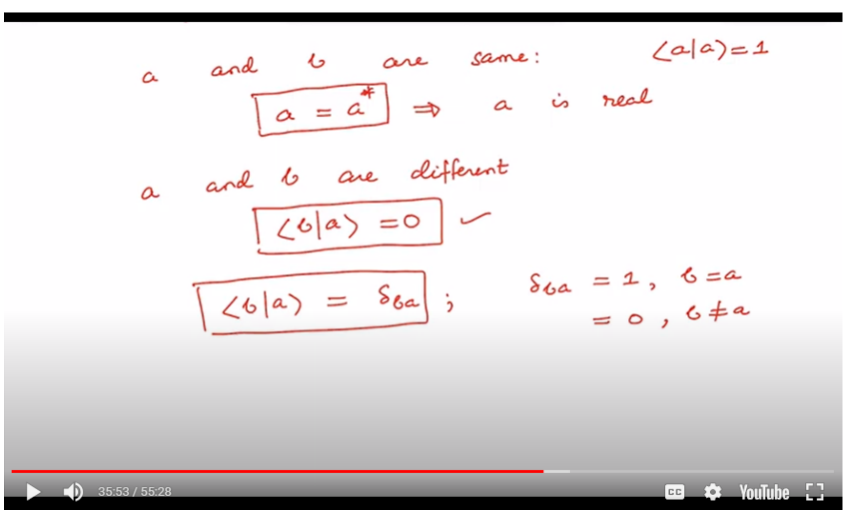

Now regarding the second part. Now let us say A and B are different. If A and B are different. A and B are different. That means if the Eigen values corresponding to the operator A are different. Then you will get this inner product would be equal to 0.

So this clearly shows the result that the Eigen kets the second part. The Eigen kets the second part. The Eigen kets of A corresponding to different Eigen values are orthogonal. So this is a very important theory. Let's say one of the basic theory related to quantum mechanics. It's a convention to write the inner product of two Eigen kets in this form.

We can write the inner product as this. This is delta BA is the so called Kronecker delta. Where this Kronecker delta is equal to 1 if B is equal to A. And that means the Eigen kets are same. And it is equal to 0 if the Eigen kets are different. Right? And this is already we have seen. This is a useful expression.

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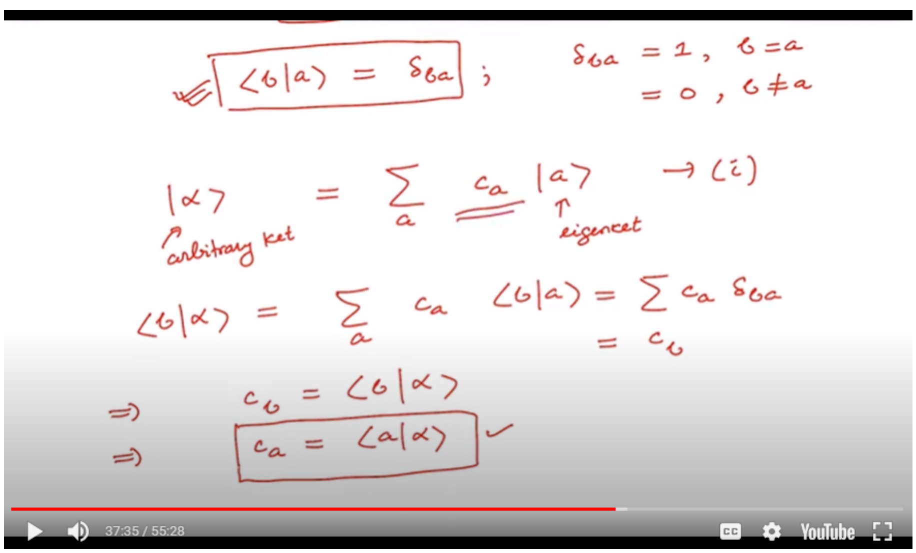

Let me now discuss how Eigen kets can be used as basis kets. In fact, I have already discussed similar things earlier. Any arbitrary ket, any arbitrary ket, this is an arbitrary ket. Can be expanded or expressed as a superposition of Eigen kets.

