Quantum Technology and Quantum Phenomena in Macroscopic Systems
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Lecture – 7
Dressed States: Introduction to Density Matrix

Hello, welcome to lecture five of this course. In today's lecture, we are going to conclude our discussion about two level atom and also we are going to discuss the so called density matrix formulism which is going to be very very useful later on. So, let us begin.

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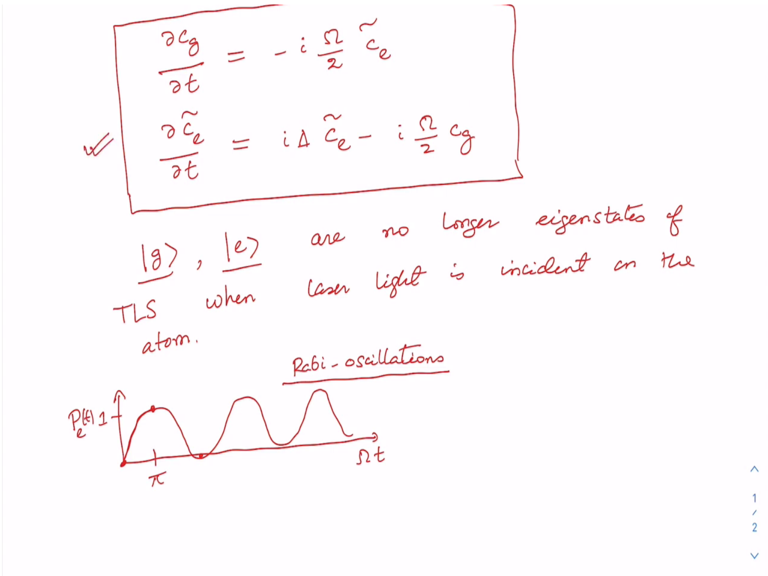

Let us first discuss dress states in the context of level system and this is a very useful concept. In the last class we saw that say, you have we have a two level atom having ground state represented by this ket g and the excited represented by this ket e and transition frequency between this level is omega 0 and a laser pulse having frequency or laser radiation having frequency omega is getting incident on the atom. This laser radiation may be different from this transition frequency of the atom by some amount.

So, that is defined as the detuning and when we have written down the Schrodinger equation for this two level system ,we obtain these equations for the two level system this couple equations for the two level system in the rotating frame of reference as we discussed in the last class. So, these were the equations that we obtained .Okay. So, let me just write it here.

So, this is what we got in our fourth lecture. All right. Now one thing that can be ascertained from our earlier discussion that when the laser radiation is incident on the atom, two level atom this ket g and e, g and e are no longer no longer eigenstates, eigenstates of two level system when laser light is incident on the atom on the atom ,because we have seen from our Rabi flopping discussion that the atom basically go from the ground state to the excited state and excited state to ground state like this.

So, suppose initially it was in the ground state when we have walked out the probability. So, the atom goes from the ground state to the excited state and after say, omega t is equal to pi, the atom would be in the excited state. So, if I just draw the probability versus omega t then this discussion we have already done in great details in the last class. So, Rabi oscillations this is the fact that this ground state and the excited state are no longer eigenstate of two level system when laser light is incident on the atom.

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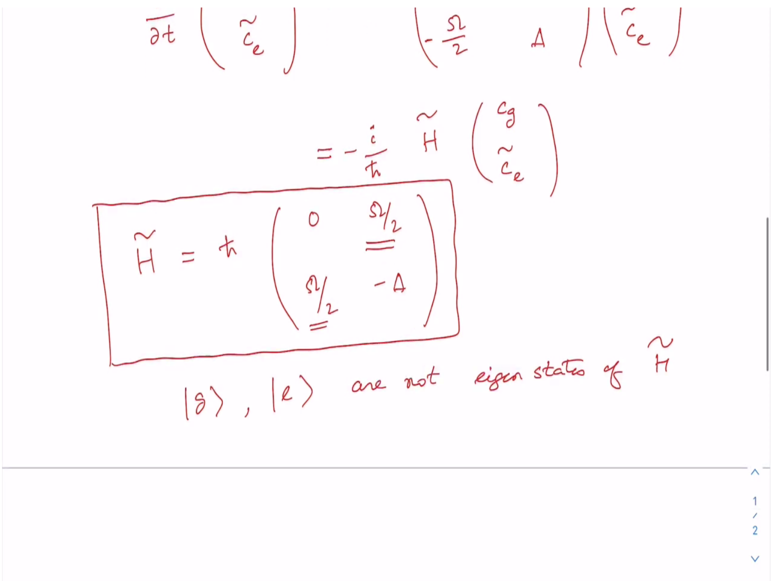

