Quantum Entanglement: Fundamentals, measures and application

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Week-01

Lec 1: Introduction

Welcome to the first lecture of the course. In this lecture, I am going to give you a brief introduction to the phenomenon of quantum entanglement and its significance. I will also tell you a bit of history behind the term called entanglement. Quantum mechanics is one of the most successful theories of physics and science and in fact it is at the root of tremendous technological development we see all around us today. But however, quantum mechanics is extremely bizarre, it is a strange animal. In fact, people or scientists who were responsible for developing this exciting field of area were quite baffled by quantum mechanics.

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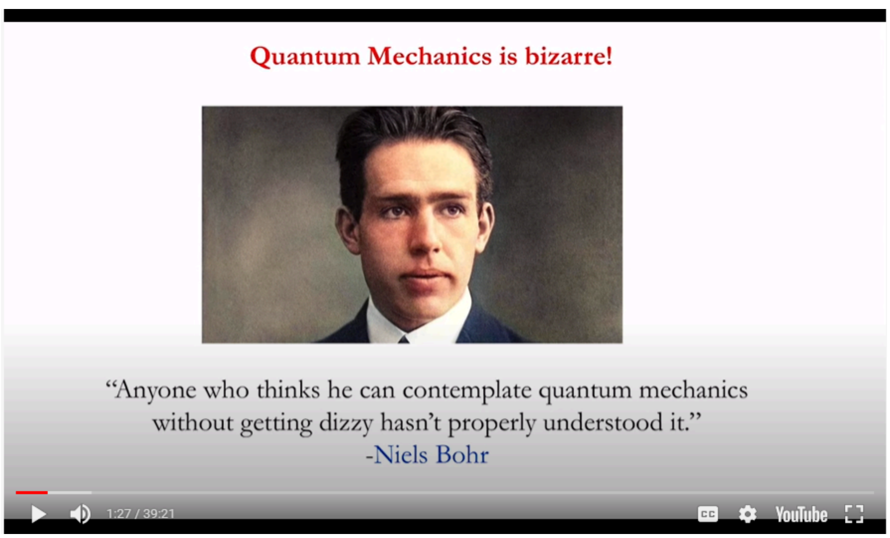

For example, Niels Bohr commented that anyone who thinks that anyone who thinks he can contemplate quantum mechanics without getting dizzy has not properly understood it. Another legend of quantum mechanics, Richard Feynman said that anyone who thinks they know quantum mechanics does not. In fact, the most noteworthy comment was made by Erwin Schrodinger who has contributed significantly towards quantum mechanics. He said that I don't like it, he said it in the context of quantum mechanics he said that I don't like it and I am sorry I have ever had anything to do with it.

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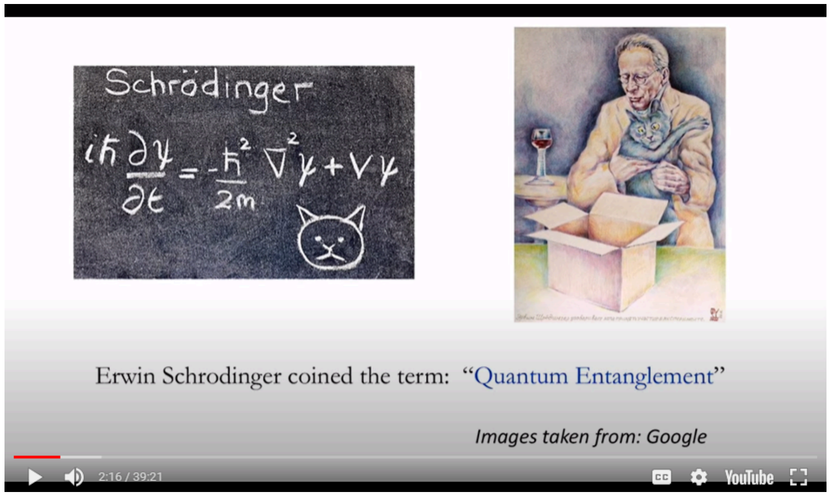

And it is remarkable to note that it is Schrodinger who coined the term quantum entanglement. In spite of all bizarreness, quantum mechanics useful and it is considered to be the most successful theory in physics so far. The principles of quantum mechanics you know that it is at the play for the technological development in lasers, atomic clock, entire field of electronics including computers, internet and mobile communications, GPS and many more. These technologies are also referred to as the first generation of quantum technology which is triggered by the principles of quantum mechanics. In the first quantum revolution the principle of quantum mechanics were used as put by 2022 Nobel laureate in physics.

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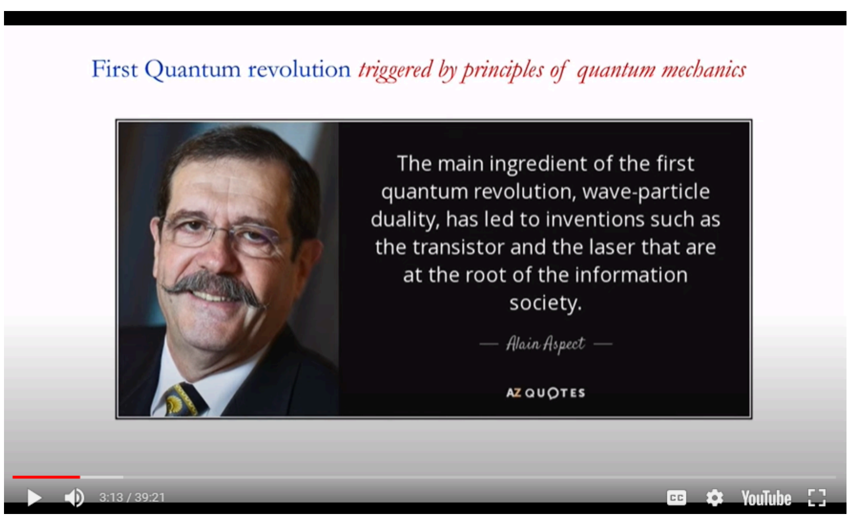

Let me read his statement. He said that the main ingredient of the first quantum revolution wave particle duality has led to inventions such as the transistor and the lasers that are at the root of the information society. Today we are said to be under the second quantum revolution and at the core of second quantum revolution is the control of individual quantum systems and engineered quantum systems. To understand it let me give you a quick background let me lead you to a quick history of quantum mechanics to begin with. So quantum mechanics begin with the Planck's explanation of black body radiation followed by Einstein's explanation of the photoelectric effect.

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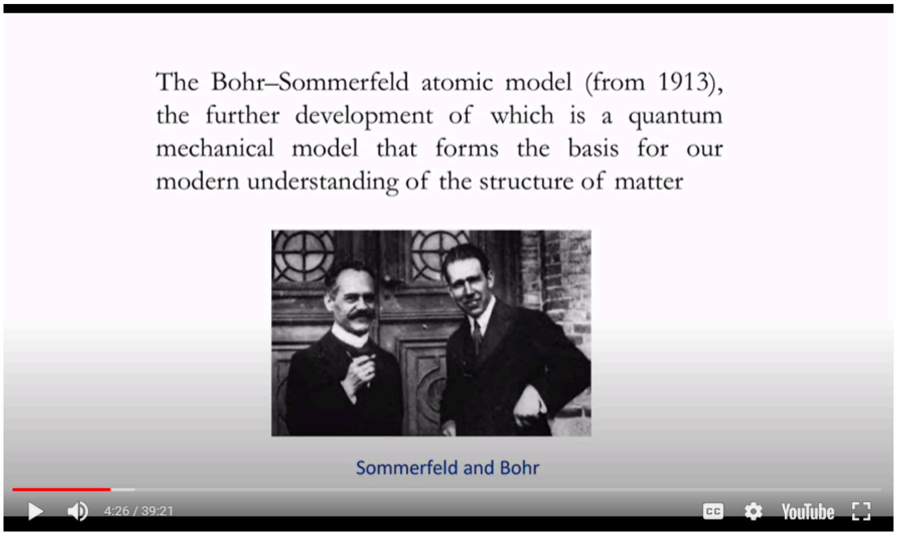

In fact Einstein got the Nobel prize in physics in 1921 for his work on the photoelectric effect not for his work on general relativity or special relativity. Then in 1913 Bohr Sommerfeld came up with their atomic model which helped us to understand the structure of matter and in 1924 the French physicist de Broglie came up with his matter wave hypothesis and for which he also got Nobel prize later on.

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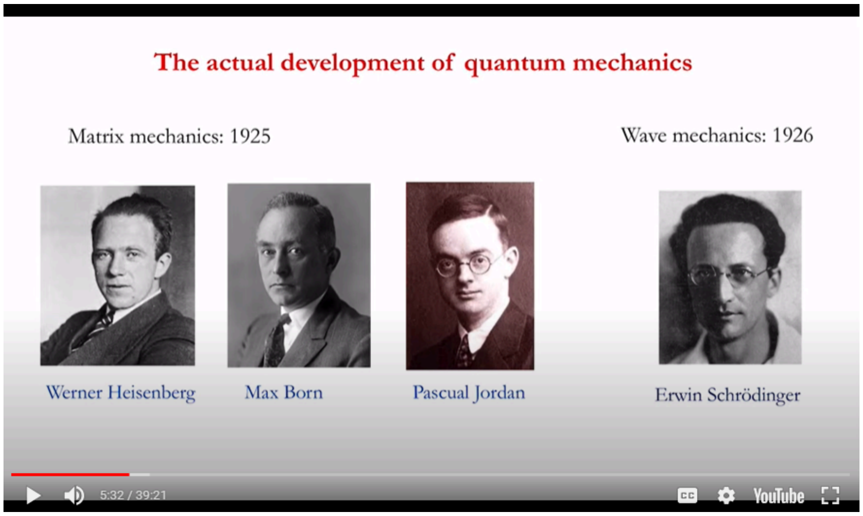

In fact the actual development of quantum mechanics began which we know of as today occurred in 1925 and 1926 in the form of two seemingly different formalism one is called the so called matrix mechanics it was developed by Werner Heisenberg, Max Born and Jordan while the second one is called the wave mechanics and it was put forward by Erwin Schrodinger but soon it turned out that both the formalism are in complete agreement with each other and they represent the same thing. The quantum mechanics continued to expand into 1930s as an independent and coherent conceptual framework.