Here this is the Eigen ket. As a superposition of Eigen kets. Let me say this is an important expression equation 1. I can now utilize this condition, orthonormality condition it is called. If I multiply equation 1 by say bra B.

Then I will get here. This would be CA bra BA. Then I can write this expression because of this relation that I have written here. I can write it as CA delta BA. This delta BA is equal to 1 if B is equal to A.

So it will result in only CB. So I have CB is equal to B alpha. And clearly, similarly I can write CA is equal to A alpha. This one, this coefficient, this is just a number, it is a complex number. I can use this expression here in this equation. Because it is a number I can write it this side also.

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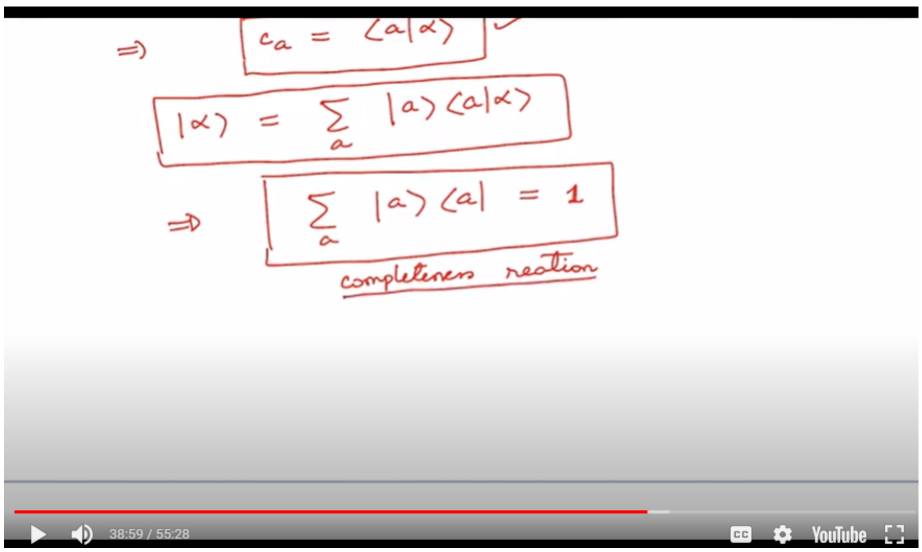

So if I do that, then I have this ket alpha is equal to sum over A. Let me first write here this ket A and the number this A alpha. Right? And this is very useful. And from this you can immediately see that this is going to lead us to this particular equation. And this equation is a famous expression in quantum mechanics.

This expression is called, the equation is called the so-called completeness condition. Completeness relation. What it effectively means is that if this relation is satisfied, this means that this eigen kets A, all the eigen kets span the linear vector, this Hilbert space. Right? This is what we mean by the completeness relation. Now, let me quickly talk about the matrix representation of an operator. And you will immediately see the usefulness of this completeness relation there.

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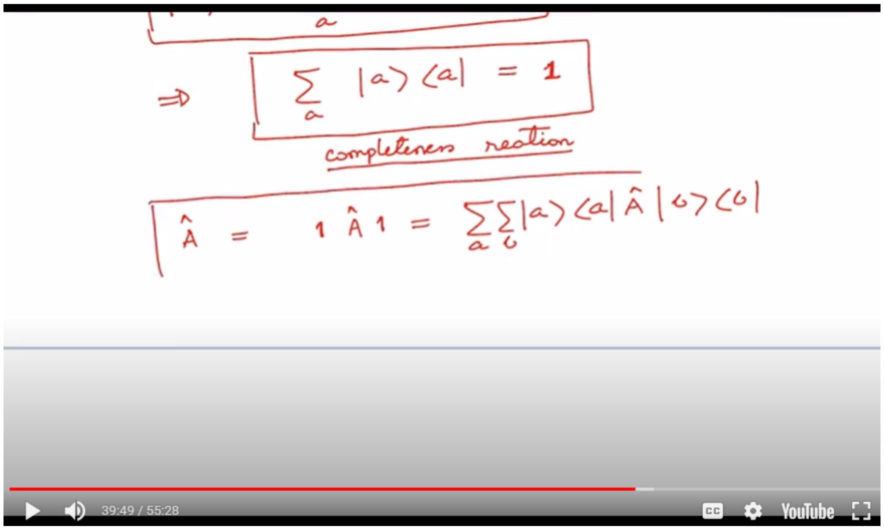