Now this couple set of differential equation I can immediately write it in the matrix form .If I write it in the matrix form ,let me just show you I can express this set of couple differential equation like this is c g c e tilde that would be equal to, i have here okay, I have here, I ,you will have 0 here and then you will have delta ,you will ,you will have a delta here and you will have omega by 2 and omega by 2 like this. Okay.

So, this is this is what I can write, I hope I am doing it correctly okay, there is a minus sign ,okay. So, or in other words I have I can also write it as minus i by h cross say, H tilde c g c e tilde here and h is the effective Hamiltonian and h is equal to h cross 0 here omega by 2 omega by 2 and I have minus delta. So, this is the effective Hamiltonian for this two level system plus the laser light in the rotating frame of reference.

So, this atom is now coupled by the so-called Rabi frequency and in this case here g and e are they are not eigenstates of this Hamiltonian H.

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$$= \underbrace{\begin{pmatrix} \frac{\hbar\Omega}{2} & -\frac{\hbar\Delta}{2} \end{pmatrix}} \quad \begin{pmatrix} 0 & -\frac{\hbar\Delta}{2} \end{pmatrix}$$

Eigenvalues:

$$E = -\frac{\hbar\Delta}{2} \pm \sqrt{\left(\frac{\hbar\Delta}{2}\right)^2 + \left(\frac{\hbar\Omega}{2}\right)^2}$$

$$\boxed{\begin{gathered} E = -\frac{\hbar\Delta}{2} \pm \frac{\hbar\tilde{\Omega}}{2} \\ \tilde{\Omega} = \sqrt{\Delta^2 + \Omega^2} \end{gathered}}$$

But what we can still do, we can still find out the eigenstates and eigenvalues of this Hamiltonian .We can find eigenvalues eigenvalues of H and also we can find eigenstates of H tilde and this Hamiltonian is kind of it is as if it is dressed by this laser light. So, it is a kind of dressed Hamiltonian and in this formalism what we are going to say these eigenstates of the eigenstates of the Hamiltonian H tilde will be known as dress states.

So, this is what we are going to do or work out first to find out the eigenvalues and the eigenstates. To find the eigenvalues ,it is very trivial. Let us first find out the eigenvalues. So, H tilde let me I have again, it is omega by 2 omega by 2 minus delta okay, this I can write in this form. If I write it a little bit differently I can always write this guy as, okay, I can write it as say h cross delta by 2 h cross omega by 2 h cross omega by 2 then I have minus s cross delta by 2.

Then I just have to add here minus s cross delta by 2 0 0 and here it would be minus s cross delta by 2. Now you see by this clever way if I do it I know that the eigenvalue of this matrix is simply minus h cross delta by 2 and on the other hand this already we see from our last to last class I think third class we worked out that for the general case .Suppose you have here epsilon z is there then here you have minus epsilon z is there and epsilon x plus epsilon y and epsilon x minus i epsilon y.

In this general Hamiltonian the eigenvalues we actually worked it out it was plus minus modulus of epsilon where epsilon is you have this is your epsilon x. So, we can just write it like this okay. So, therefore it was simply square root of epsilon x square plus epsilon y square plus epsilon g square. So, using this immediately you see the eigenvalue. So, let me just remove this now ,so we can immediately write down eigenvalues as this e is equal to from here from this part I get minus h cross delta by 2 and from this part plus minus I will have s cross delta by 2 mod square whole square plus h cross omega by 2 whole square.