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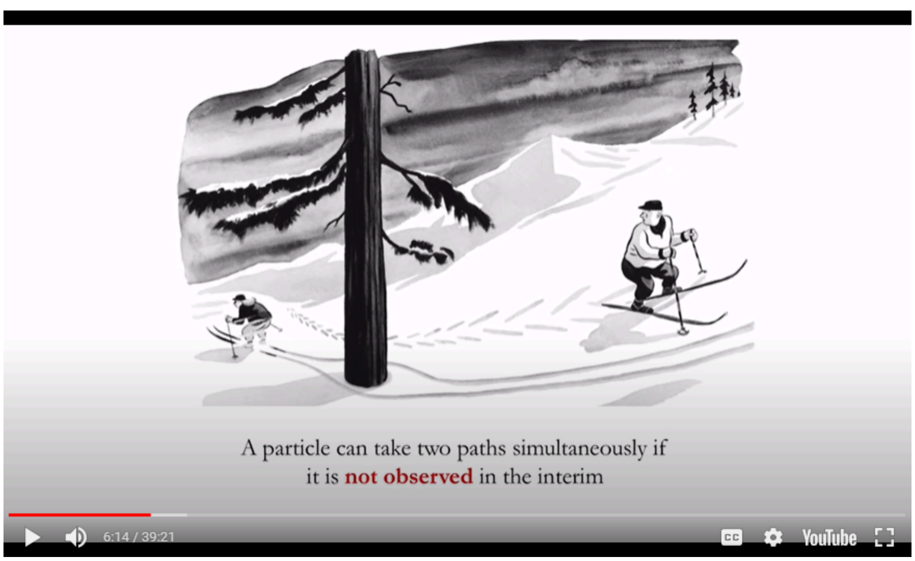

Even after almost a century now, principles of quantum mechanics are difficult to understand as they often have no analogies to our day-to-day life. This is the reason why there is so much fascination and unfortunately misinformation in popular science media. As you know in quantum mechanics a particle can take two parts simultaneously if it is not observed in the interim. This is depicted in this cartoon picture how it would be if it is extended to our daily life situations. Because of the entirely different view of the world, quantum mechanics was doubted by many.

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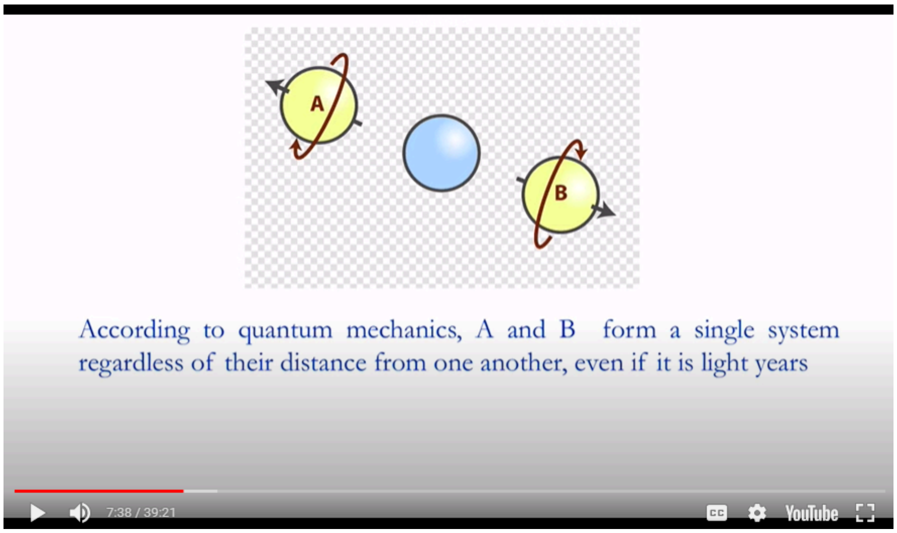

One epic example, in fact a landmark work was a publication by Einstein, Podolsky and Rosen, now it is known as the so called EPR paradox in this work they considered a quantum mechanical system comprising of two particles arising from the same source but subsequently separated and sent to different directions. Now as for quantum mechanics they form both say particle A and B form a single system regardless of their distance from one another even this distance is in light years. Even they are separated by billions and billions of years they are still going to form a, considered to form a single system and say they have a fixed total momentum as long as neither of particles A and B is disturbed. But the distribution of the total momentum because the two particles has not yet been determined only when a measurement is performed on one particle say A to know a particular property, the corresponding property of the other particle B is immediately determined. For example, if the total momentum of the pair of particles A and B are known then measuring the momentum of one particle reveals the momentum of the other particle.

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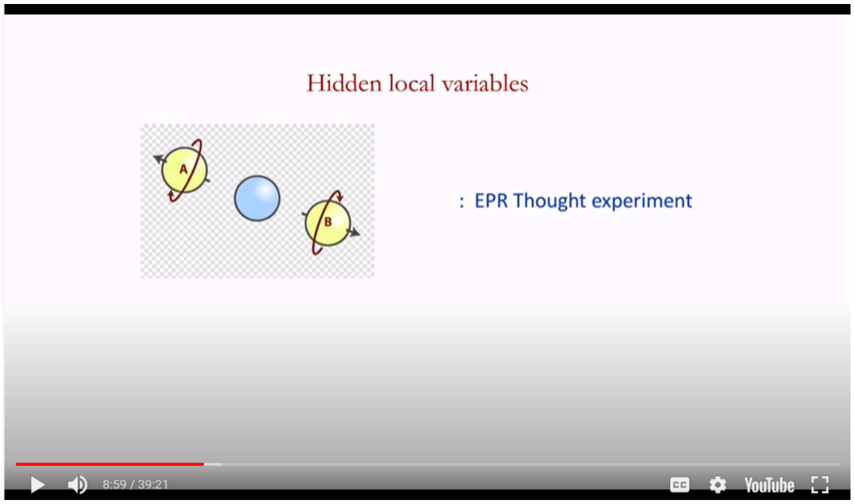

Okay, Einstein, Podolsky and Rosen conclude that from this that the momentum of the two particles must already be fixed prior to the measurement because otherwise the particles which are now far apart from one another would have to communicate at more than the speed of light. So Einstein never approved this kind of a situation and he referred to as spooky action at a distance in quantum mechanics and we will talk more about EPR paradox actually in some details in a later class. To explain EPR kind of experiment from the beginning the so called hidden local variable theory were proposed. This went on for many years without any conclusion. However, in 1964 John bell proposed an inequality which shows that the classical view in the form of a theory of hidden local variable satisfies certain conditions which are violated by quantum physics so hidden variable theory has to be abandoned.

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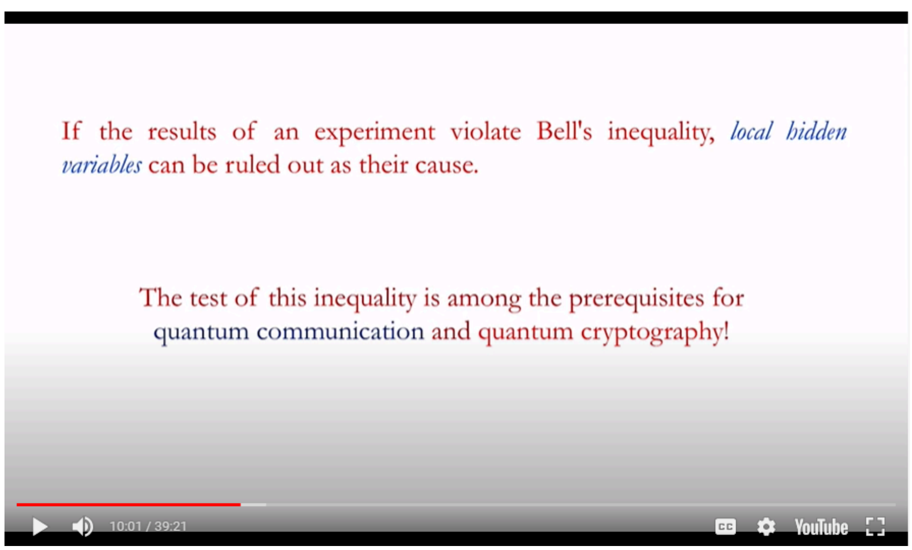

Most importantly it is possible to check this inequality by experiments. Since 1970 physicists have performed increasingly refined experiment for this surprisingly what was aimed to test the fundamentals of quantum mechanics eventually resulted in advancement of quantum technology. In fact the test of Bell's inequality is one among the prerequisites for secure message transmission using quantum systems also called quantum communication and quantum cryptography. By the way not a single experiment till date has violated Bell's inequality this means that quantum mechanics is still correct even with his so called weirdness or bizarreness. One of the most important quantum phenomena is the so called phenomena of quantum entanglement which is the subject matter of this course.

