Quantum Entanglement: Fundamentals, measures and application

Prof. Amarendra Kumar Sarma

Department of Physics

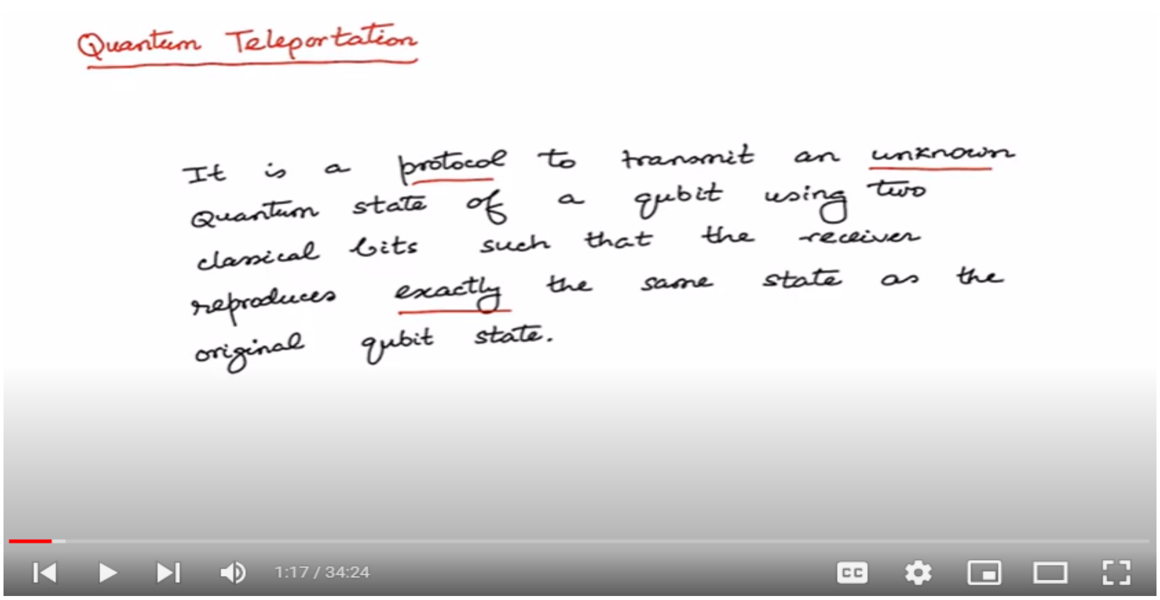
Indian Institute of Technology-Guwahati

Week-04

Lec 16: Applications of Quantum Entanglement-II

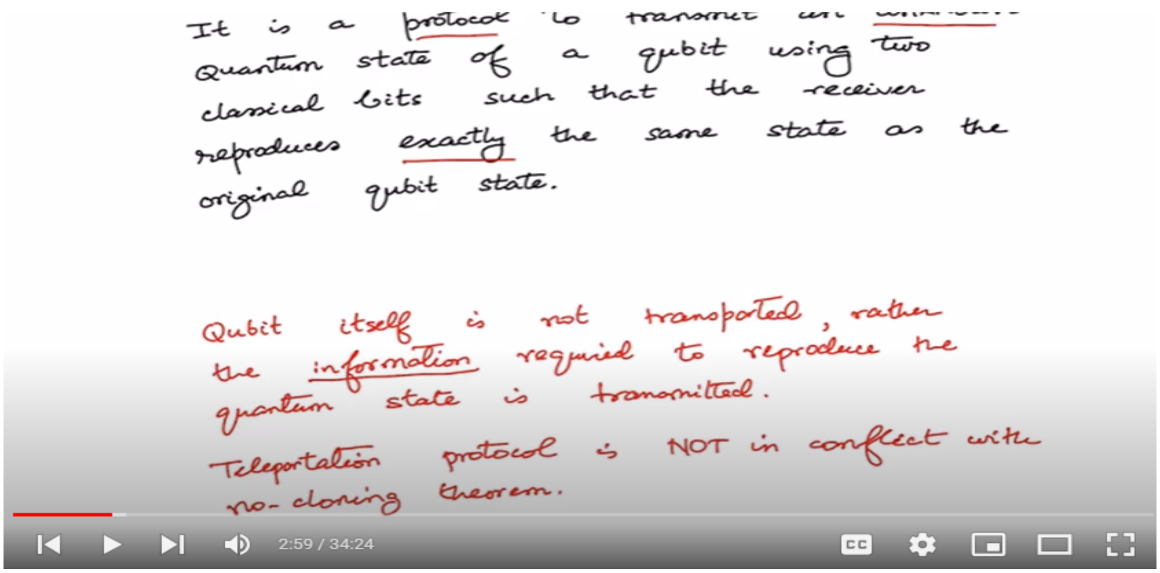
. Hello, welcome to lecture 2 of module 4. This is lecture number 12 of the course. In this lecture, we will continue discussing applications of quantum entanglement. In the last lecture, we will discuss super dense coding. In this lecture, we will discuss quantum teleportation.

(Refer Slide Time: 01:17)



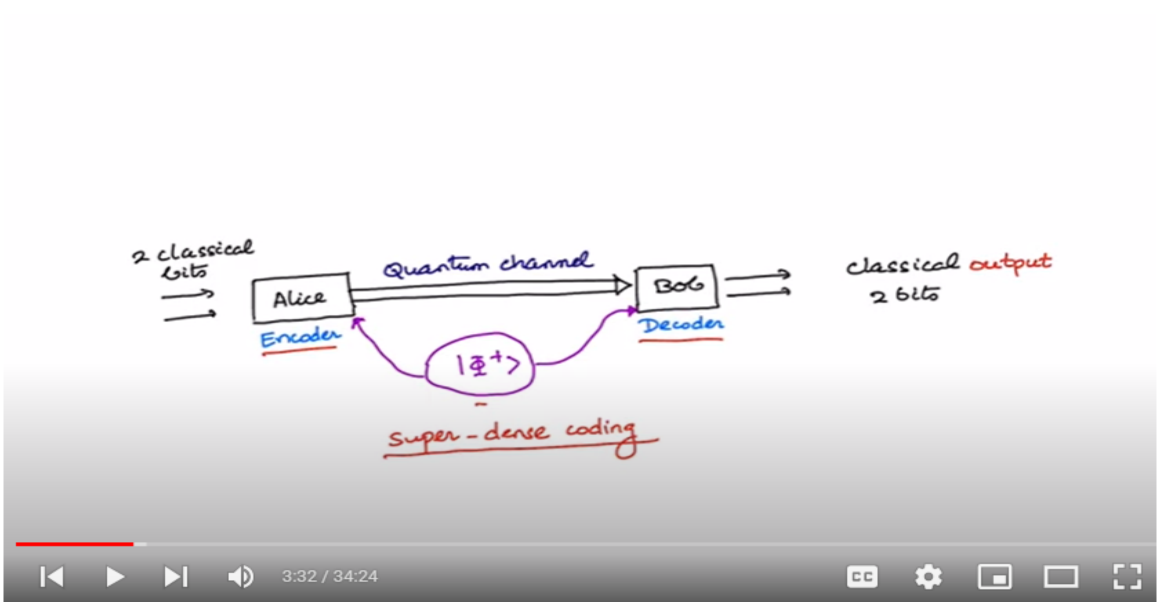
Let us begin. Quantum teleportation is one of the most fascinating topic and draws interest even from general public. Technically speaking, quantum teleportation is a protocol or scheme to transmit an unknown quantum state of a qubit using two classical bits such that receiver reproduces exactly the same state as the original qubit state. Here, you need to note that qubit itself is not transported.

(Refer Slide Time: 02:59)



This is very important. Qubit itself is not transported, is not transported rather the information required to reproduce the quantum state is to reproduce the quantum state is transmitted. This is very important and in fact because of this original state is actually destroyed such that quantum teleportation is not in conflict with no cloning theorem. So teleportation protocol is not in conflict or against the so called no cloning theorem because as per the no cloning theorem, you know that a quantum state cannot be copied or cloned. An arbitrary quantum state or unknown quantum state cannot be copied or cloned.