Or better let me write it as minus h cross delta by 2 plus minus h cross omega tilde by 2 where omega tilde is the generalized Rabi frequency and it is equal to delta square plus omega square the Rabi frequency okay. So, we got the eigenvalues. Now the next thing that we have to do is to find out the corresponding eigenvectors.

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$$b = -\cos\theta$$

$$|-\rangle = \sin\theta\,|g\rangle - \cos\theta\,|e\rangle$$

$$E_{\pm} = -\frac{\hbar\Delta}{2} \pm \frac{\hbar\tilde{\Omega}}{2}\ ; \quad \tilde{\Omega} = \sqrt{\Omega^2+\Delta^2}$$

$$\left.\begin{aligned} |+\rangle &= \cos\theta\,|g\rangle + \sin\theta\,|e\rangle \\ |-\rangle &= \sin\theta\,|g\rangle - \cos\theta\,|e\rangle \end{aligned}\right]\ \text{Dressed st.}$$

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So, let us do values corresponding to say e plus which is minus s cross delta by 2 plus h cross omega tilde by 2 let me say, this is represented by this ket and I can express any ket vector any eigenstate or any state as a superposition of basis states. So, because I know that this ground state and excited state this g and e are the basis states for the two level system. So, therefore I can express it, let me simply write it as say this is say cos theta here and this coefficients we are going to work it out it is cos theta and sine theta.

I have taken it in such a way because I know that because I know that this coefficient mod square if I sum them that has to be equal to 1 and cos square theta plus sine square theta is equal to 1. So, this is what I have and I am going to work it out. On the other hand for the other case for the other eigenvalue that is minus h cross delta by 2 minus h cross omega tilde by 2. So, let us say it is represented by this ket here.

And let us say that is a g plus say b e is where again I have to find out this coefficient a and b. Now the eigenstates ket plus and ket minus are orthogonal .That means the scalar product of bra plus and ket minus has to be equal to 0. This means that we can write an equation bra plus will give us cos theta bra g plus sine theta bra e and ket minus will we have a linear combination of a ket g plus b ket e is equal to 0 ,because ket g and ket e are basis states.

So, we have this equation a cos theta plus b sine theta is equal to 0. Of course we have also this constraint that a square plus b square has to be equal to one .Now from this equation we can take a is equal to say, sine theta .Then to satisfy that equation. we must have b is equal to minus cos theta. Therefore we have the state minus this ket is equal to sine theta ket g minus cos theta ket e and we have also.

So, already what we are having all the results we have now we have this eigenvalues e plus minus is equal to minus h cross delta by 2 plus minus s cross omega tilde by 2 where omega tilde is the generalized Rabi frequency and that is equal to omega squared thats the Rabi frequency plus the detuning parameter square delta square. And the states this eigenstate ket plus and ket minus we have them as ket plus is equal to cos theta ket g plus sine theta ket e and ket minus is equal to sine theta ket g minus cos theta ket e. And in fact this ket plus and ket minus these are called the dress states.

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θ is known as the Stückelberg angle.

$$E_\pm = -\frac{\hbar\Delta}{2} \pm \frac{\hbar\tilde{\Omega}}{2}$$

$\Rightarrow$ coupling to the field causes an avoided crossing in the energy level structure of the atom.

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Now what about the angle theta . Let us find it out. What about angle theta. Let us find it out in terms of realistic parameters. You see we have this eigenvalue equation with us that is H tilde ket plus if we will get eigenvalue would be e plus and then I get ket plus here .If we take if we take ket g as the column vector 0 1 and ket e as the column vector 1 0 then, we can write this equation in the matrix form and let me write it the Hamiltonian in the matrix form would be h cross 0 omega by 2 omega by 2 minus delta and this ket plus we will have it as sine theta cos theta and we have here e plus is the eigenvalue and this ket plus is sine theta cos theta.