Let me consider this matrix, say operator A. I can multiply both sides of these operators by this identity operator. By the way, here it looks like 1. It's basically the identity operator.

So I can multiply both sides of this operator by the identity operator. And by the end of this completeness relation, I can write it as in terms of the basis kets or the eigen kets. Say A, A, this is equal to 1. And then for the other one, I can write sum over B eigen kets. So this is what I have. Right?

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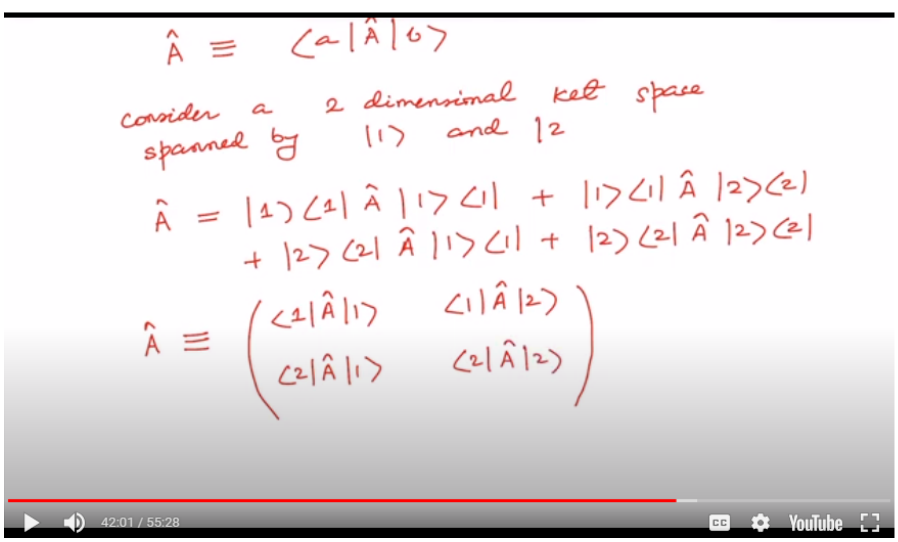

When I say, when we talk about the matrix representation of an operator A, so what actually we mean by the matrix representation is this particular expression.

So this one we mean. Let me explain it by using a simple two-dimensional ket space. Consider, let me say, consider a two-dimensional ket space. Alright? Let us say, spanned by the, spanned by eigen kets, eigen kets 1, ket 1 and ket 2.

Let me write here. Ket 1 and ket 2. These are the eigen kets. Users need to have only two eigen kets because it's a two-dimensional ket space. Then I can write this expression. I can just expand it and then I can write, let me write all the terms.

I'll have first term would be this. Say this one. Then the second term would be 1, 1, A, 2, 2. Then I'll have 2, 2, A, 1, 1. I think you can easily see. This is 2, 2, A, 2, 2. Right? If I expand it, this is what I'm going to get. So now as regards matrix representation is concerned, then I can write it as a 2 by 2 matrix with the elements of the matrix would be this. First term would be 1, A, 1. Second term would be 1. This is the first row and then this is going to be my second row and first column. And this is the second row, second column. I think you can see it. What you see that in this representation, in this matrix representation, here, this particular one, this bra is referring to the rows and this is referring to the column.