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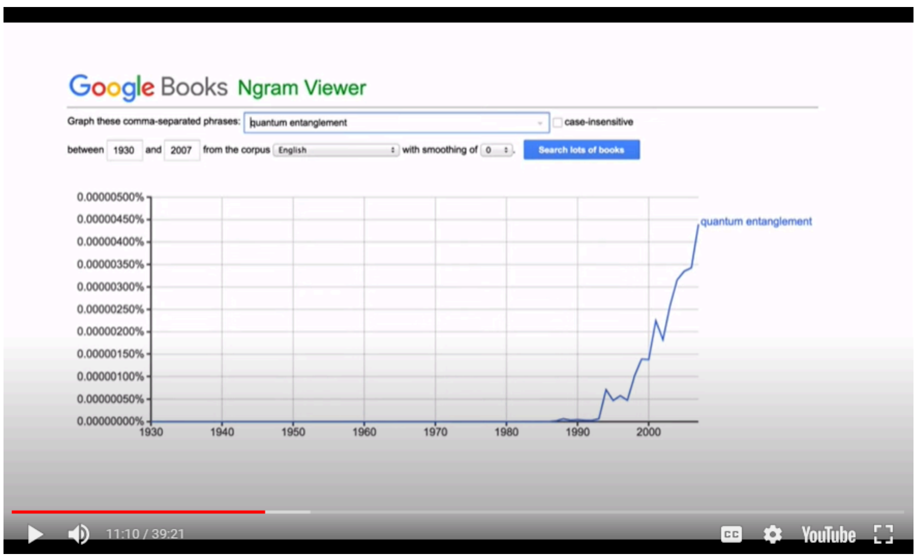

Interestingly entanglement was introduced in 1935 by the Austrian physicist Erwin Schrodinger as I have already told you it came as a result of his response to the so called EPR paradox. However it is astonishing that it took roughly 60 years for the term entanglement to come into common use after Schrodinger introduced it in 1935. If you look at Google books Ngram viewer it shows that the phrase quantum entanglement does not occur at all in this large database of books until 1987. And there are then a very small number of occurrences through 1993 after which the number rises repeatedly and particularly of course it's going to increase 1000 fold after 2022 physics Nobel prize. As I can to quote from an article in American Journal of Physics the decisive year for the term entanglement was probably 1987.

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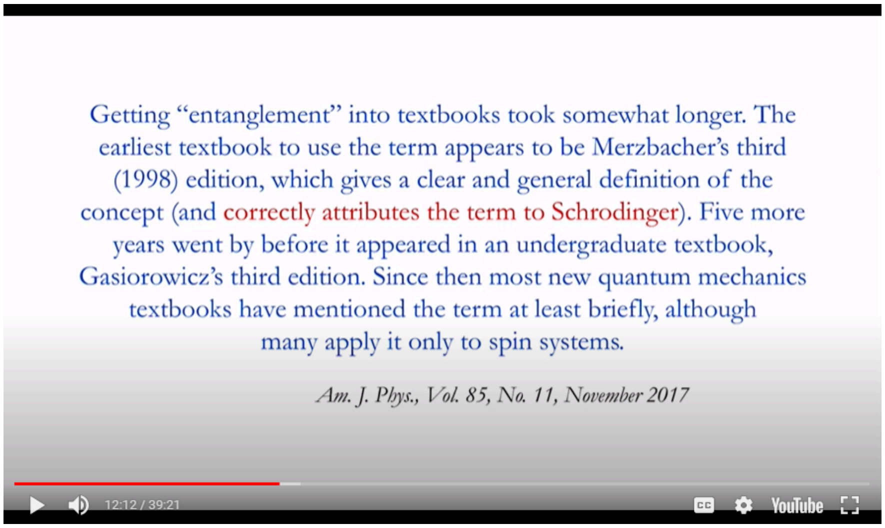

A conference was held in London that year to celebrate Schrodinger's 100th birthday and Bell's contribution to the confidence proceedings highlighted Schrodinger's phrase quantum entanglement even using it as a section title. Another thing is that the getting entanglement into textbooks took somewhat longer again I am quoting from that same article in American Journal of Physics the earliest textbook to use the term appears to be Merzbecher's famous quantum mechanics book in third edition 1998 which gives a clear and general definition to the concept, and it also correctly attributes the term to Schrodinger. And after five more years passed when this term quantum entanglement appeared in undergraduate textbook as well for example there is a book called quantum physics by Gasiorwicz it's for undergraduate and since then most new quantum mechanics textbook have mentioned the term at least briefly although many apply it to only to spin systems.

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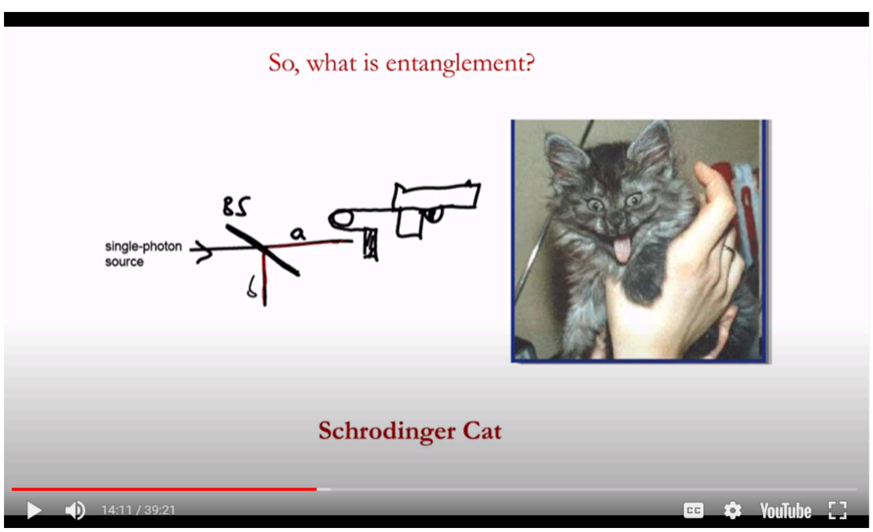

In fact we are going to discuss spin quantum systems to illustrate and to explain quantum entanglement to you in the beginning later on we will consider other systems as well. Now what is entanglement? The phenomenon of entanglement can be illustrated with a thought experiment which has become known as the Schrodinger cat. Consider a cat is confined in a box a photon is incident on a 50-50 beam splitter that means the photon can take either the path A or path B with equal probability if the photon takes the path A then it triggers the pistol thereby killing the cat making it dead on the other hand if the path A is empty that means there is no photon there in other words the photon is taking the path B the pistol can't be obviously triggered and the cat is alive.

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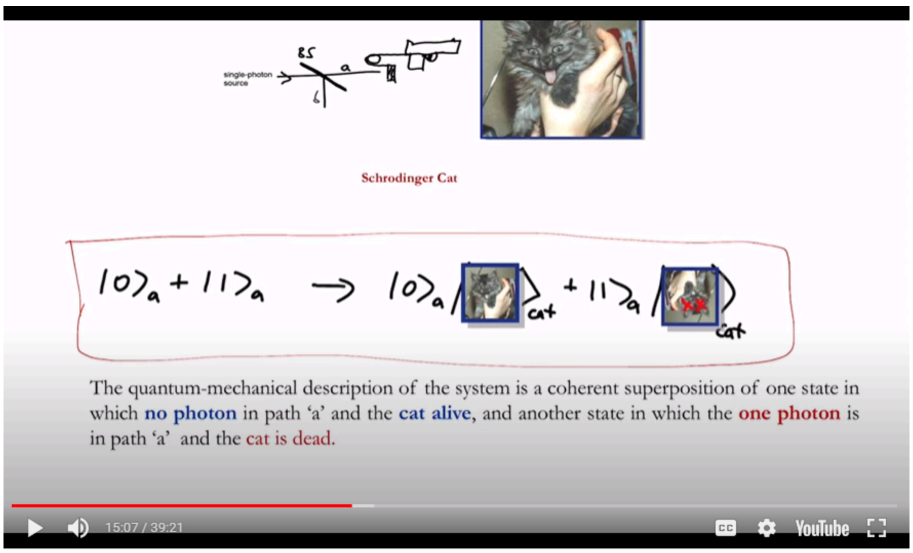

This situation can be captured mathematically in this superposition state so I will explain what it is. You see as long as you don't open up the box the cat is alive and dead at the same time. This is basically the essence of this equation here. In physical terms the cat has two states alive and dead. The same is true for the photon in the path A either there is zero photon and one photon the state of the cat and the state of the photon are not independent of one another.