We can get the equation one equation is sufficient to solve to find the angle .We have this equation h cross omega by 2 cos theta is equal to e plus sine theta and from here it is trivial to see that we will get tan theta is equal to h cross omega by 2 divided by e plus if I write the value of e plus here that is minus h cross delta by 2 plus h cross omega by 2 here I have h cross actually this of h cross omega tilde by 2 and here I have h cross omega by 2 then I can have omega Rabi frequency divided by generalized Rabi frequency omega tilde minus the detuning parameter delta.

In fact using this trigonometric formula tan 2 theta is equal to 2 tan theta divided by 1 minus tan square theta we can show that tan 2 theta is equal to ,it is very trivial and please try it, it will be equal to omega divided by delta. So, omega is the Rabi frequency ,delta is the detuning parameter and which are our realistic parameters and with this condition that here when I am writing it I am taking theta to be lying between the angle 0 and pi by 2 .This angle theta is known as the Stuckelberg angle.

Now we can see from this eigenvalues that e plus minus is equal to minus h cross delta by 2 plus minus h cross omega tilde by 2 that coupling to the field, let me write here this actually implies that coupling to the field causes an avoided crossing avoided crossing in the energy level structure of the atom.

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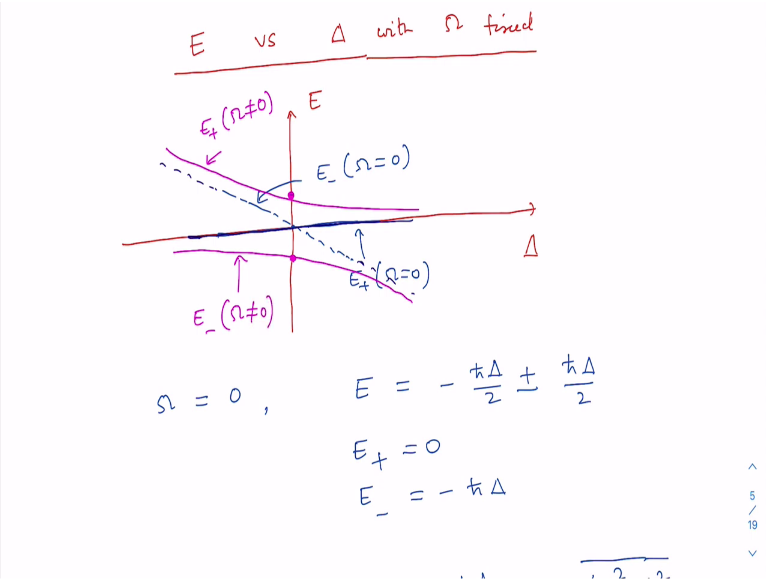

If you are not getting it, let me show it little bit more clearly. To see that let us plot e as a function of the detuning parameter delta eigenvalue e as a function of the detuning parameter delta with say the Rabi frequency fixed. We have done similar kind of stuff when we discussed two level atom that is a general characteristics of any two-state system. So, what we have here let us say this is e in the y-axis and the tuning parameter delta is in the x-axis.

First of all say we have here ,let me consider first the case when there is no coupling that means the Rabi frequency is 0 omega is equal to 0 then e energy eigenvalue e would be equal to minus h cross delta by 2 plus minus h cross delta by 2. So, this means I have e plus is equal to 0 and e minus is equal to minus h cross delta. So, for e plus we will get say this plot that is e plus is equal to 0. So, this is what I will have e plus for when Rabi frequency is 0 and e minus minus h cross delta. So, it has a negative slope. So, I will have a something like this. I will have here.

So, this is what I will have this is for e minus where omega is equal to 0 up there is no coupling. Now when the coupling is there when omega is not equal to 0 then we have e plus minus is equal to minus s cross delta by 2 plus minus h cross omega square plus delta square.

Now you see that at delta is equal to 0 here I will have e plus is equal to as you can see we will have h cross delta.

So, this would be the point say, this would be the point here and for e minus I will have my minus h cross omega. So, these are the points and because of that I will get a plot of this type, actually let me do it extend it. So, I will have a plot of this type and this is e plus when omega is not equal to 0 and this one is e minus where omega is not equal to 0 that means when the coupling is there.