(Refer Slide Time: 03:32)

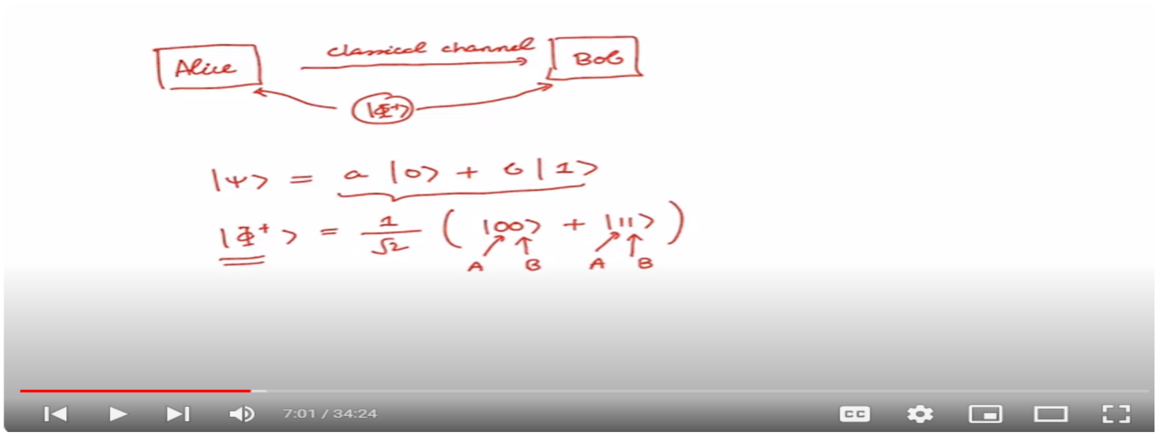


As you saw in the previous class on super dense coding, two classical bits are transmitted from Alice to Bob using a quantum channel. Here, Alice is the encoder and Bob is the decoder. The critical component of the protocol was to use a shared entangled state between Alice and Bob and the entangled state is preferably a Bell state and we have used the Bell state phi plus to illustrate this super dense coding protocol in the previous class.

(Refer Slide Time: 04:54)

Quantum teleportation protocol is also similar in nature. However, here a quantum state is reproduced at Bob's place after receiving information from Alice. So in this protocol, say Alice want to transfer a quantum state to Bob and what Alice does as per the teleportation protocol, he basically transfers some information to Bob through a classical channel. Alice transfers the information about the qubit she wants to send to Bob through a classical channel and Bob after receiving the information do some operation on his qubit and then reproduces the quantum state in his lab. And just like in super dense coding, here also critical thing is that both Alice and Bob has to share an entangled state and preferably a Bell state such as say phi plus.

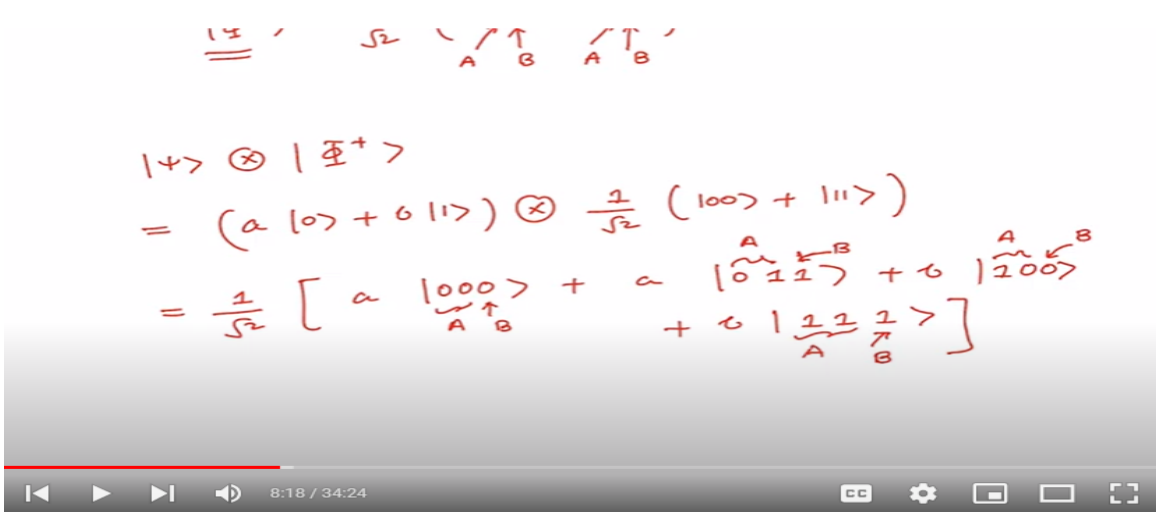
(Refer Slide Time: 07:01)



Okay, let me now discuss this teleportation protocol in some detail. Say Alice has a quantum state or a qubit whose state she does not know and she wants to transfer this quantum state or qubit state to Bob and she does that by using a classical channel.

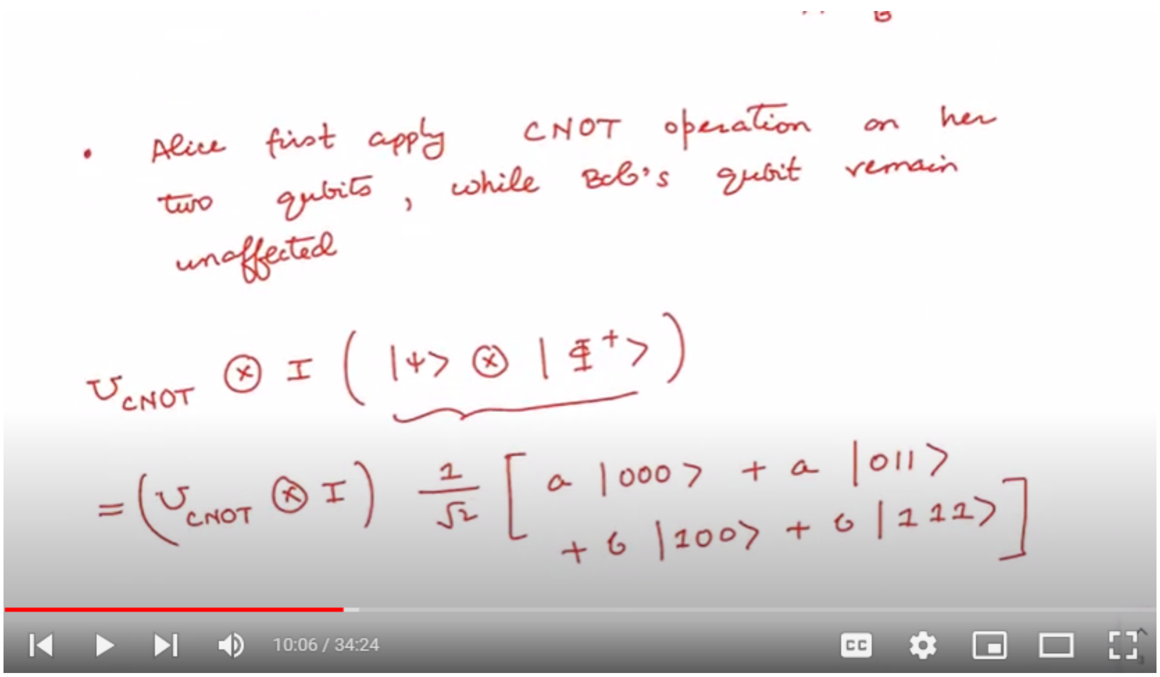
Let me for illustration purpose, let us say Alice has this particular state say ket psi is equal to a ket 0 plus b ket 1. This state belongs to say Alice initially and Alice want to send information about it to Bob so that Bob can reproduce this state in his lab and Alice and Bob share the entangled state phi plus at the beginning. So Alice and Bob share this entangled state phi plus and you know phi plus this Bell state is 1 by root 2 ket 0 0 plus ket 1 1. Now Alice applies just like super dense coding some decoding states I mean some unitary operations in her qubits. Now as you can see Alice contains two qubit one is this particular qubit and the another one is the qubit from this shared entangled pair. As you see this shared entangled pair this Bell state the first qubit belongs to Alice the second qubit belongs to Bob first qubit belongs to Alice and second qubit belongs to Bob.