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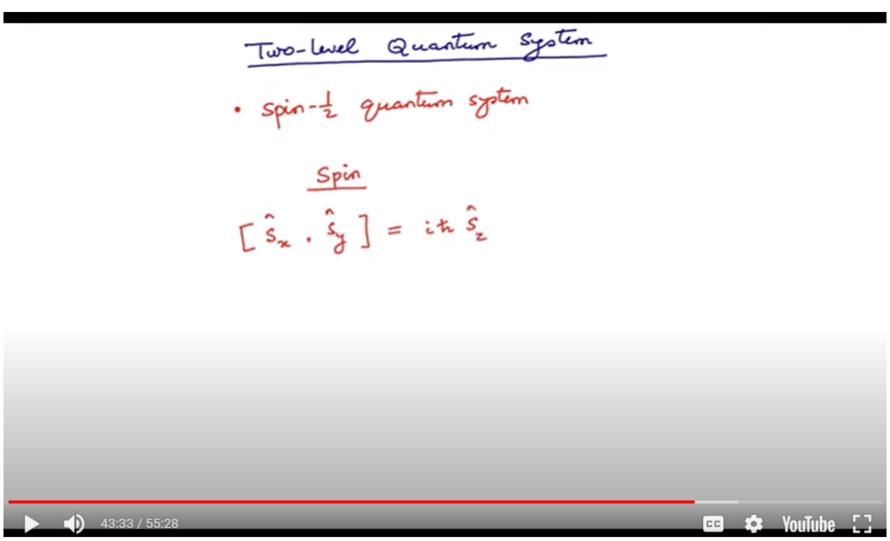

Now we'll discuss the case of a two level quantum system. As an example, we are going to consider the so-called spin-half quantum system. You will find this very useful later on when we discuss the so-called EPR paradox. But before that, let me make some general comments about spin. And I'm sure all of you have studied about spin in an elementary quantum mechanics course. You know that spin is an intrinsic angular momentum of a quantum system, particularly elementary particles, and it has no classical counterpart.

And one cannot measure the components of spin simultaneously. For example, you cannot measure the x component and the y components of the spin simultaneously. And this is revealed in the so-called commutation relation between the operators. You know the commutation relation between the spin operators, x component of spin, and y component of spin is going to result in this expression.

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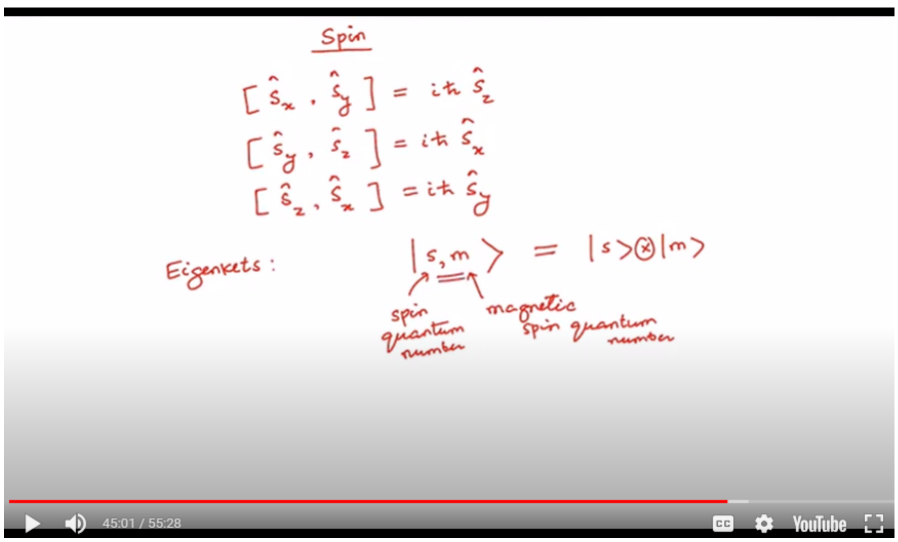

We have other two relations, similar relations, commutation relations for the other components. Say, as commutation between Sy and Sz is ih cross Sx. And we also have Sz Sx is equal to ih cross Sy. And we know that the eigenvectors or the eigenkets of a spin system is represented in Dirac notation by a ket, which is characterized by two quantum numbers S and M. S refers to the spin quantum number. It is called a spin quantum number or simply it is called spin. This is spin quantum number and M is the so-called magnetic spin quantum number. Actually, this ket state is a direct product of two ket states, the spin state and the magnetic quantum number. So direct product is written by this and in short hand notation, this is how we represent the ket state.