So, you see there is a clear indication of avoided crossing. So, far in our discussion of two level system we discussed a closed two-level system that is a two-level system not interacting with the surrounding and for discussion of such system the state vector formalism is sufficient. We can discuss a two-level system using the so called density matrix formalism as well.

The advantage of this formalism that is the density for matrix formalism is that we can describe a two-level system interacting with the surrounding as well that means a an open quantum system can be described using the so called density matrix formulism. So, let me discuss density matrix formulism in some what details in the context of a two level system.

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In state vector form

- $|\psi\rangle = c_g|g\rangle + c_e|e\rangle$ $\left[i\hbar \frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle \right.$

$$\frac{\partial c_g}{\partial t} = -i\frac{\Omega}{2}\tilde{c}_e \rightarrow (i)$$

$$\frac{\partial \tilde{c}_e}{\partial t} = i\Delta\tilde{c}_e - i\frac{\Omega}{2}c_g \rightarrow (ii)$$

- The state of TLS can also be represented by

$$\boxed{\hat{\rho} = |\psi\rangle\langle\psi|}$$

- density operator
- density matrix

We will discuss density matrix formulism of two level system or two level atom. So, recall that in the state vector formalism in state vector formalism the state of the two level system is described by this state vector psi who is we wrote it as a superposition of the basis states ground state as well as the excited state. And when we actually did that in the rotating frame of reference, this coefficients depended on time by this equations and c e tilde delta t we found it as delta c e tilde minus i omega by 2 c z.

So, these equations are basically we got it from the so called Schrodinger equations. So, in the state vector formalism the wave function or the state vector as well as the Schrodinger equation is suffice or sufficient to describe the two-level system. And generally this we always can know about the state vector of a quantum system if the if the quantum system is pure.

On the other hand the state of the TLS, two level system can also represented by a quantity say rho is equal to this ket and this notation. This is if you look at it this is basically an operator ,by this operator and this operator is called density operator it is called density operator or it is called also known as density matrix.

So, either you represent the two level system by in this formulism or this and the good thing about as I said is that this particular way of representing the state of the system has many advantages .One advantages we will see later on that by this formalism we can discuss a two level system interacting with the surrounding very easily.

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$$\rho = |c_g|^2 \, |g\rangle\langle g| + c_g \tilde{c}_e^{*} \, |g\rangle\langle e| + \tilde{c}_e c_g^{*} \, |e\rangle\langle g| + |\tilde{c}_e|^2 \, |e\rangle\langle e|$$

$$\boxed{\rho \equiv \begin{pmatrix} c_g c_g^{*} & c_g \tilde{c}_e^{*} \\ \tilde{c}_e c_g^{*} & \tilde{c}_e \tilde{c}_e^{*} \end{pmatrix}}$$

Now if I expand it in terms of these states as we have written it as a superposition of ground state and the excited state in the rotating frame, if I just write it like this c g and c it is a complex quantity. So, if I take the bra part there you will get c g star c e plus c e tilde star e.

So, if you open it up then let me just expand it. So, you will get c e mod square this ket g bra g. So, this let me just write all the combinations here, you will get c g c e tilde star you will have g e then you will have c e tilde c g star e g.

And finally you will have c e tilde mod square e e. So, this is what you have and this can be written in the in a matrix form very easily in the basis state this ground state and ket c and ket e if you take it as your this is then you will be able to write it as c g c g star c g c e tilde and you just look at this expressions here and from here again here it would be c e tilde c g star and finally you have c e tilde c e tilde star.

These are the matrix element if now this is the density matrix. It is an operator we are now writing it in the matrix form. So, in the matrix form these elements are like this.