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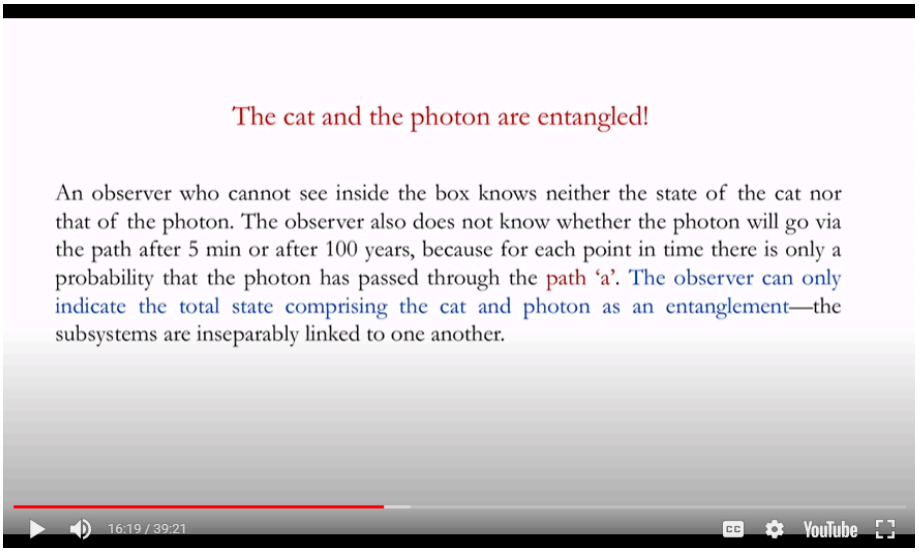

The cat is alive and the photon does not go via path A or the cat is dead and the photon goes through path A. So basically, it means that the cat and the photon are entangled they are said to be entangled in fact an observer who can't see the box has no idea or no clue about the state of the cat or the state of the photon. The observer also has no knowledge whether the photon will go via path A after five minutes or after hundred years because for each point in time there is only a probability that the photon has passed through via path A it can go via path A or not. The observer can only indicate the total state comprising the cat and the photon as an entanglement. The sub-systems here the sub-systems mean the system the cat as one system and the photon as the another sub-system.

So photon and the cat are two sub-systems and both cat and the photon comprises a total system. So these sub-systems are inseparably linked to one another.

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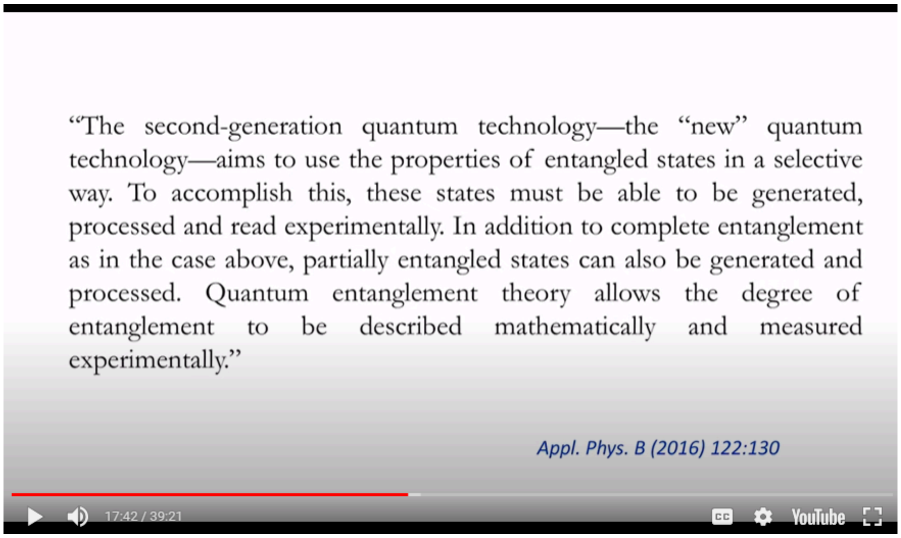

Now the second generation of quantum technology is heavily impacted by the phenomenon of quantum entanglement. In fact let me quote from a nice research article on quantum technology it says that the second generation quantum technology the new quantum technology aims to use the properties of entangled states in a selective way to accomplish this, these states must be able to be generated, processed and read experimentally. In addition to complete entanglement as in the case above which we have already discussed partially entangled states can also be generated and processed Quantum entanglement theory allows the degree of entanglement to be described mathematically and measured experimentally This particular quote also explains the purpose of this course because one of the major goals of this course is to quantification or measure of quantum entanglement which we are going to do in great details starting in module 2 of this course.

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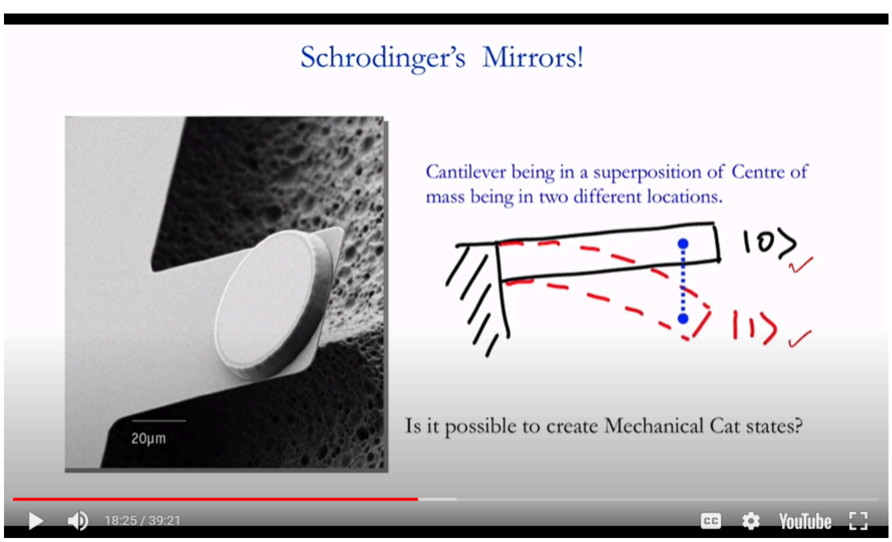

Now today, you know the Schrodinger cat I described earlier was a purely thought experiment today we are in a situation because of the advancement in technology to really test Schrodinger's idea in variety of ways. As an example consider this cantilever shown here, it is a mechanical vibrator of micrometer size, it is possible to create a superposition state of the cantilever being into different positions denoted by states say 0 and state 1. Now the question is, is it possible to create mechanical Schrodinger cat state?

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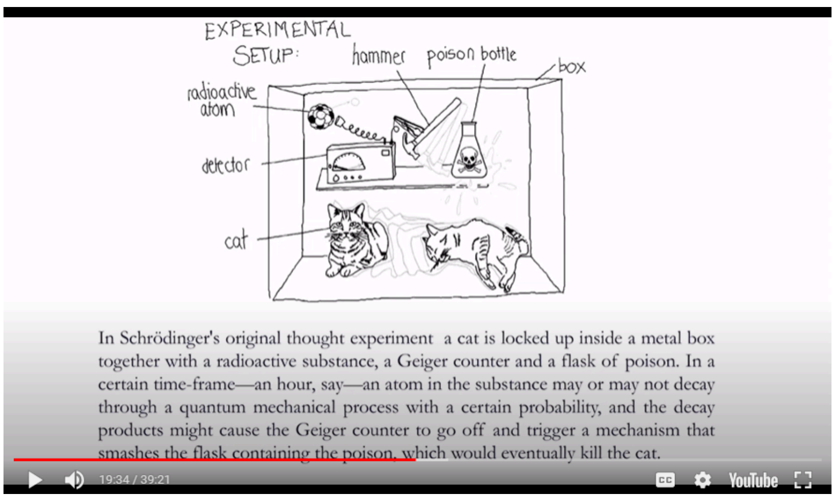

The answer is yes researchers have recently reported similar states in laboratory in a recent science experiment reported in science journal, a team of scientists created a substantially heavier Schrodinger cat by putting a small crystal into a superposition of two oscillation states however to explain it first let me talk about another version of Schrodinger's thought experiment this one is said to be the original thought experiment not the one I told involving beam splitter in this experiment a cat is locked up in this thought experiment the cat is locked up inside a metal box together with a radioactive substance, a Geiger counter and a flux of poison in a certain time frame say an hour or so say an atom in the substance may or may not decay through a quantum mechanical process with a certain probability and decay produces, decay products might cause the Geiger counter to go off and trigger a mechanism that smashes the flux containing the poison which would eventually kill the cat so this was a thought experiment and since an outside observer can't know whether the atom has actually decayed he or she does not know whether the cat is alive or dead according to quantum mechanics which governs the decay of the atom the cat should be in an alive or dead superposition state.

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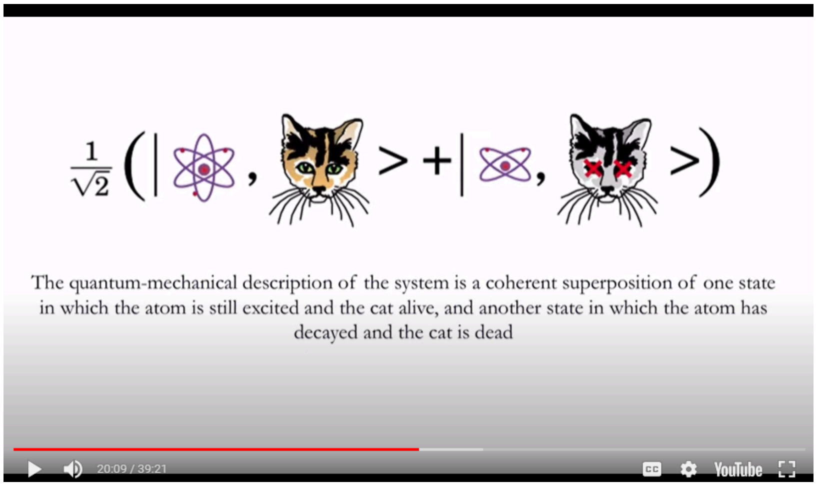

Now this is again described in this picture here the quantum mechanical description of the system is a coherent superposition of one state in which the atom is still excited and the cat is alive and another state in which the atom has decayed and the cat is dead now coming back to the recent science paper, the team leader Yiwen Chu says that it is clear that it is not possible to realize the exact thought experiment of Schrodinger in laboratory.