(Refer Slide Time: 08:18)



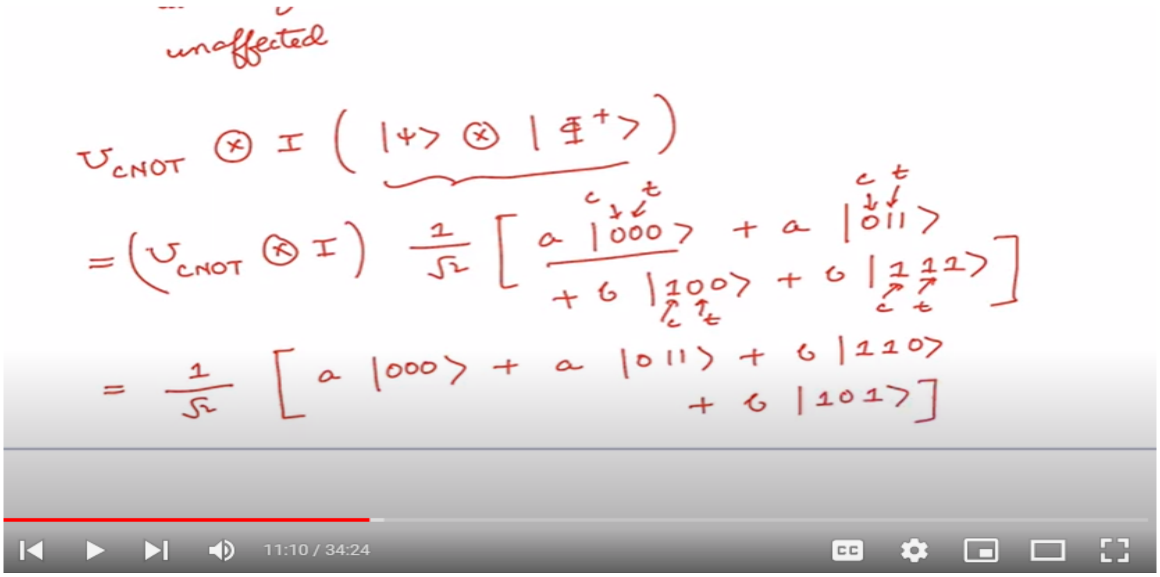
So effectively Alice contains two qubits in her lab and Bob has only one qubit in his lab right. So Alice and Bob together start with the state ket psi tensor product ket phi plus so if I write it so ket psi is a 0 plus b 1 and phi plus is 1 by root 2 ket 0 0 plus ket 1 1. So if I open it up so you will get 1 by root 2 a 0 0 0 plus a 0 1 1 plus b 1 0 0 plus b 1 1 1 right. Now this first two qubits belongs to Alice this third one belongs to Bob here this two qubits belongs to Alice this third one belongs to belongs to Bob and here the first two qubit belongs to Alice the third one belongs to Bob and finally here the first two belongs to Alice and the third belongs to Bob.

(Refer Slide Time: 10:06)



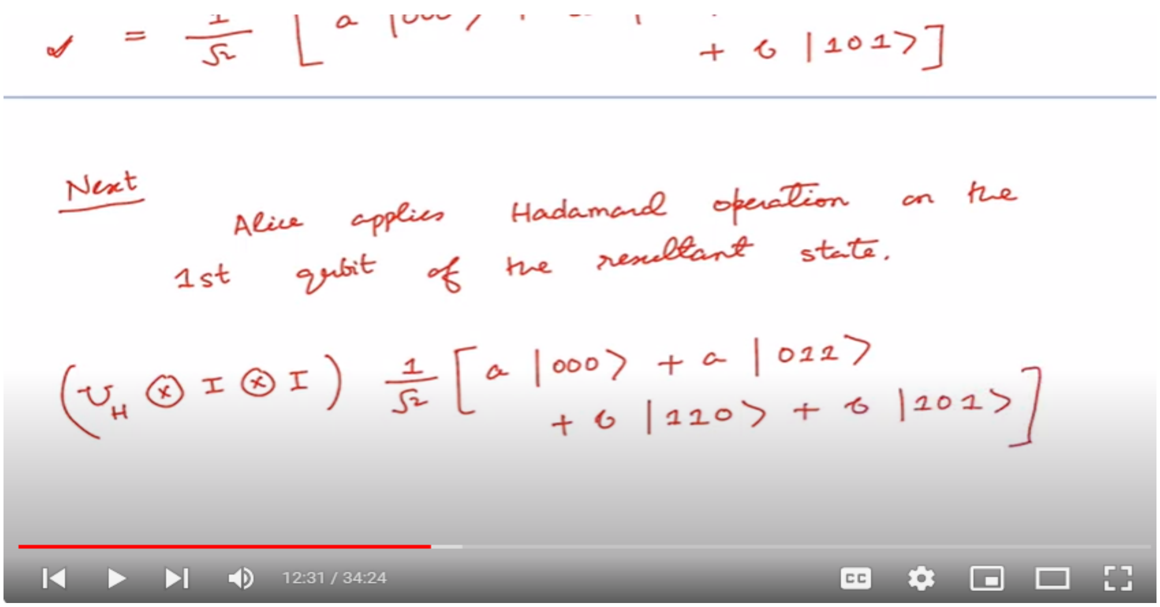
Okay Alice first apply now Alice has two qubits in her lab now as per the protocol Alice first Alice first apply CNOT operation CNOT operation and you know that this is a two qubit operation on her two qubits on her two qubits while Bob's qubit remain unaffected while Bob's qubit remain unaffected remain unaffected because Alice can't do any measurement on Bob's qubit naturally so the this operation mathematically speaking I will write it like this u CNOT is the operation made by Alice on her two qubits and identity refers to the fact that no operation is done on Bob's qubit so we have started with this particular state initially and now let us see what would be the result of this operation the CNOT operation let me just write it again u CNOT tensor product I this is the operation and this already we have written the full form of it is 1 by root 2 we have already written let me write it again a 0 0 0 plus a 0 1 1 plus b 1 0 plus b 1 1 1 right this is what we'll have you see just to you know the what happens because of the CNOT operation and we have already discussed CNOT operation in great detail.

(Refer Slide Time: 11:10)



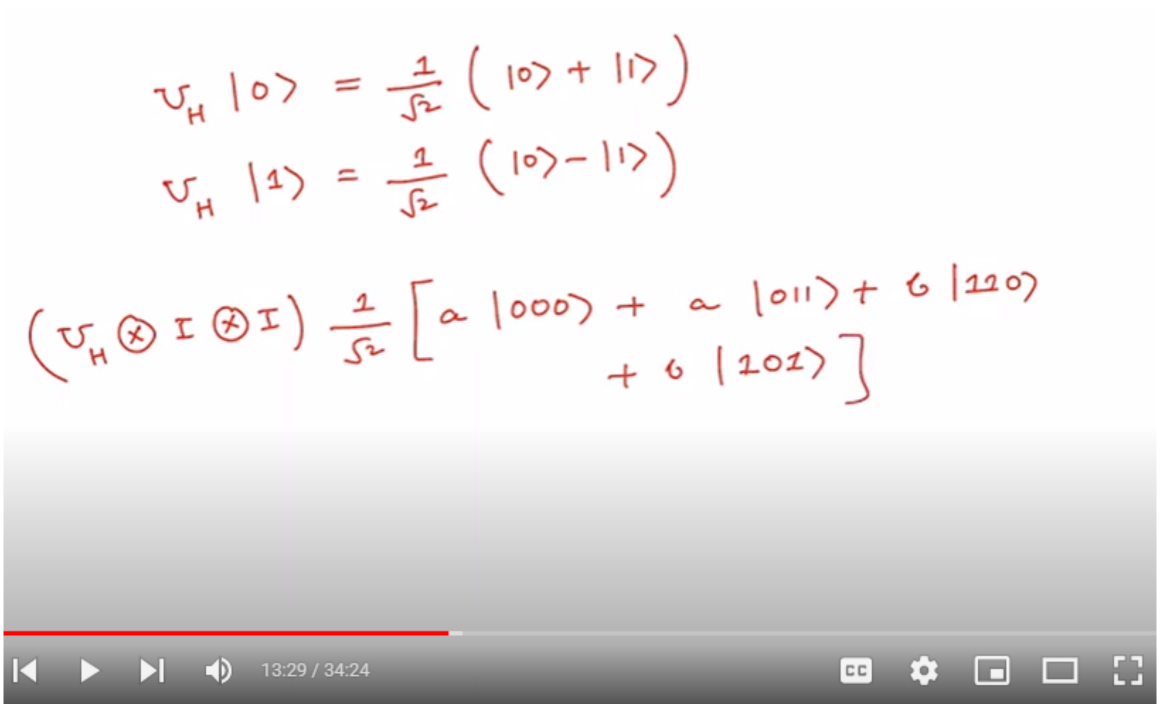
So let us let me write down the result of this CNOT operation on the first two qubits of Alice so because of the CNOT operation the first term would become a 0 0 0 and here just to remind you this is going to be my control qubit and this is going to be my target qubit here this is going to be the control qubit and this is going to be the target qubit this is going to be the control this is going to be the target this is going to this is going to be the target and we know as per the CNOT operation the target qubit gets flipped only if the control gate is 1 right so using that we will have these terms so second term would be a 0 1 1 and third b would be this would become b 1 1 0 and last term would become b 1 0 1 so this would be the result of CNOT operation