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- $$\frac{\partial \rho_{ge}}{\partial t} = -i\Delta\rho_{ge} - i\frac{\Omega}{2}\left(\rho_{ee}-\rho_{gg}\right) \rightarrow (1)$$
- $$\frac{\partial \rho_{eg}}{\partial t} = i\Delta\rho_{eg} + i\frac{\Omega}{2}\left(\rho_{ee}-\rho_{gg}\right) \rightarrow (2)$$
- $$\frac{\partial \rho_{gg}}{\partial t} = i\frac{\Omega}{2}\left(\rho_{ge}-\rho_{eg}\right) \rightarrow (3)$$
- $$\frac{\partial \rho_{ee}}{\partial t} = -i\frac{\Omega}{2}\left(\rho_{ge}-\rho_{eg}\right) \rightarrow (4)$$

$$\frac{\partial(\rho_{gg}+\rho_{ee})}{\partial t} = 0$$

$$\Rightarrow \boxed{\rho_{gg}+\rho_{ee} = \text{const.} = 1}$$

So, if I defined my quantities say matrix elements rho gg. So, first of all say rho gg this first element here rho easy this guy is c g c g star and then this one is rho g e is equal to c g c e tilde star and accordingly I will have rho e g that would be c e tilde c g star and I will have rho e e that would be c e and c e star. So, these are the density elements of the density matrix and it has some physical meaning first of all if you see the diagonal elements here these, let me just pick up here.

If you look at this diagonal elements of the matrix this represents the probabilities ,right. So, rho gg which is c mod square this gives the probability of finding the tool system in the ground state rho ee represents the probability of getting the system in the excited state and so therefore diagonal elements physically speaking, it has it represents the population in the either in the ground state or in the excited state.

And this particular terms ,off diagonal terms are base represents the coupling between the ground state and the excited state and they are actually called coherent terms coherences. These are termed as coherences. So, these are called populations they represents populations .On the other hand this rho ge and rho eg they represents the coherence. Because basically what it says that if rho ge and rho eg are non-0 then you can create a superposition state between the ground state and the excited state.

So, more about all these things later on.Now another thing you can get the time evolution of this density matrix elements as well and one can very easily get it .Just let me show one example. Say if I want to know how this density matrix element rho g evolves in time. So, you just have to work out this you have to just expand it and open it up and if you okay you will get c g delta t c e star here and you will have c g c e tilde star delta t and then if you use these equations here I think yeah here, I have written use these equations equation 1 and 2.

If you use it here I encourage all of you to do it and if you do it you will get this equation for this density matrix element delta rho g e delta t would turn out to be i delta rho g e minus i omega by 2 rho ee minus rho gg. And similarly you will get delta actually, the other equation is very simple because what turns out that you can immediately you see because rho g g is equal to c g is c e tilde star and rho e g is equal to c e tilde c g star. So, this clearly implies that rho g e is equal to rho e g star okay complex conjugate.

So, therefore this equation is immediately you just have to take the complex conjugate of this equation then you will be able to get rho e g. So, that will be i delta rho e g plus i omega by 2 you will get rho e e minus rho gg all right ,and similarly you will get equations for delta rho gg delta t that would be equal to i omega by 2 rho g e minus rho e g. Please kindly verify this equations and you will get rho ee delta t that would be equal to minus i by omega 2 rho g e minus rho e g. In fact if you let me say this is my equation say 1, 2, 3 and 4.

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$$\boxed{\begin{aligned} \frac{\partial u}{\partial t} &= \Delta v \\ \frac{\partial v}{\partial t} &= -\Omega w - \Delta u \\ \frac{\partial w}{\partial t} &= \Omega v \end{aligned}}$$

optical Bloch equation for a closed TLS

If you look at equation from three and four you can immediately see that if I just add three and four you will get rho gg plus rho ee delta t that would be simply equal to 0 and this means that the total probability or the population this implies that rho gg plus rho ee is constant and which is for a closed two level system is obviously equal to one and here these equations that I have written here are for a closed two level system and these equations are also termed as optical bloch equation.

Because I am considering the introduction of a two level system with a optical radiation generally these are called bloch equation. And we can incorporate the its interaction of the two level system with surrounding as well more I think I will deal those kind of things little bit later. Now these four equations can be written in a very compact form if I introduce. So, let us introduce three variables one variable is say u is equal to rho g e plus rho e g.