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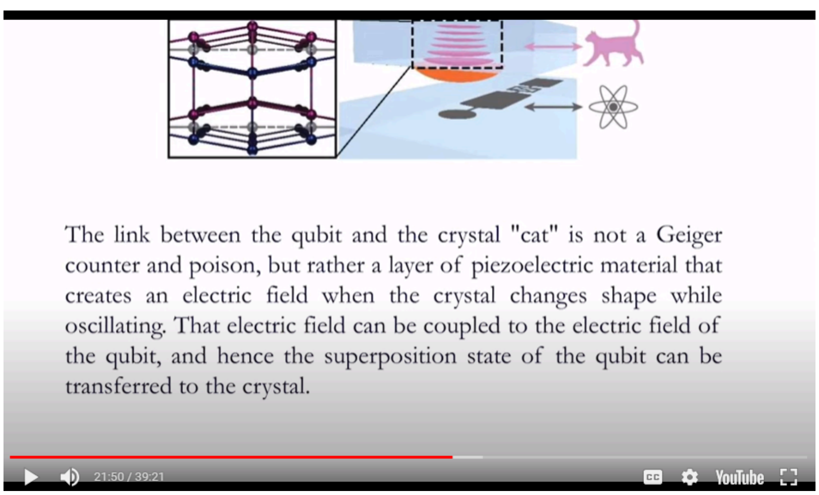

But what they did, they were able to generate a cat state using an oscillating crystal that represents the cat and while a superconducting circuit representing the original atom, that circuit is essentially a quantum bit or qubit taking the state 0 or 1 or superposition of both states 0 and 1 the link between the qubit and the cat is not a Geiger counter and poison but rather a layer of piezoelectric material that creates an electric field when the crystal changes shape while oscillating that electric field can be coupled to the electric field of the qubit and hence the superposition state of the qubit can be transferred to the crystal. As a result of this transfer of state from the superconducting circuit to the crystal, the crystal can now oscillate simultaneously in two directions that means at the same time it can oscillate in two opposite directions, that is up and down or down and up, those two directions represents the alive or dead states of the cat, for example say the up state corresponds to the alive state of the cat and the down state corresponds to the dead state of the cat by putting the two oscillation states of the crystal in a superposition, the team have effectively created a Schrodinger cat weighing 16 micrograms, clearly this is, this weight of the cat is extremely small, it is roughly the mass of a fine grain of sand and nowhere near that of the real cat but it is still several billion times heavier than that of an atom or molecule making it the fattest quantum cat reported till date in laboratory, these brilliant experimental results make it possible to explore the boundary between classical and quantum worlds. I hope you got some idea about what is quantum entanglement, we will start discussing it in great depth from lecture 4 onwards.

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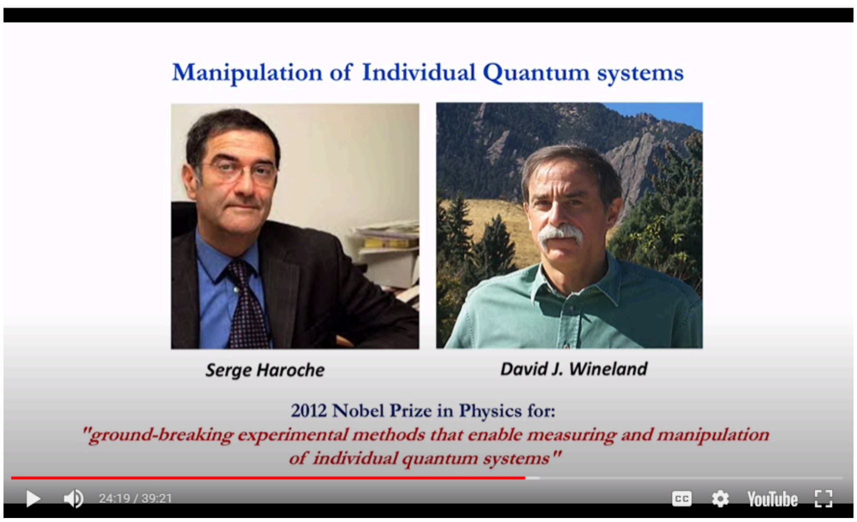

Quantum entanglement is an integral part of current quantum technology where it is used as a resource, it is used as a resource for various applications like quantum cryptography and quantum teleportation. By the way, quantum technology primarily as I said earlier resulted from the control of or manipulation of individual quantum systems. In this context, the works of Serge Haroche and David Wineland are most noteworthy. Both Serge and Wineland got 2012 physics Nobel prize for their ground breaking work which enabled control of individual quantum systems. They were awarded the Nobel prize in physics for ground breaking experimental methods that enable measuring and manipulation of individual quantum systems.

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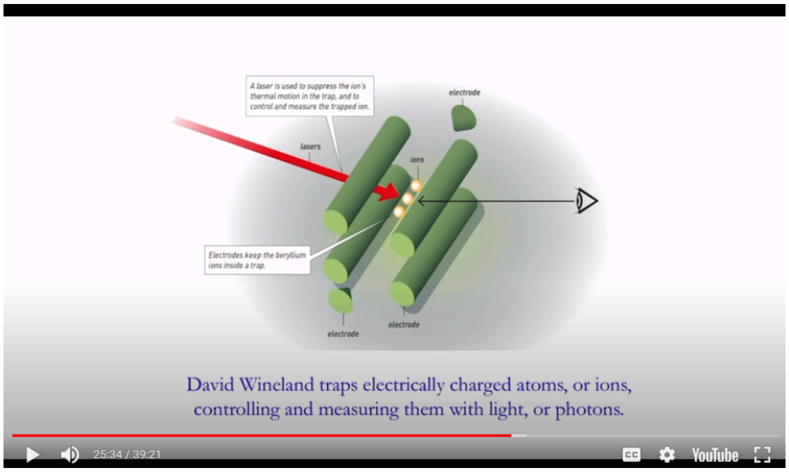

Through their ingenious laboratory methods they have managed to measure and control very fragile quantum states enabling their field of research to take the first steps towards building a new type of super-fast computer based on quantum physics. This was basically declared by the Nobel prize committee, that's what I have just read it for you. Both the methods suggested by Wineland and Haroche were different. In David Wineland's laboratory in Colorado, what he did, he trapped electrically charged atoms or ions in fact those were kept as you can see from this diagram here, those were kept inside a trap by surrounding electric fields. One of the secrets behind Wineland's breakthrough is the mastery of the art of using laser beams and creating laser pulses. A laser is used to put the ions in its lowest energy state thereby enabling the study of quantum phenomena with the trapped ion.

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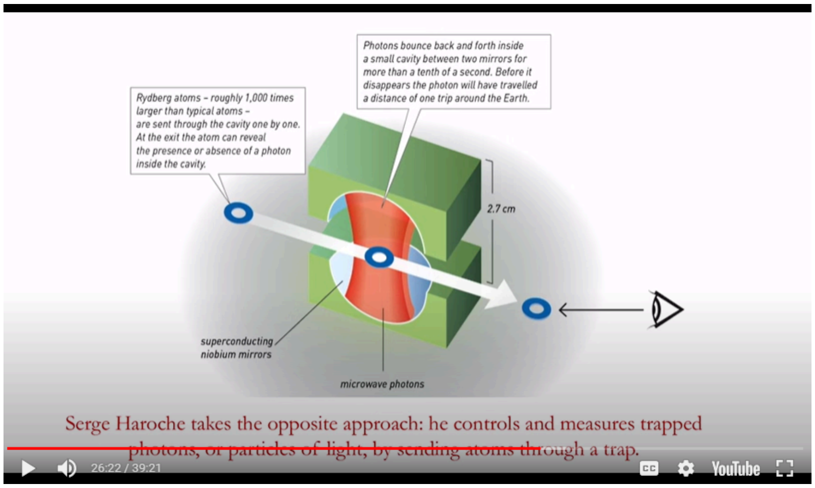

On the other hand, in the case of the Haroche's method, he took the opposite approach where he controls and measures trapped photons or particles of light by sending atoms through a trap in his laboratory in Paris in a vacuum and at room temperature of almost absolute zero, the microwave photons bounce back and forth inside a small cavity between two mirrors as depicted in this diagram here. The mirrors are so reflective that a single photon stays for more than a tenth of a second before it is lost. During its long lifetime, many quantum manipulations can be performed with the trapped photon without destroying it.