(Refer Slide Time: 12:31)



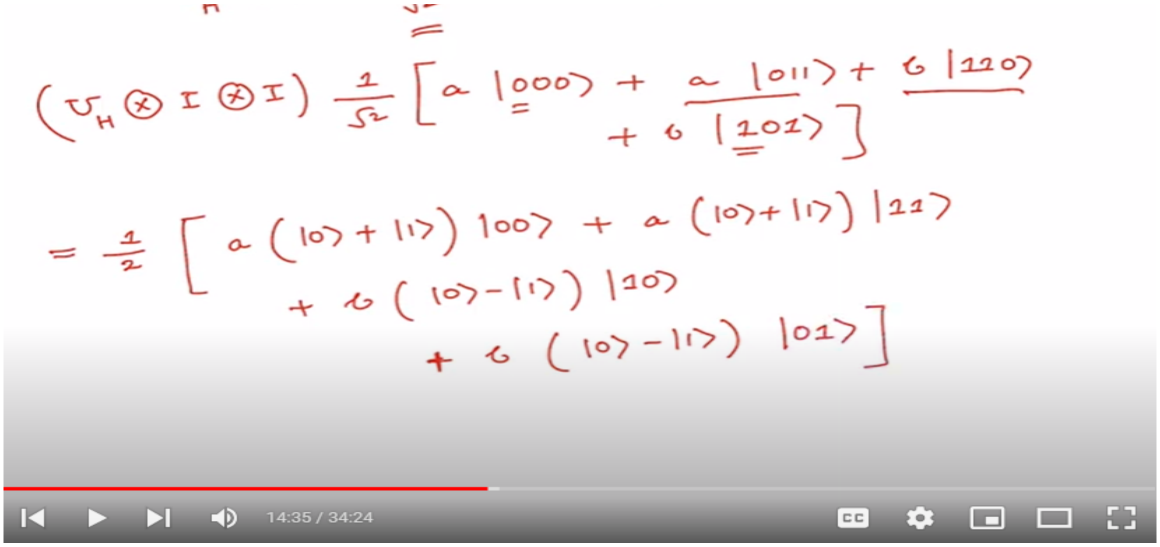
Now next step what Alice is going to do Alice applies Hadamard operation on the first qubit of the resultant state I mean by resultant state mean the state that Alice obtained after the CNOT operation is done and here now second qubit and the third qubit would remain unaffected so mathematically speaking what Alice is doing she is basically making a Hadamard operation on the first qubit and the second and the third qubit remaining unaffected and he is doing this operation on the state which is obtained here because of the CNOT operation so let me write it again so you will have a000 plus a011 plus b110 plus b101 okay this is what it will have.

(Refer Slide Time: 13:29)



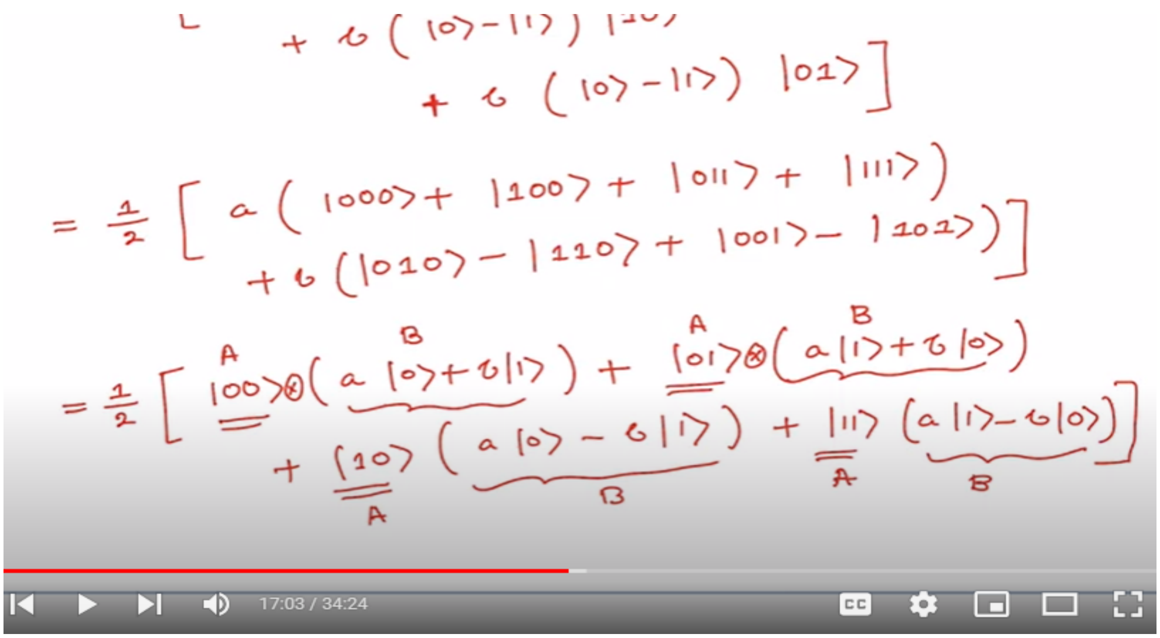
Just now recall we will require that the Hadamard operation is a single qubit operation and if the Hadamard gate is applied on qubit single qubit ket 0 we will get 1 by root 2 ket 0 plus ket 1 and if Hadamard operation is done on the qubit 1 then we will get 1 by root 2 ket 0 minus ket 1 okay so Alice Hadamard operation let me write once again Alice Hadamard operation on the first qubit let me write it once again here so that I can write the full thing so the operation is done on this particular state a000 a011 plus b110 plus b101.

(Refer Slide Time: 14:36)



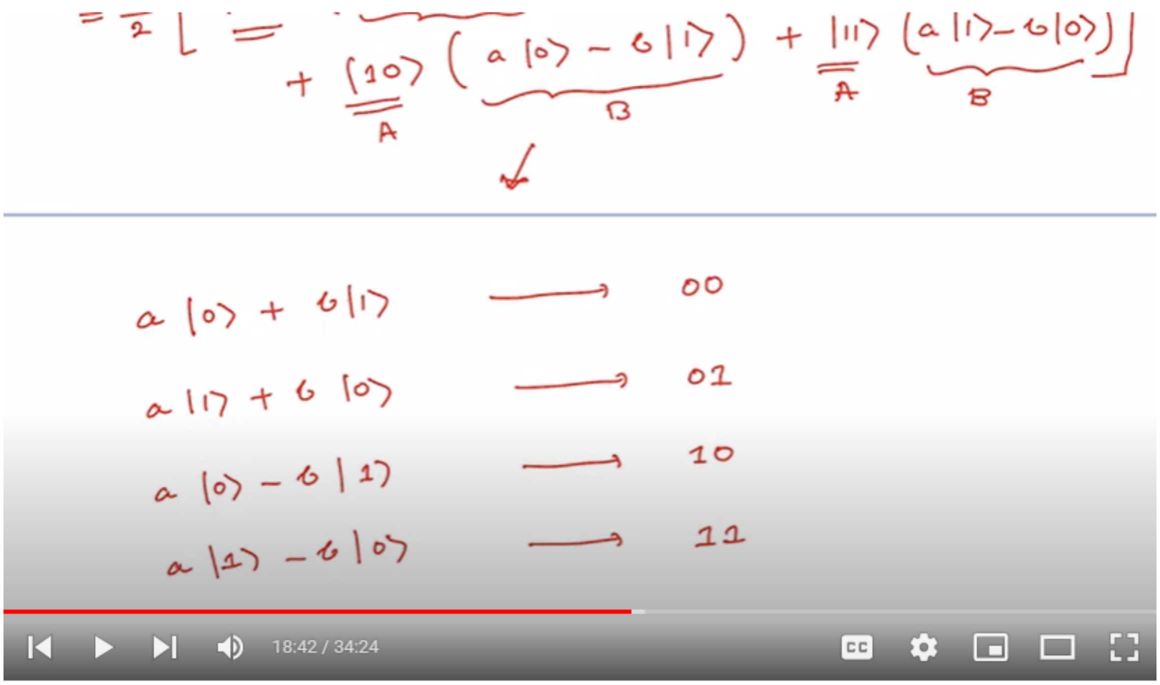
Now if Alice does the Hadamard operation on the first qubit this will give me 1 by root 2 will be there so that will become this half I can take common and then here the first one this one because of the operation of the Hadamard it would become a let me write here it would be ket 0 plus ket 1 then you will have 00 right then the second term let us look at it look at it then it will also become a ket 0 plus ket 1 1 1 and then the third term let me look at look at here then B you will get ket 0 minus ket 1 1 0 right and finally finally you will get B if you apply Hadamard operation here you will get ket 0 minus ket 1 0 1 so this is what would be the result of this Hadamard gate of operation on the first qubit.