In fact if you do a close look this will this should remind you about the pauli matrix sigma x. If you look at the you know the matrix element here even in the density matrix case if you see sigma x is equal to 0 1 1 0 and now if you look at it rho g e and rho e g that is basically the off diagonal elements in the you see off diagonal elements. So, here it is you have rho g z rho g e rho e g rho e e.

So, hopefully you can see the connection that you have this off diagonal elements there. So, this is something like your Pauli matrices .It should remind you about that. And another variable say v i can say that is defined as i rho e g minus rho g e and it should remind you about the pauli matrix is sigma y that was 0 minus i i 0 they are not equivalent but just for it is to remind you they are linked actually and the other one last one w we can define as rho e g minus rho g g and that is the so-called population difference or population inversion.

And sigma g it should remind you about that it is 1 0 0 -1 this is this is population inversion. So, using this is population inversion using this, these are called bloch vectors again and obviously they are related to this Pauli matrices. So, u v w are also known as bloch vector u v w they are called bloch vector the components of the block vector. Three components of the bloch vector and one can immediately using this density matrix equation one can write the equations for del u del t equation for del v del t and equations for del w del t.

It is very simple you can just it is some straightforward algebra please. So, that you are going to get it as delta v here and you will get minus capital omega w minus delta u and you will get it as omega v, alright. So, these are called optical bloch equation also this is called optical bloch equation for in for a closed two-level system. So, this nothing extra actually we are learning here because state vector formulism also told us the same thing but this is advantage as I said.

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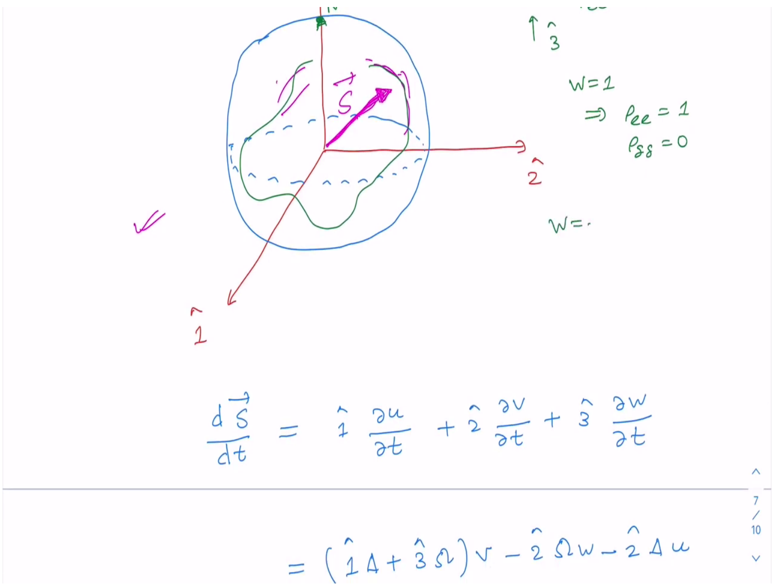

And now I can define the vector this is called the bloch vector and that has a component along this one direction one cap and then say I have this two and three and I can represent it in a diagram soon in the so-called bloch sphere. So, that is what I am going to do now. These vectors it is one cap two cap and three cap these are unit vectors and they are mutually orthogonal to each other in a ficticious three dimensional space.

Let us say vector 1 is let us say vector 1 is directed along this direction 2 is along this and 3 is along this direction ,then I can draw a sphere three dimensional fictitious sphere say like this. So, this should remind you about what we discussed in the generic treatment of two level systems and this vector s. If I just okay, let me show one thing let me take a time derivative of this vector ds dt then if I take it using this bloch equation.

So, I have one okay let me just show you del u del t plus 2 cap del v del t plus 3 cap del w del t and I encourage you to just put this equations from here del u del t all these values you put there then you will be able to show that this ds dt will be equal to 1 cap delta plus 3 cap omega v minus 2 cap omega w minus 2 cap delta u. So, this is what you will get. Now let me define a vector let me define a vector say q as this, say 1 ket omega minus 3 cap delta.