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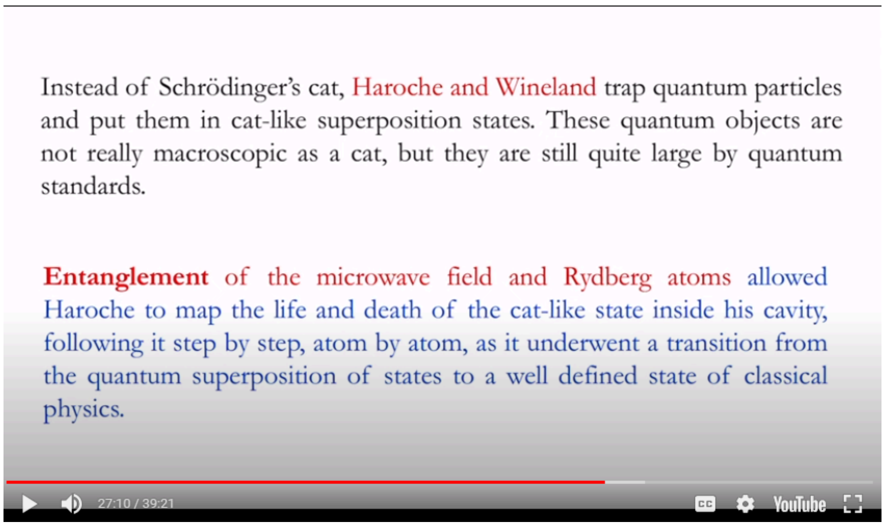

Instead of Schrodinger's cat, Haroche and Wineland trapped quantum particles and put them in cat-like superposition states and these quantum objects are not really macroscopic as a cat sometime back what I have described but they are still quite large by quantum standards. In fact, in the Haroche's experiment, there is an entanglement between the microwave field and the Rydberg atoms and it allowed Haroche to map the life and data of the cat-like state inside this cavity, following its step-by-step, atom-by-atom as it underwent the transition from quantum superposition of states to a well-defined state of classical physics.

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Quantum entanglement is already used in technology a decade ago, particularly in quantum cryptographic protocols. Data encryption using quantum cryptography is already done in 2004. The first bank transfer using a protocol with quantum cryptography was made in Vienna. In 2007, National Council election was conducted for the first time using quantum cryptography methods at Switzerland.

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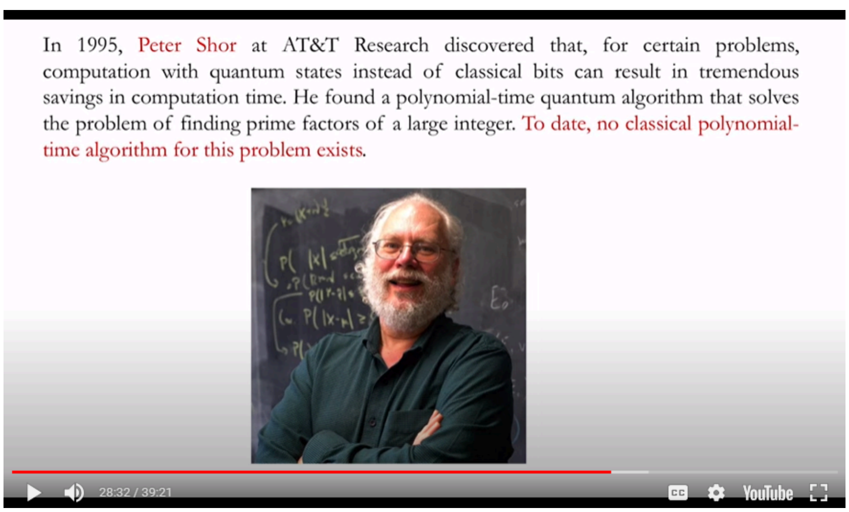

Another domain where quantum entanglement plays prominent role is in quantum computers. Apparently today all of us know that what kind of craze is there for quantum computers. In fact, this story began with the work of particularly current state of affairs that we have seen in quantum computers, began with the work of Peter Shor in 1995. In a work he discovered that for certain problems, computation with quantum states instead of classical bits can result in tremendous savings in computation time. Peter Shor found a polynomial time quantum algorithm that solves the problem of finding prime factors of a large integers. Till that no classical polynomial time algorithm for this problem exists.

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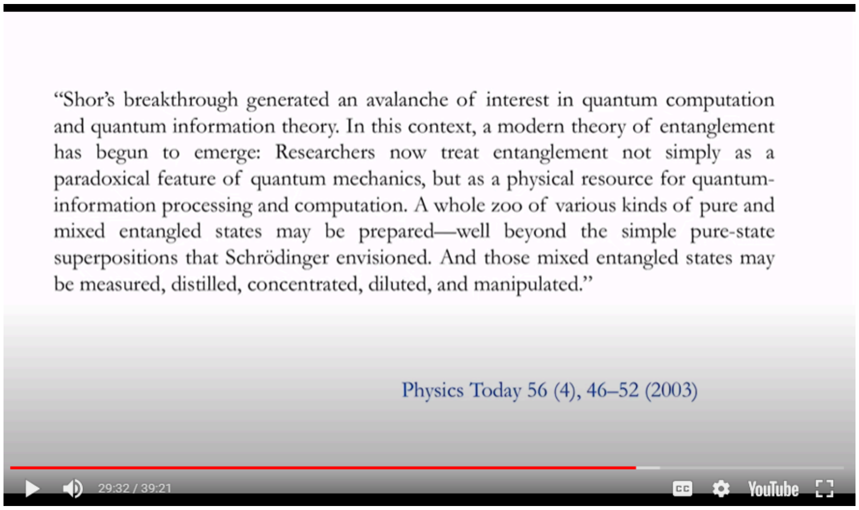

As put in a nice article in Physics Today way back in 2003, it's written there, Shor's breakthrough generated an avalanche of interest in quantum computation and quantum information theory. In this context a modern theory of entanglement has begun to emerge. Researchers now treat entanglement not simply as a paradoxical feature of quantum mechanics but as a physical resource for quantum information processing and computation. A whole zoo of various kinds of pure and mixed entangled states may be prepared well beyond the simple pure state superpositions that Schrodinger envisioned. And those mixed entangled states may be measured, distilled, concentrated, diluted and manipulated.

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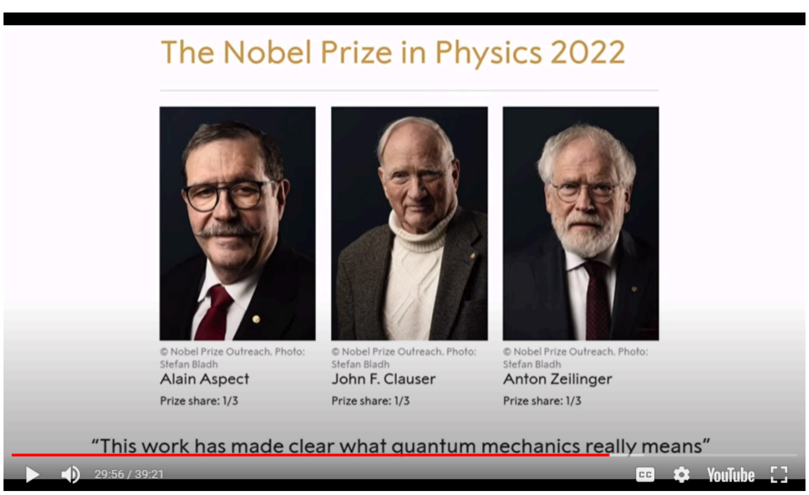

Entanglement has become such a realistic phenomenon that it has even been recognized by the Nobel Prize Committee. 2022 Nobel Prize in Physics went to three physicists, namely Alain Aspect, John F. Clauser and Anton Zeilinger. Their work has made it clear what quantum mechanics really means. In fact, the Nobel Prize Committee observed in their statement it says the Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger for experiments with entangled photons establishing the violation of Bell's inequalities and pioneering quantum information science.

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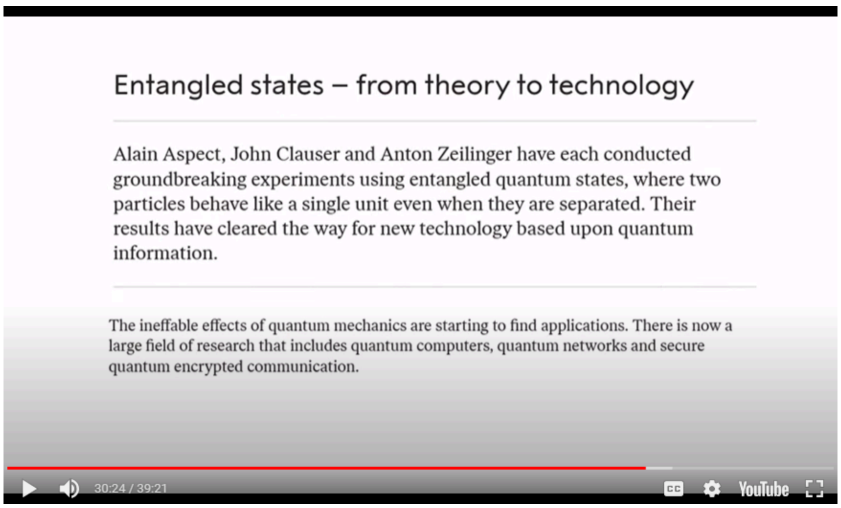

Alain Aspect, John F. Clauser and Anton Zeilinger have each conducted groundbreaking experiments using entangled quantum states where two particles behave like a single unit even when they are separated. Their results have cleared the way for new technology based upon quantum information. This was observed by the Nobel Prize Committee. Also they observed that the ineffable effects of quantum mechanics are starting to find applications. There is now a large field of research that includes quantum computers, quantum networks and secure quantum encrypted communication.