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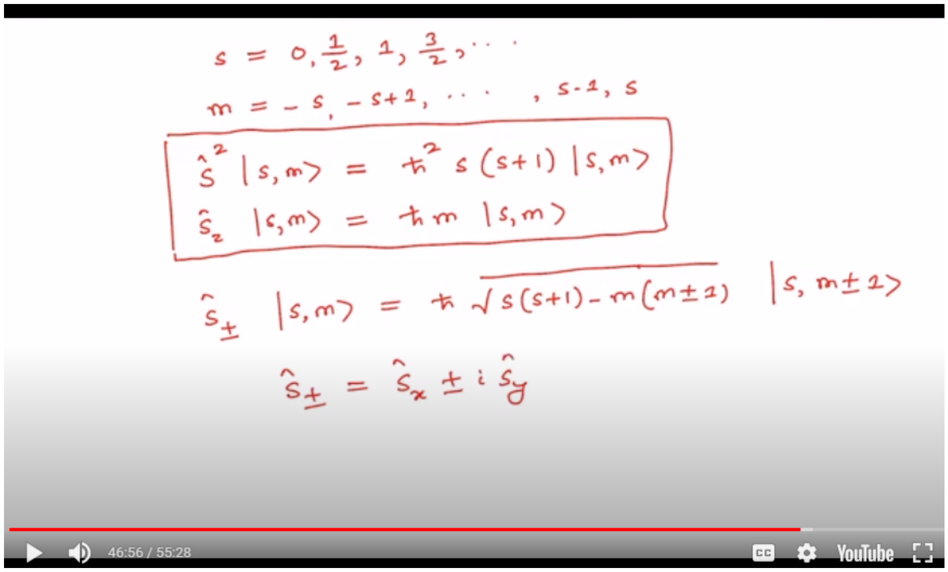

S can take half integers value and these are values, say 0, a half, 1, 3 by 2 and so on. On the other hand, this magnetic quantum number can take value from minus S to plus S. So minus S then minus S plus 1 and you will have S minus 1 and finally you will have S. Now, S regards the operators and eigenvectors are concerned corresponding to the total spin angular momentum operator.

The square of the spin angular momentum and the z component of the spin satisfy this eigenvalue equation. So let me write it down and these eigenvalue equations we can utilize for our analysis later on. We will see that. So this first one is going to give us h cross square S into S plus 1 S M. So clearly this is an eigenvalue equation and the next one is going to give us h cross M S M. So apart from these two important equations, we have another equation where let me first write it S plus minus S M is equal to h cross square root of S into S plus 1 minus M into M plus minus 1 and here we will have S M plus minus 1. And in this case S plus minus refers to S x plus minus i S y.