(Refer Slide Time: 17:03)



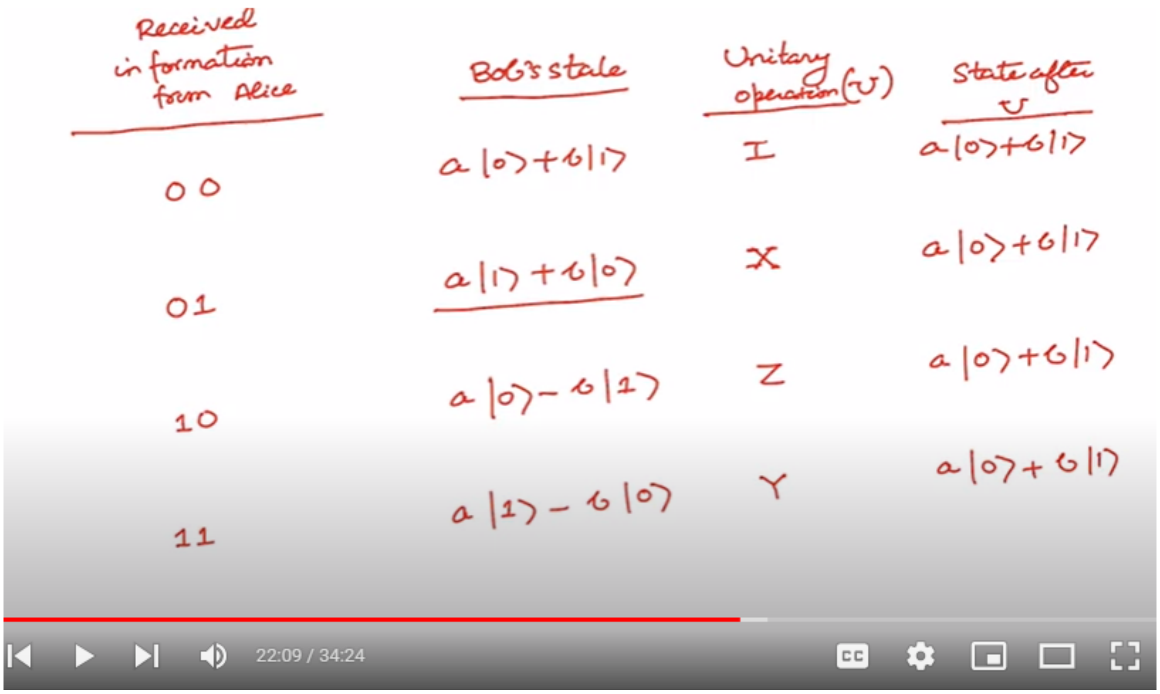
And if I open it up and then I can write a compact relation here if I collect all the quantities associated with a I will have a 0 0 0 plus 1 0 0 plus 0 1 1 plus 1 1 1 and then I'll have B 0 1 0 minus 1 1 0 plus 0 0 1 minus 1 0 1 I think you can easily verify it this is what we'll get in fact I can write it further in this form 1 half I can write 0 0 ket 0 0 a ket 0 plus B ket 1 then I have plus 0 1 a ket 1 plus B ket 0 and ket 1 0 a ket 0 minus B ket 1 and plus ket 1 1 a ket 1 minus B ket 0 okay this is what we get so what you actually see is this if Alice measures her two qubits in hand after these two operation that that is first CNOT operations followed by Hadamard operation she will obtain one of the state ket 0 0 ket 0 1 ket 1 0 or ket 1 1 with equal probability that is the probability would be 1 by 4 right probability of getting ket 0 0 ket 0 1 ket 1 0 ket 1 1 now as you see after I write it in this form this means that ket 0 0 now belongs to Alice ket 0 1 belongs to Alice while this belongs to Bob write this belongs to Alice this belongs to Bob this belongs to Alice this belongs to belong to Bob these belong to in least these belong to Bob and this belong to Alice and this qubit belongs to Bob because of this operation made by Alice.

(Refer Slide Time: 18:42)



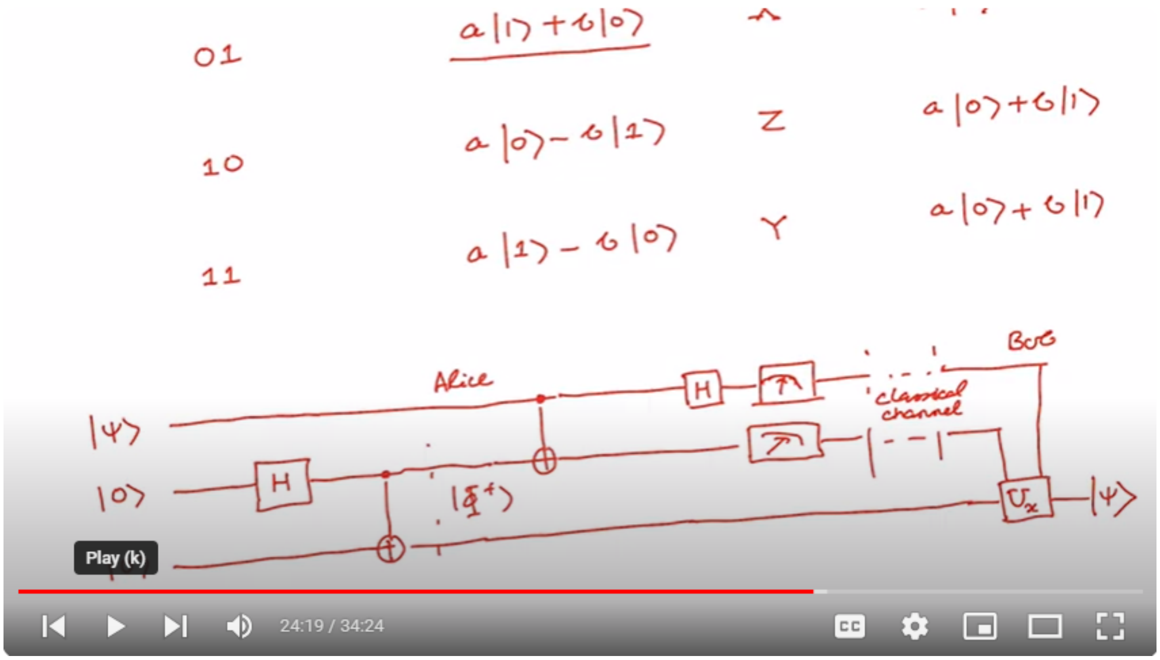
and this is very important so interestingly what is happening is that Bob's qubit which was a qubit from Bell state initially collapses to you know let me just write here Bob's qubit getting collapse to a0 plus b1 if Alice measures her qubit as 0 0 and Bob's qubit I'm just writing all these things from here summarizing in a tabular form and if you know Alice measures her qubit to be say 0 1 then Bob's qubit collapses to a ket 1 plus b ket 0 if Alice measures her qubit to be 1 0 Bob's qubit become a ket 0 minus b ket 1 and if Alice measures her qubit to be 1 1 then Bob's qubit becomes a ket 1 minus b ket 0 right so you you might be noting that must be noticing this that Alice has totally destroyed the initial qubit state ket psi upon her measurement this makes quantum teleportation consistent with the no cloning theory now what Alice does after doing this two operation Alice informs Bob about her measurement result through a through classical communication and based on the information received from Alice Bob now carry out certain unitary operation on his qubit.