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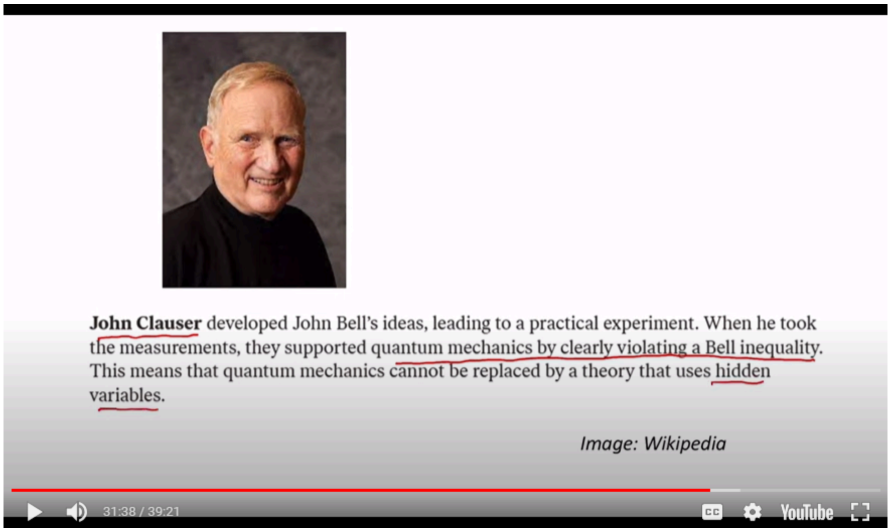

Let me now tell about the individual contribution one by one of these three physicists. For example, John F. Clauser, he developed John Bell's ideas leading to a practical experiment. When he took the measurements, they supported quantum mechanics by clearly violating a Bell inequality. This means that quantum mechanics cannot be replaced by a theory that uses hidden variables that I think I touched upon this earlier.

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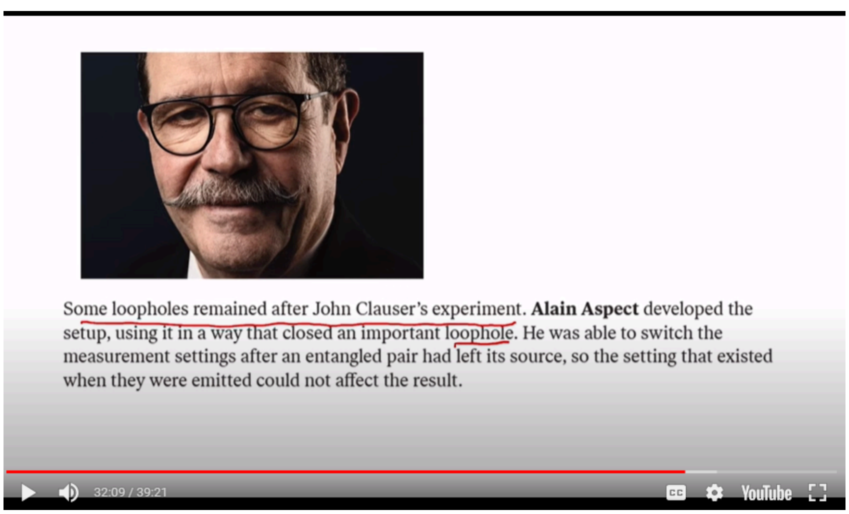

Then Alain Aspect worked, he basically filled up some loopholes, some loopholes were remained after John F. Clauser's experiment. He, Alain Aspect, closed it. He developed the setup using it in a way that closed an important loophole. He was able to switch the measurement settings after an entangled pair had left its source. So the setting that existed when they were emitted could not affect the result.

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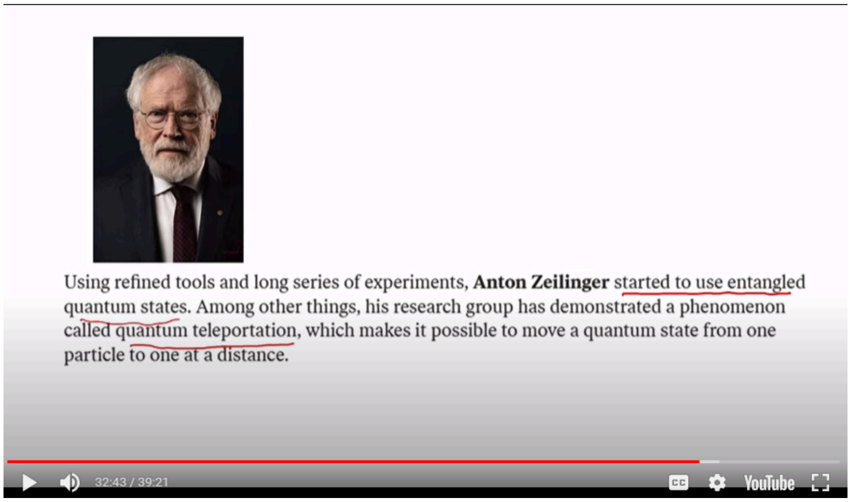

Finally, Anton Zeilinger, he refined tools and did long series of experiments and Zeilinger started to use entangled quantum states. Among other things, his research group has demonstrated a phenomenon called quantum teleportation. We will talk about quantum teleportation later in this course. And quantum teleportation makes it possible to move a quantum state from one particle to another one at a distance.

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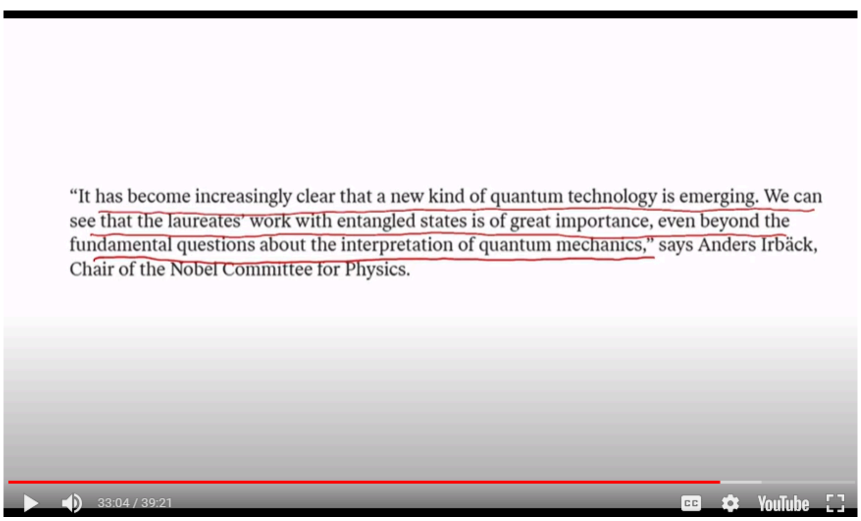

The chair of the Nobel Committee for Physics observed that it has become increasingly clear that a new kind of quantum technology is emerging. We can see that the Loretz work with entangled state is of great importance even beyond the fundamental questions about the interpretation of quantum mechanics.

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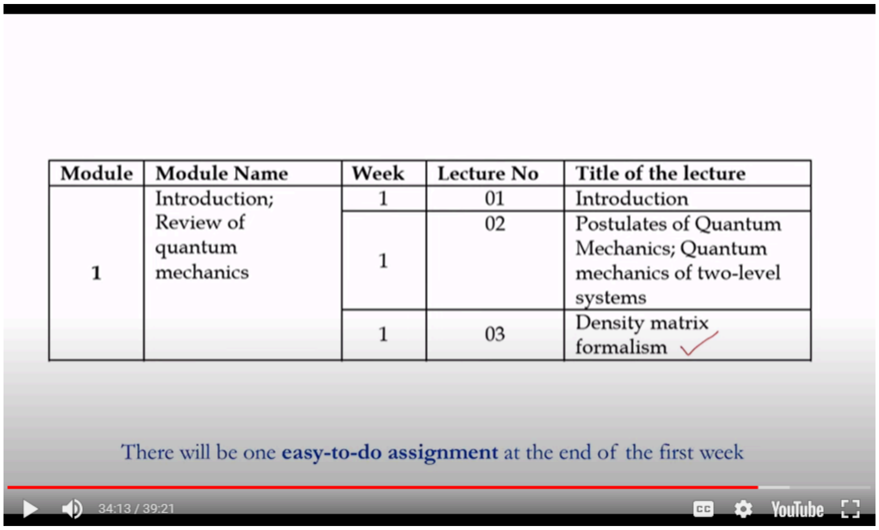

Module	Module Name	Week	Lecture No	Title of the lecture
1	Introduction; Review of quantum mechanics	1	01	Introduction
		1	02	Postulates of Quantum Mechanics; Quantum mechanics of two-level systems
		1	03	Density matrix formalism

I think I have spoken enough as regards the introduction to quantum entanglement and its significance. Now let me talk briefly about the structure of the course and what you can expect and what you should not expect from the course. This NPTEL MOOC course on quantum entanglement fundamentals, measures and applications is of a 10 hour duration course. But I may give you extra materials which might increase the duration by a couple of hours or so. I have divided the course into four modules and in the first week, I will cover the first module which is basically going to be a discussion on some elementary quantum mechanics relevant for quantum entanglement along with some mathematical tools in particular the so-called density matrix formalism I am going to discuss.