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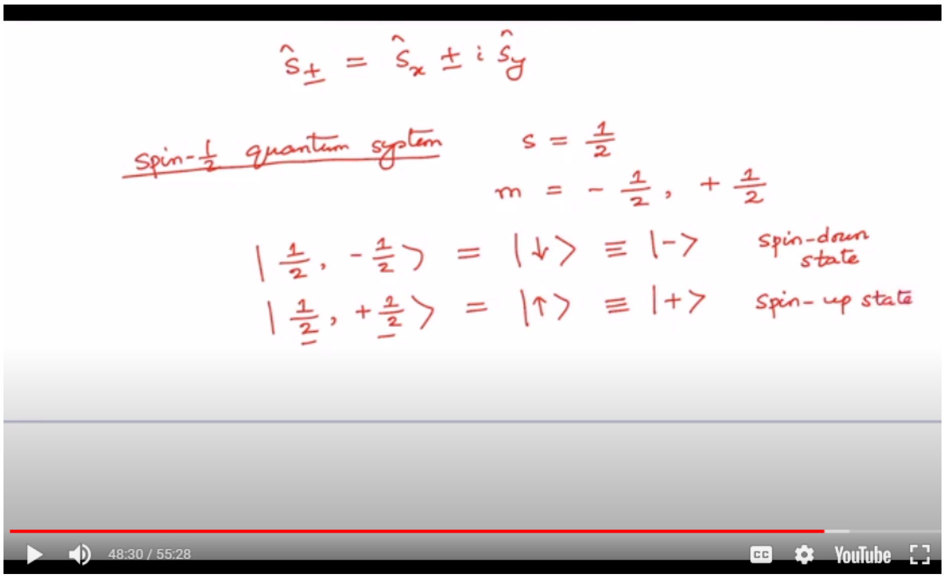

Now let us discuss spin half quantum system. Spin half quantum system. In such systems S is equal to the spin quantum number S or spin is equal to S is equal to half. So clearly M can take value minus half and plus half.

This will result in two eigenstate. One eigenstate would be S is equal to half M is equal to minus half. And another eigenstate would be S is equal to half and M is equal to plus half. In fact, the first one S is equal to plus half and M is equal to minus half. This is called the spin down state and represented by this notation and sometime this notation is also used. You just put a minus there. This is called spin down state. Spin down state. And this particular case S is equal to plus half and M is equal to plus half refers to spin up state. And it is sometime also this notation is also used. This ket notation is used. It is called spin up state. Alright.

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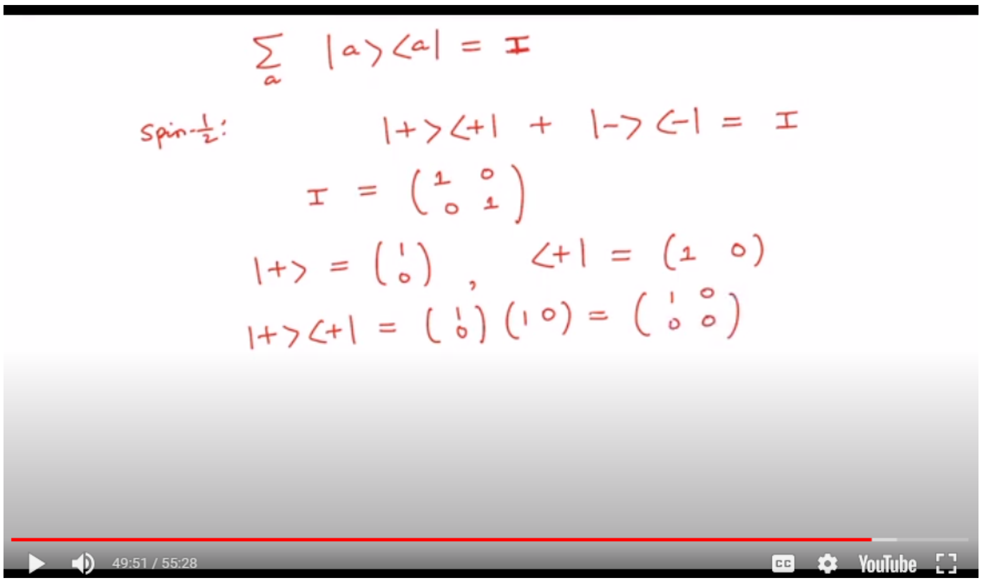

And these two eigenstates plus and minus can be taken as the basis vectors to describe a spin half quantum system. So, but before that let me talk about the so called completeness relation that we discussed sometime back. The completeness relation if you recall it was this relation where I is the identity operator. And this is I is the identity operator.

And for the spin half system, for spin half system I can break it down like this. And identity operator in this case it is a two state system. So, it is the unit matrix 1 0 0 1 in matrix representation. Now, if we take the up state as by represented by this column vector 1 0. And this will have the corresponding bra would be this row vector 1 0.