(Refer Slide Time: 22:09)



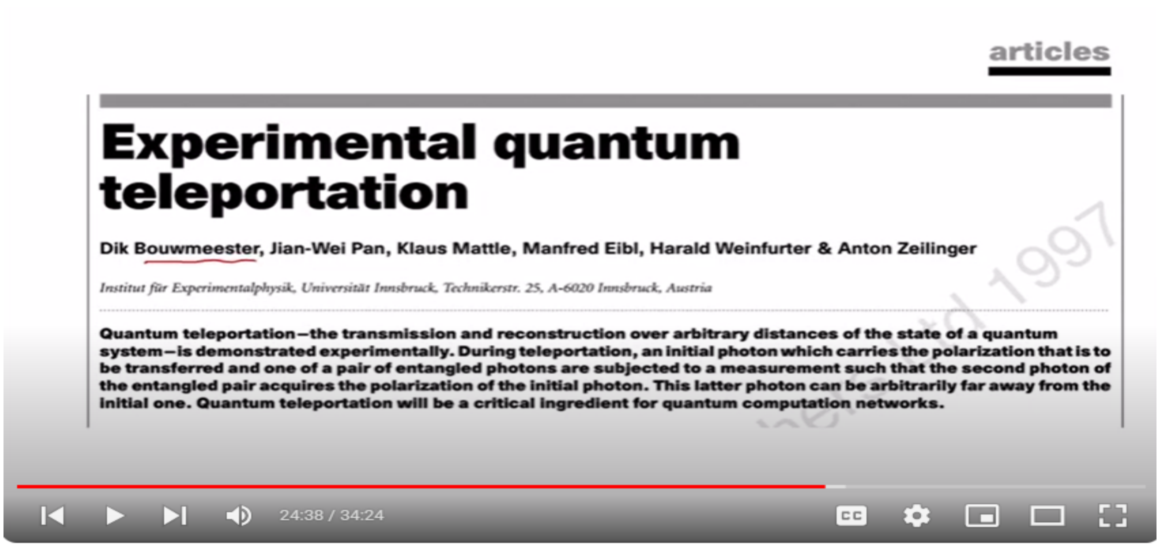
Let me now write little bit clearly what Bob does in his lab after receiving the information from Alice so received let me show it in a tabular form received information information from Alice as part of protocol based on this information Bob is going to do something suppose Alice inform Bob that she got 0 0 as a result of this experiment as our experiment then Bob state Bob state Bob has no idea what is it Bob state is a 0 plus b 1 and in this case the unitary operation if Bob say get the information 0 0 from Alice then the unitary operation Bob does is the identity operation that means Bob is going to do nothing on his qubit so therefore state after the unitary operation state after unitary operation would be simply a ket 0 plus b ket 1 and as you can see this is the original quantum qubit state which is Alice intended to send to Bob now now say Bob get the information that Alice got 0 1 then the state in Bob Bob state would be a 1 a ket 1 plus b ket 0 and Bob is going to make the operation X let me again tell you that Bob has no idea that this is the state he is having but Bob know that he has a qubit state and then he make the operation X which is the not operation because of this not operation he is going to get a 0 plus b 1 once again you see this is the intended state Alice wanted to send to Bob and now say Bob get the information that Alice got 1 0 then corresponding Bob state would be a ket 0 minus b ket 1 and in this case Bob is going to make the Z operation and this will result in the state a ket 0 plus b ket 1 which is the originally Alice wanted to send to Bob and then finally if the information received by Bob about Alice measurement is that Alice got 1 1 then the corresponding Bob state is a ket 1 minus b ket 0 and Bob here makes the Y gate operation on his qubit on his quantum state then as a result of this he will get a 0 plus b ket 1 and this is the original state that Alice intended to send to Bob so you immediately see and clearly see that Bob reconstruct the initial state that Alice wanted to send by applying unitary operation on his qubit based on the information received from Alice this is what quantum teleportation protocol is

(Refer Slide Time: 24:19)



Now let me quickly show this protocol in a schematic so this is the original state that ket psi Alice wanted to send and as part of protocol both Alice and Bob has to share a quantum state and in this case we saw that so called phi plus state has to be shared between Alice and Bob so phi plus state this Bell state can be created by using a Hadamard gate operation with the input as 0 0 followed by a CNOT operation this will result in this will result in the state phi plus right and then Alice is based on this Alice after this state is shared between Alice and Bob Alice is going to make on his cube on her qubit he she is going to make the CNOT operation followed by a Hadamard operation right this already we discussed and then he she makes the measurement on her qubits this is the measurement I'm just showing this is the measurement on her two qubits that Alice is having and this information is passed to through a classical channel through a classical channel this is this is the classical channel classical channel this information is passed to Bob and then what Bob does finally Bob makes unitary operation on his qubit based on the information received from Alice and then he reconstruct the original state ket psi and this is the so called quantum teleportation protocol is so I hope you have understood it.

(Refer Slide Time: 24:38)


articles

Experimental quantum teleportation

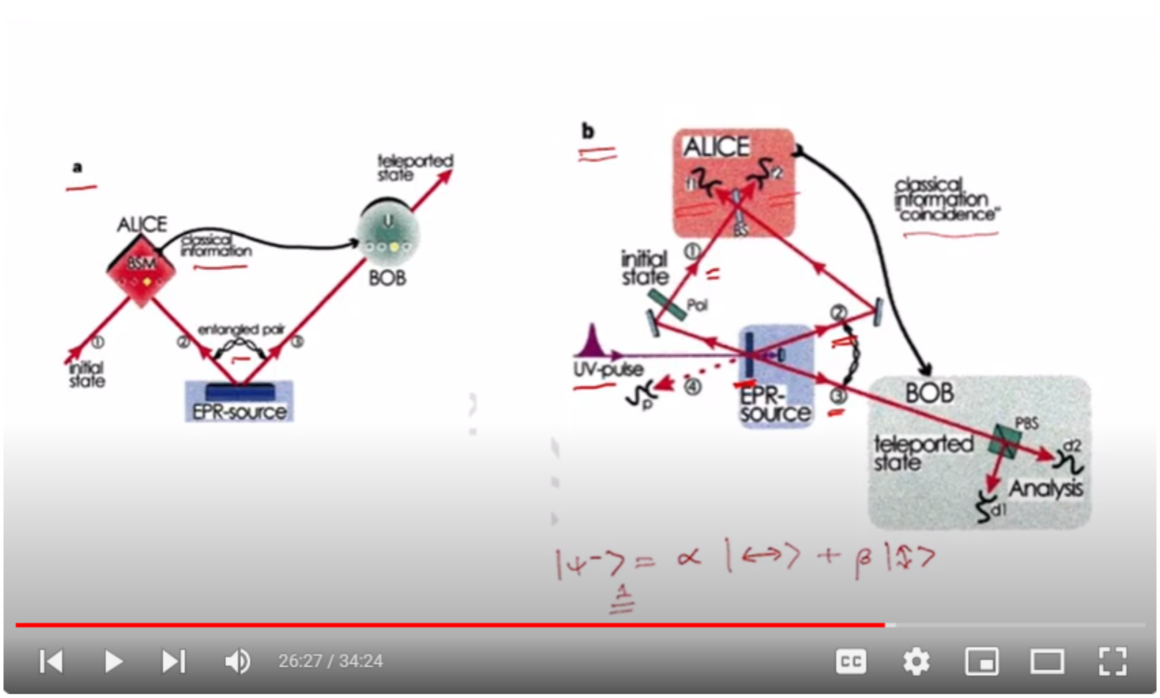
Dik Bouwmeester, Jian-Wei Pan, Klaus Mattle, Manfred Eibl, Harald Weinfurter & Anton Zeilinger

Institut für Experimentalphysik, Universität Innsbruck, Technikerstr. 25, A-6020 Innsbruck, Austria

Quantum teleportation—the transmission and reconstruction over arbitrary distances of the state of a quantum system—is demonstrated experimentally. During teleportation, an initial photon which carries the polarization that is to be transferred and one of a pair of entangled photons are subjected to a measurement such that the second photon of the entangled pair acquires the polarization of the initial photon. This latter photon can be arbitrarily far away from the initial one. Quantum teleportation will be a critical ingredient for quantum computation networks.