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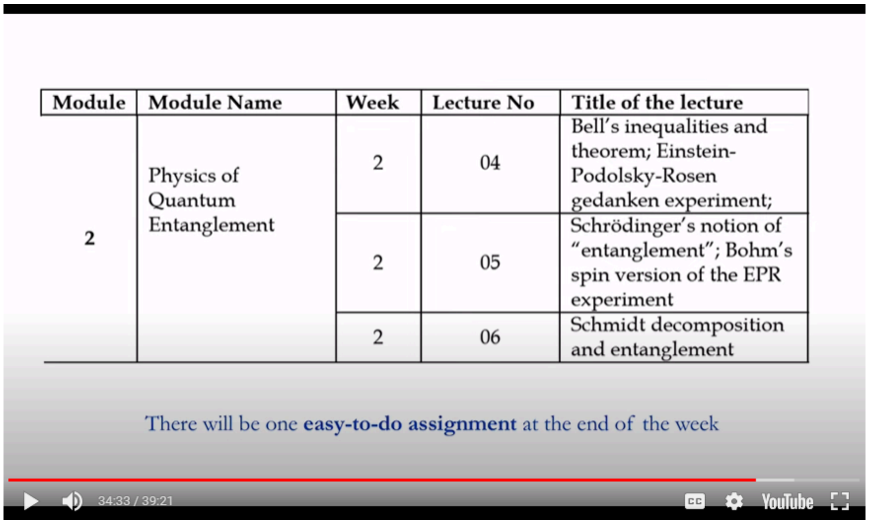

Module	Module Name	Week	Lecture No	Title of the lecture
2	Physics of Quantum Entanglement	2	04	Bell's inequalities and theorem; Einstein-Podolsky-Rosen gedanken experiment;
		2	05	Schrödinger's notion of "entanglement"; Bohm's spin version of the EPR experiment
		2	06	Schmidt decomposition and entanglement

There will be one **easy-to-do assignment** at the end of the week

In the second module, in fact every module at the end of every module there is going to be an easy to do assignment and in the second module I will discuss the physics of quantum entanglement here topics like Bell's inequalities, Bohm's spin version of EPR experiments, Schmidt decomposition and so on are going to be discussed.

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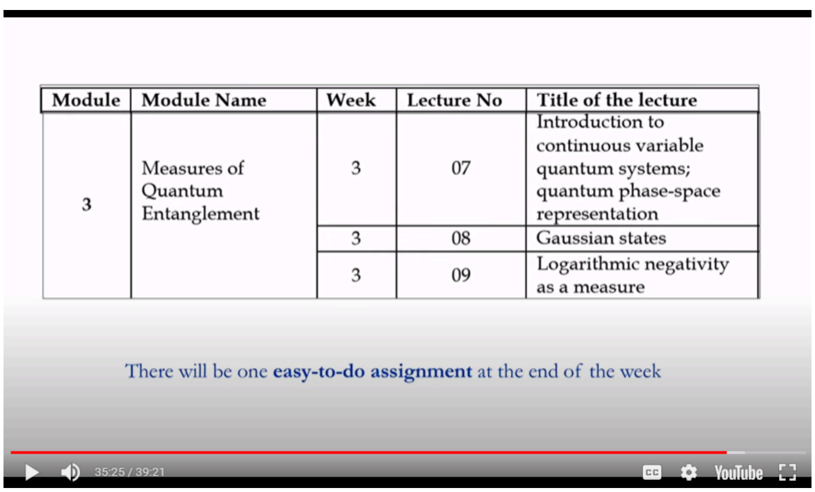

Module	Module Name	Week	Lecture No	Title of the lecture
3	Measures of Quantum Entanglement	3	07	Introduction to continuous variable quantum systems; quantum phase-space representation
		3	08	Gaussian states
		3	09	Logarithmic negativity as a measure

There will be one **easy-to-do assignment** at the end of the week

In the third module that is in the third week we are going to discuss measures of quantum entanglement. Here I am planning to cover both the discrete variable as well as the continuous variable entanglement in fact this module is going to be the longest duration one because this lecture maybe I will, these three lectures that we are mentioning here apart from these three main lectures there are going to be many supplementary lectures would be required because the nature of this particular topic is such and here I am going to discuss about particularly I am going to confine to Gaussian states only and as regards continuous variable is concerned and I am going to discuss about the so-called logarithmic negativity as a measure for quantum entanglement.

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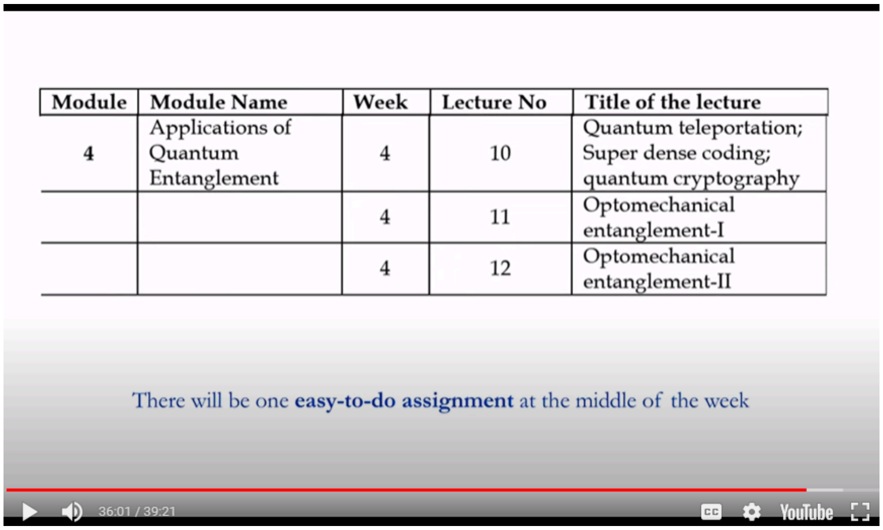

In the final module some applications of quantum entanglement is going to be discussed and particularly quantum teleportation, super dense coding and quantum cryptography. Apart from that as an example for continuous variable entanglement I will show you how the measures like logarithmic negativity can be applied to quantify the entanglement between light and mechanics in the context of an optomechanical system.

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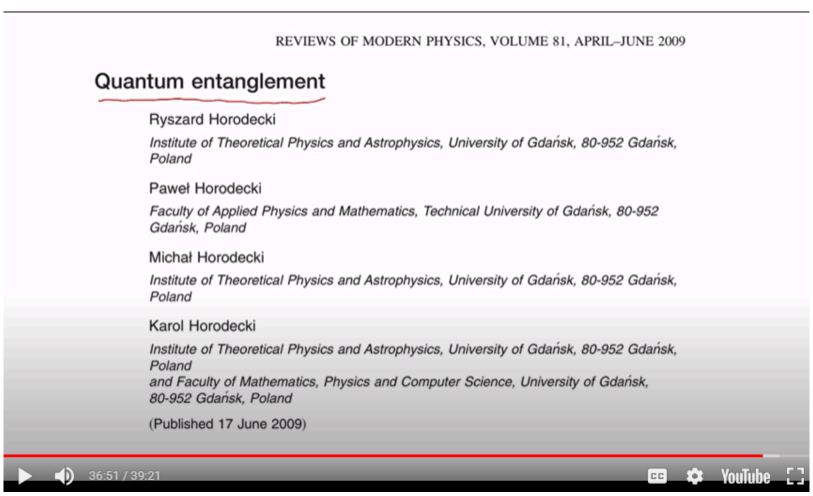

Now as regards books are concerned there are a couple of books here I am giving the example of three books as you can see here these three books has a chapter on each of them contain some chapter on quantum entanglement you can read them but as regard the material of this particular lecture is concerned, course is concerned the main article or the reference is going to be this particular article called quantum entanglement that is published in Review of Modern Physics and it's a very long article but this is going to be the main article on which this course is based and for continuous variable systems this particular topical review is going to be used.

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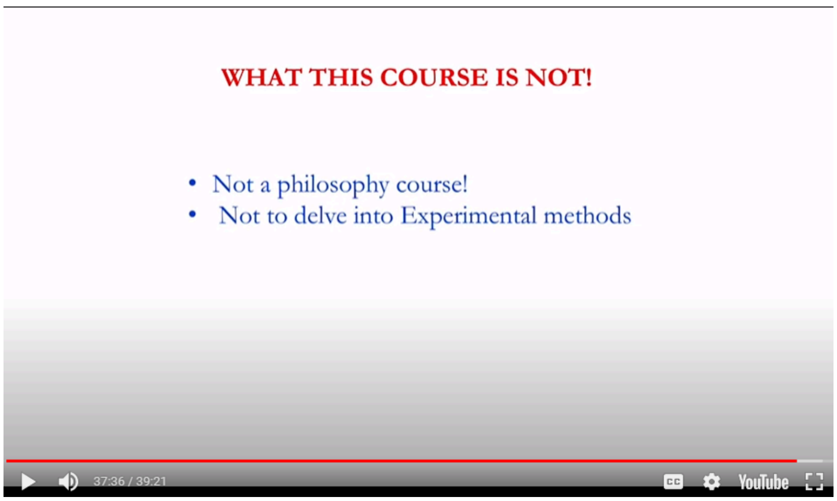

Now what this course is not definitely this is not a philosophy course and I am not going to talk much about the philosophical aspect I think I mentioned that earlier also I am not going to discuss any philosophical implication of quantum entanglement and so on and moreover discussing the experimental methods related to quantum entanglement is beyond the scope of this course so that's also we are not going to discuss but what you can expect is that everything I am going to start from ultra basics so basically I will take a bottom up approach I will break down the concepts and mathematical steps as much as possible and lots of supplementary materials and beyond not mentioning this course structure is also going to be provided assignments will be very easy to do so you do not have to worry too much about examination, basic idea is here is to so that you can learn something and also main objective of this course is to so that you can read and understand current literature on quantum entanglement and finally to appreciate and wonder about the world of quantum entanglement Let me stop for today in this lecture I have given you an introduction to quantum entanglement and also I briefly discuss about its significance also I have given you an overview of the course in the next two lectures I am going to revisit elementary quantum mechanics that is necessary for understanding quantum entanglement as well as I will discuss some mathematical tools so see you in the next class. Thank you so much.