Then if I take the outer product you can clearly see this would be 1 0 matrix multiplication of this. And this is going to give us simply 1 0 0 0.

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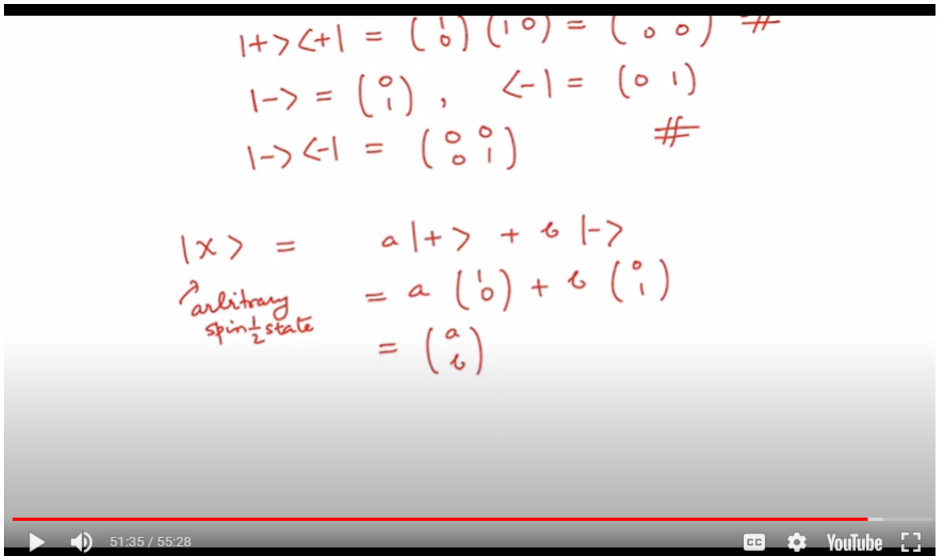

Similarly, if I take the down state minus here down state if I take it as 0 1. And then the corresponding bra would be the row matrix row column row vector 0 1.

And if I take the outer product we will get simply 0 0 0 1. So, as you can see from this one, this one and this one. This is going to if you put it here you will get the so called identity matrix. Now, this representation is going to help us a lot whenever we want to write any arbitrary ket in terms of a column vector. As you can see that if I have a arbitrary ket a spin ket.

This is an arbitrary spin ket. Arbitrary spin ket spin state right spin up state actually here. This I can write as a linear superposition of the eigenkets say a plus and then say b minus. So, this is the linear superposition of the eigenkets. Now, because of the matrix representation of plus state that is your 1 0 and down state is 0 1. So, I can express it as a column vector like this a b.

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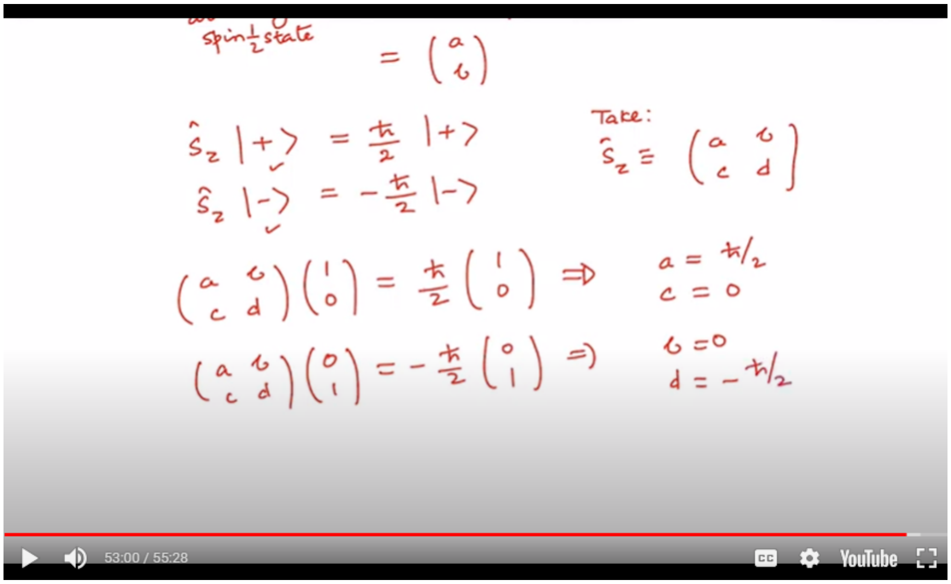