Quantum teleportation is not just a theoretical proposal it has been realized in many different setups realized under various laboratory conditions one of the first reported results were from Baumister and Anton Zellinger group where they have demonstrated quantum teleportation of a photon polarization state of a photon.

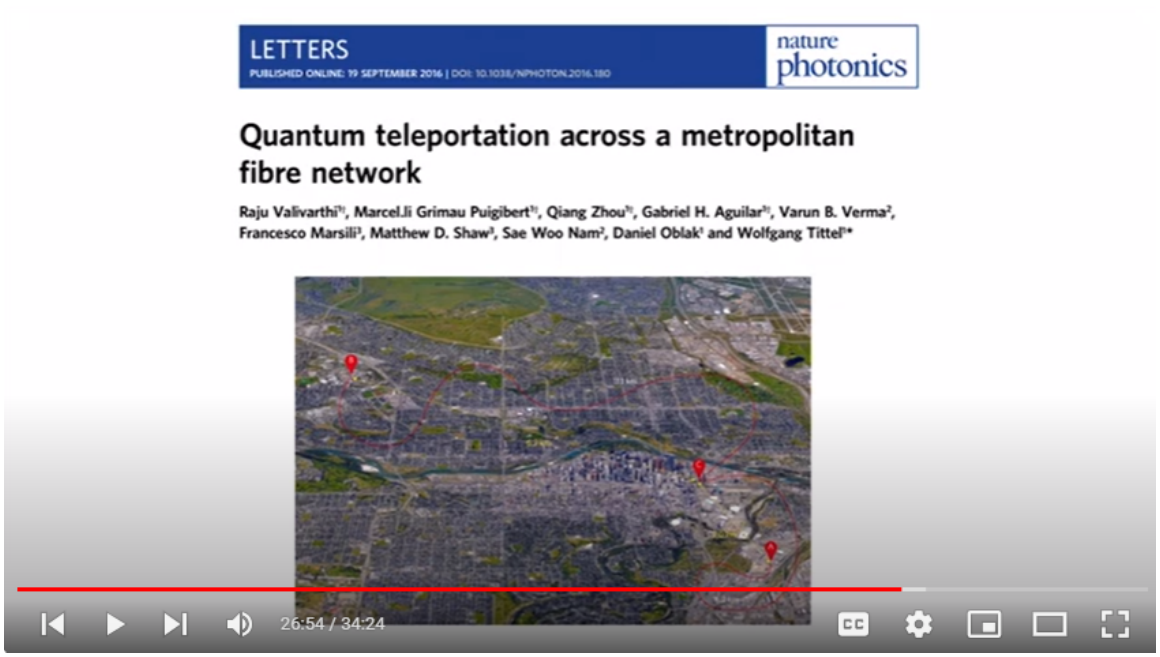
(Refer Slide Time: 26:27)



I have picked up two figures from their paper in figure a the teleportation protocol is schematically explained Alice and Bob are sharing entangled pair of photons between them and Alice using classical channel informs Bob about her joint measurement results and thereby Bob reconstruct the quantum state and which Alice wanted to send to Bob in figure B the actual experimental scheme used by the research group is shown

now here what they have done is just passing a ultraviolet pulse through a nonlinear crystal they created entangled pair of photons say 2 and 3 where the photon number 2 belongs to Alice and photon 3 belongs to Bob and they have created using this nonlinear crystal another photon 1 which is to be teleported and this photon number 1 is in the state it is basically in either it is in a horizontal polarization state or it is in the vertical polarization state this is I am talking about the photon 1 which is to be teleported now after Alice make some measurement call coincidence measurement using a beam splitter and detector two detectors were there to make the measurement the result of the measurement is informed to Bob using classical channel and then Bob reconstruct the quantum state in his lab.

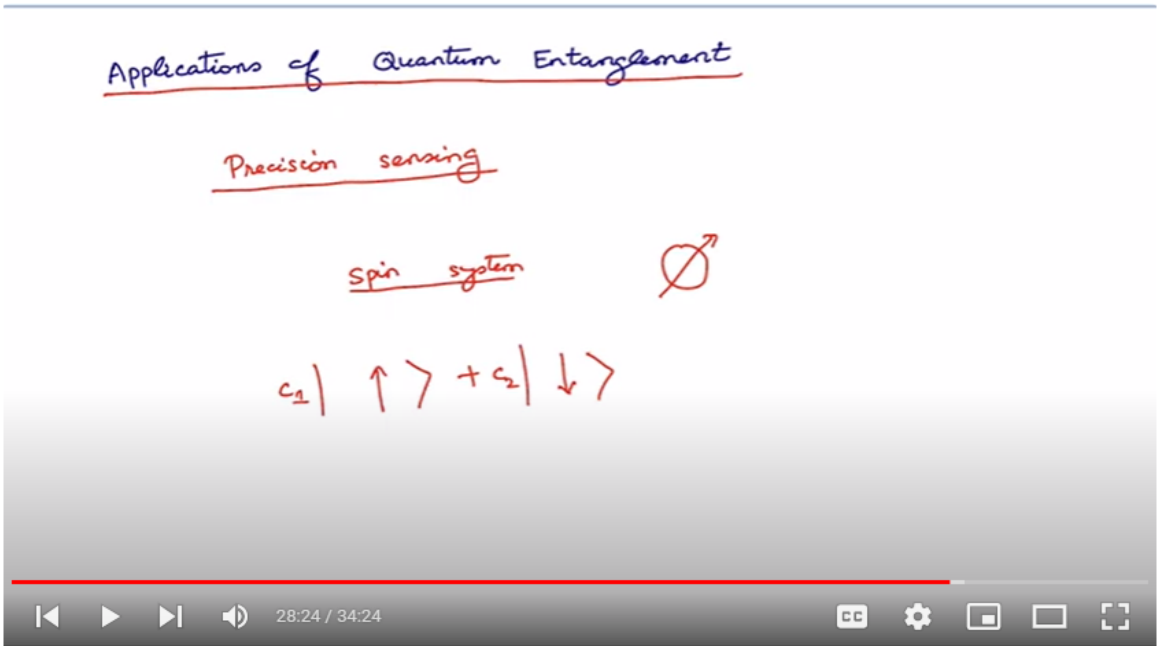
(Refer Slide Time: 26:54)



Quantum teleportation has been realized over atomic distances using nuclear magnetic resonance as reported by Nielsen group then it has been reported with atoms as well as atomic qubits and so on all these results were reported only for very short distances.

however in 2016 a research work is reported where quantum teleportation across a metropolitan fiber network was a safe and they reported quantum teleportation over a distance of 6.2 kilometers and actually such demonstration demonstrations are huge boost towards realizing quantum communication networks and so called quantum internet.

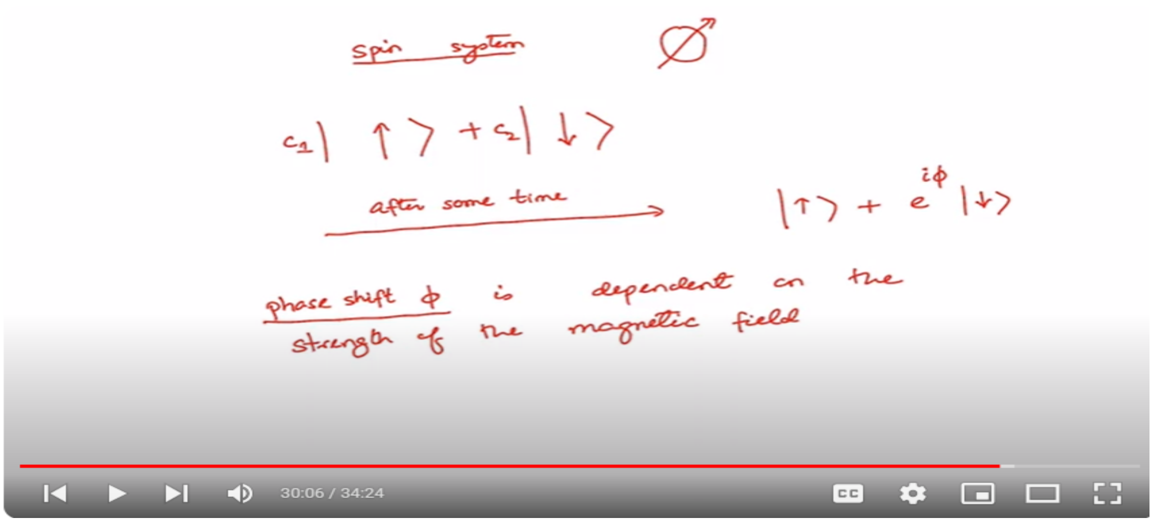
(Refer Slide Time: 28:24)



Super dense coding and quantum teleportation are not the only applications of quantum entanglement quantum entanglement could be used to realize precision sensor devices as well precision sensing is one of the important application of quantum entanglement here I will give you a brief idea about the precision sensing only.