And if I defined it actually I can show that Q the cross product of these two vectors Q cross S that if you work it out then what you are going to get is this .You will be able to show that this would be exactly equal to ds dt, alright. So, again immediately you see that ds dt okay let me write here like this ds dt is equal to Q cross S and similar kind of equations recall from our discussion of the generic two level system where we got this equation for the Pauli matrices right or this was S is there also we said that that is the bloch vector and this epsilon vector was there.

If you recall twice e and then you have this equation in the generic case that we considered earlier. So, here this quantity is the role of this quantity is played by this vector Q and immediately you can see that Q cross S is perpendicular okay perpendicular to S. So, the effect of Q is only to rotate S. Let me write here the effect of Q the effect of this vector Q is to rotate is to rotate s about the about the direction of direction of Q alright.

So, it actually what does it mean that it can only rotate it cannot lengthen or shorten the vector s and that is very clear because if you just take the time derivative of s square okay, which is in fact I can write it as 2 s dot ds dt I hope you get it because s square is equal to s dot s. So, you will get this equation now as I said that ds dt is equal to Q cross S so immediately you see because s is perpendicular to this.

So, therefore the dot product is going to be equal to 0. So, therefore this quantity x square is constant and I want to remind you that we are considering an isolated two-level system as yet and you can show that s square if you take it because s is equal to 1 cap u plus 2 ket v plus 3 cap w. So, s square is equal to u square plus v square plus w square. Now if you write u, v and w in terms of this density matrix elements.

So, say u rho e g plus rho g e whole square and then accordingly v if you put then you will get it because i is there so minus will be there. So, rho e g minus rho g e whole square plus w is rho e minus rho g g okay and if you open it up and write okay, you will be able to write it as rho e g rho g e plus rho e e square minus twice rho e e rho g g plus rho g g square and what I can show you that if you just write say rho e g is equal to c e tilde c g star and so on.

Then you will be able to show that this is equal to c e tilde mod square plus c g mod square and which is obviously equal to 1. So, S square turns out to be equal to 1 means that this implies that the unit length unit length of bloch vector okay unit length of bloch vector S is thus because of this does seem to be equivalent it is equivalent to the conservation to the conservation of probability in a two level system or two level atom.

So, whenever S square is equal to 1 whenever s square is equal to 1 the tip of the bloch vector uh. So, in this diagram I can if I show just say tip of the bloch vector suppose, this is the surface tip of the bloch vector always assess the surface of the sphere and the bloch vector always lies on the surface of the sphere okay and only the angle of this bloch vector changes as the two level system evolves in time.

So, that is equivalent to say that because the population is remaining conserved and we are considering an isolated system remember. So, only this angle that is made by the bloch vector is going to change with time and in effect the evolution of two-step density matrix is equivalent to changes in the orientation of the bloch vector s as this is sketch in this particular diagram. So, say this bloch vector if I just okay let me just use this so bloch vector can actually move around here in the surface of the sphere.

And this is a very useful representation and many times even in quantum computation and quantum information processing a lot of physics and ideas can be just by could be explained in terms of this bloch sphere representation. A few comments about the population inversion which is defined as w is equal to rho e minus rho g g and as you see that this population inversion is associated with this axis 3 and whenever w is equal to 1 which implies that rho e is equal to 1 and rho g g is equal to 0.

So, therefore what it mean is that the atomic population is entirely in the upper level atomic population is entirely in the upper level and bloch sphere corresponds to a fully inverted atom and the bloch vector is directed along the north pole here and on the other hand by the same reasoning if w is equal to minus 1 the bloch vector would point towards the south pole.

It would be along the South Pole and the atom is into the lower energy level. Let me stop for today in today's class we learned about the drastic picture of two level system and I also very briefly the concept of density matrix. In the next class we are going to discuss density matrix in somewhat details because density matrix is going to be used as a important tool in this course, thank you.