Now, what about the matrix representation of spin operators? For example, we know that SZ what about the matrix representation of the SZ operator? If you apply on the up state we will get h cross by 2 this up state spin up state and SZ minus is going to give us minus h cross by 2 minus right.

So, if you take say let us take SZ it is in matrix representation let me say we have these elements are a b c d. If I put SZ as a b c d here and plus state is 1 0 this is going to give me h cross by 2 1 0. This if you can easily see that this is going to result in a is equal to h cross by 2 and b is equal to 0. In fact, c here you will get c is equal to 0 from this first relation you will get this and from the other one if SZ is applied on the down state 0 1. You will get minus h cross by 2 0 1 and this is going to give us b is equal to 0 and d is equal to minus h cross by 2.

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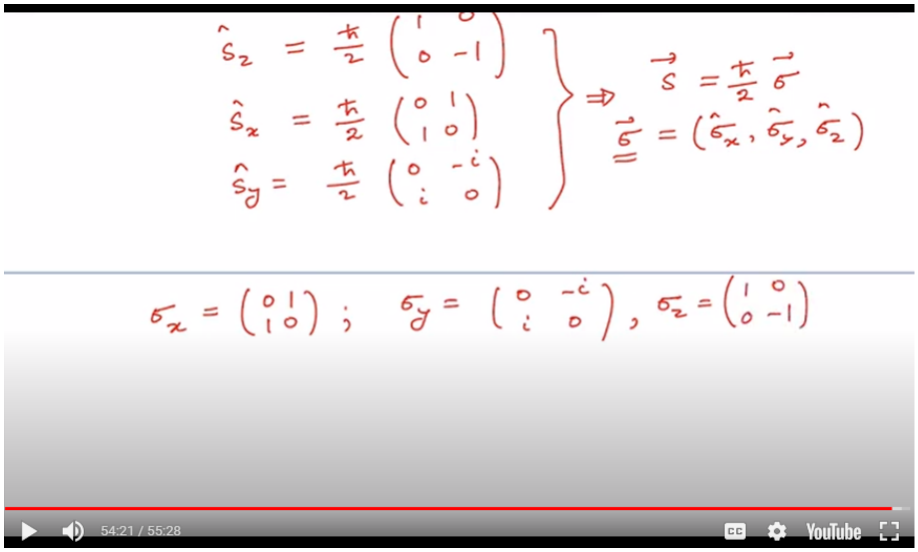

So, this means that my matrix representation for the z component the spin operator would be h cross by 2 1 0 0 minus 1 right. And similarly you can write SX the matrix representation for the x component of the spin would be h cross by 2 0 1 1 0 and SY would be equal to h cross by 2. You will have 0 minus I I 0 actually all these equations three equations we can write it in the short hand notation by this S equal to h cross by sigma. Where sigma is the so called Pauli matrices sigma has the components say sigma x sigma y sigma z sigma this is called the Pauli vector Pauli spin vector. Sigma x is equal to 0 1 1 0 sigma y is equal to 0 minus I I 0 and sigma z is equal to 1 0 0 minus 1.

These are worth remembering and we are going to use it later on. Let me stop for today. In this lecture I have touched upon the bare minimum of quantum mechanics that is necessary for starting the subject of quantum entanglement. In the next lecture we will discuss the mathematical tool in particular the so called density matrix formalism. Density matrix is an extremely important tool and we have to master it because many of the entanglement measures are based on density matrix. We are going to discuss that in great details in the next lecture. See you in the next lecture. Thank you. .