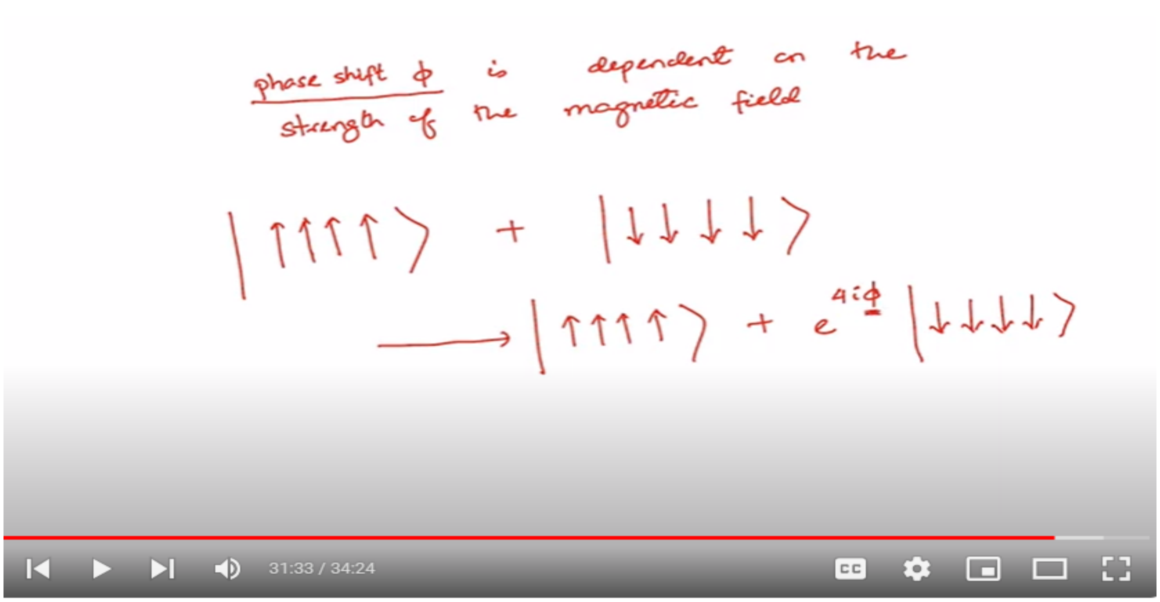
to illustrate that let me give you an example consider a spin system say we have a spin system is given to you and this spin may be in the up state or maybe in the down state so it is in a superposition state like this now as you know a spin system is very sensitive to presence of magnetic field and we want to measure them ambient tiny magnetic field that may be present around it now what you can do is that you can leave the spin in the magnetic field for some time if you leave the spin in the magnetic field for some time then it will evolve and in fact the spin will precess around the magnetic field

(Refer Slide Time: 30:06)



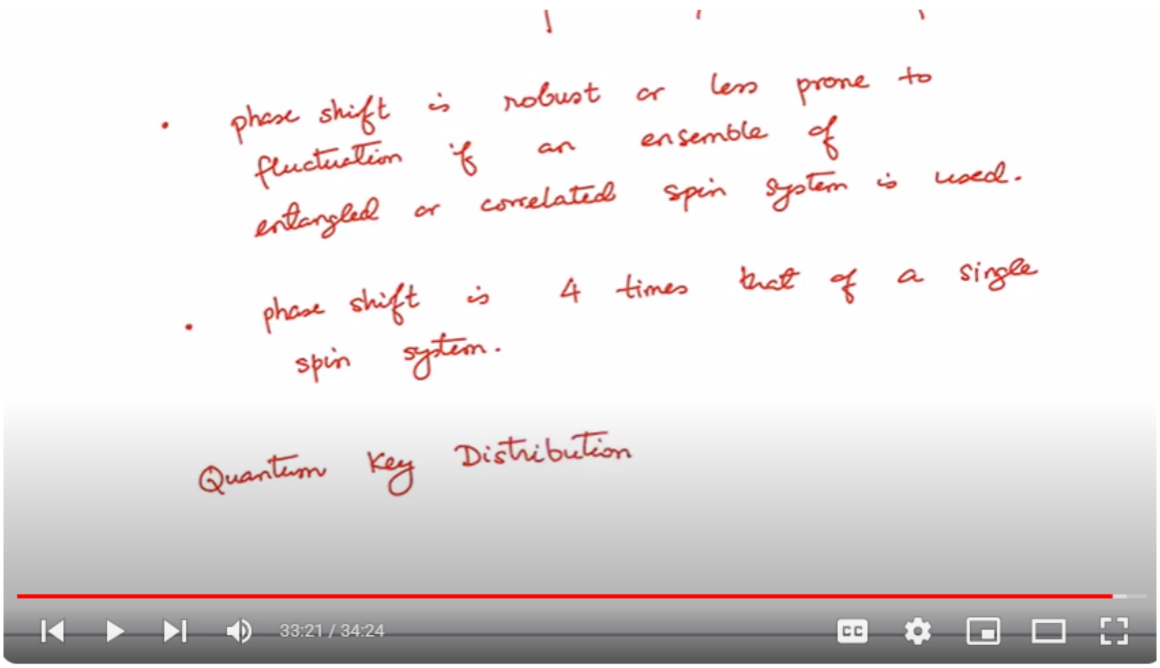
and as a result there will be there will arise a phase difference between the ups up state and the down down spin state so after some time if we leave the spin in the magnetic field after some time there will be a phase difference between the up state and the down state suppose this phase difference between up and down state is say Phi and this phase this phase Phi is very sensitive to magnetic field in fact Phi is dependent Phi is dependent on the strength of the magnetic field on the strength of the magnetic field so you can calculate the magnetic field because phase is directly dependent on or directly proportional to the magnetic field so if you measure the phase shift this is phase shift if you measure the phase shift then you can measure the magnetic field now this phase is extremely sensitive and it's prone to fluctuation very easily.

(Refer Slide Time: 31:33)



however rather than dealing with only one single spin system if we take an ensemble of correlated or entangled spin system then the sensitivity can be enhanced significantly say you take an ensemble of say for correlated spin system suppose you correlated entangled spin system you take then the this set of four correlated spin system will result in this kind of a state because it is entangled and as you leave it in the magnetic field then there would be a development of phase between this up state and the down state of the spin ensemble spin state and what is remarkable is that the phase shift that is going to be developed between the up and the down state would be four times that of the phase shift that will occur for a single spin system so therefore because this Phi is anyway a very small quantity and it's very sensitive to fluctuation if we just consider only one single spin system or a set of or ensemble of uncorrelated set of spin system

(Refer Slide Time: 33:21)



however if we take a entangled spin system or correlated spin system this phase is going to be this phase shift phase shift is robust or robust compared to or what I mean to say is that it is less prone to robust or less prone to fluctuation fluctuation if an symbol of spin system in fact ensemble of entangled or entangled or correlated correlated spin system is used is used and also as I mentioned the phase shift is four times in the example that I have given four times that of a single spin system so obviously this phase shift phase shift and hence phase shift is easy to measure and thereby will be able to calculate the magnetic field easily so this is one of the important application of quantum entanglement

and one other application measure application involving quantum entanglement is the so-called quantum key distribution which is used in quantum cryptography however I will not talk about it here because there are many materials available in later research already.

Let me stop here in this lecture we have discussed quantum teleportation protocol and gave you a brief idea how quantum entanglement can be used for precision sensing I ask all of you to go through problem solving session number four whatever I discuss till problem solving session number four is going to be part of your examination if any more lecture is posted you can consider that as additional material only. thank you so much