Quantum Technology and Quantum Phenomena in Macroscopic Systems
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Lecture-43
Quantum Optomechanics Normal-Mode Splitting

Hello, welcome to lecture 11 of module 3, this is lecture number 32 of the course. In this lecture we are going to discuss a phenomenon called normal mode splitting. This phenomenon is extremely significant because this is the definite signature of coupling between the optical oscillator and the mechanical oscillator. Then we will also discuss the physics or the principle behind how an optomechanical system can act like a transducer. That means how it can transfer information from one optical mode to the mechanical mode or vice versa, so let us begin.

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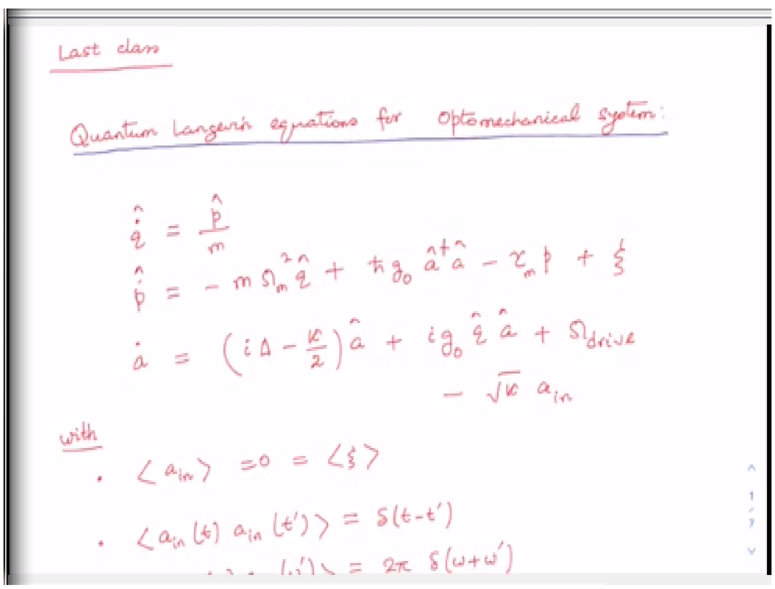

In the last class we started our discussion by writing down the quantum Langevin equations for optomechanical system.

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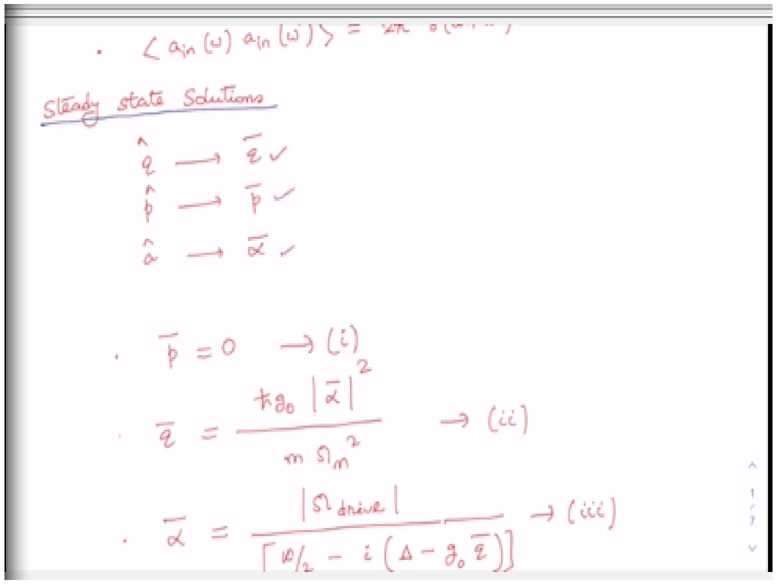

So, firstly we worked out the steady state solution for this quantum Langevin equation for various variables. The position variable and the momentum variable for the mechanical system and the optical mode of the cavity and steady state solution for the position variable is represented by q bar momentum by p bar and optical mode by alpha bar.

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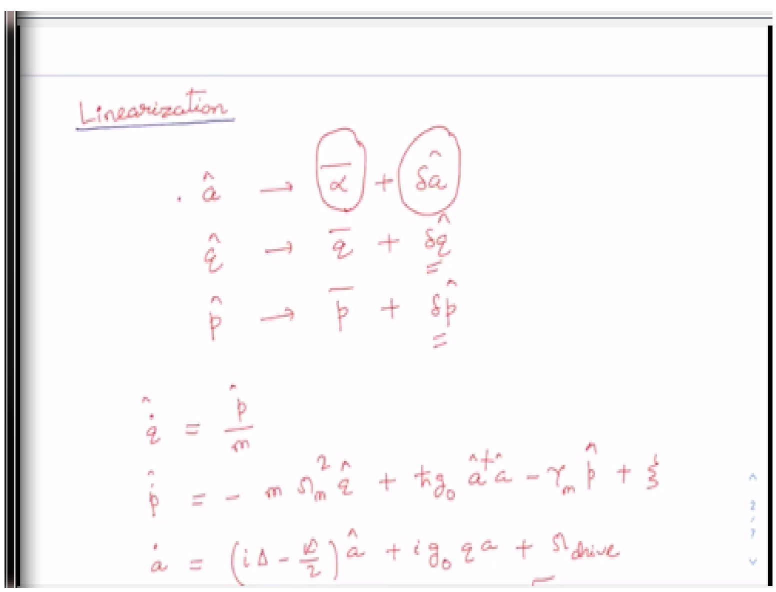

Then we went on to linearize these equations around the steady state value. Here for example for the optical mode we are writing it as alpha bar + delta a a bar and delta a cap. Here alpha bar is actually the classical you can consider it to be the classical part and delta a is the deviation from this classical one, that is the quantum fluctuation. Similarly delta q and delta p are the corresponding quantum fluctuation for the position and the momentum variable of the mechanical oscillator.

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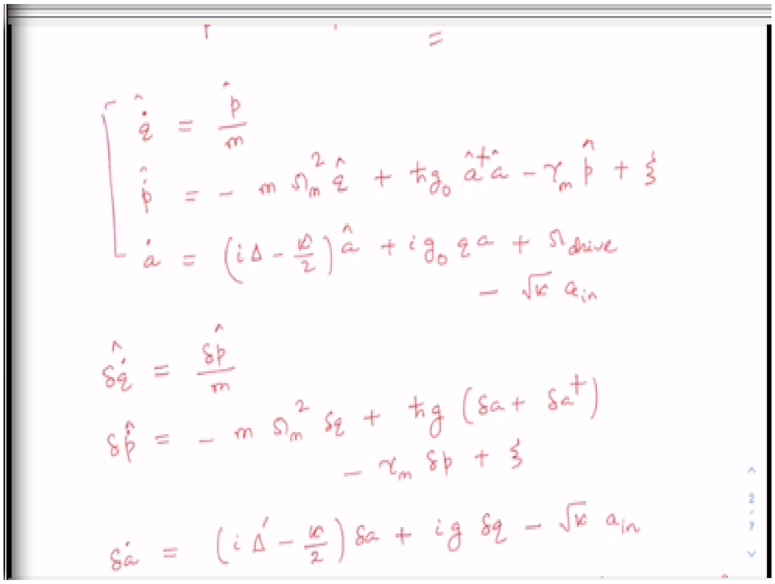

Putting them in the quantum Langevin equations.

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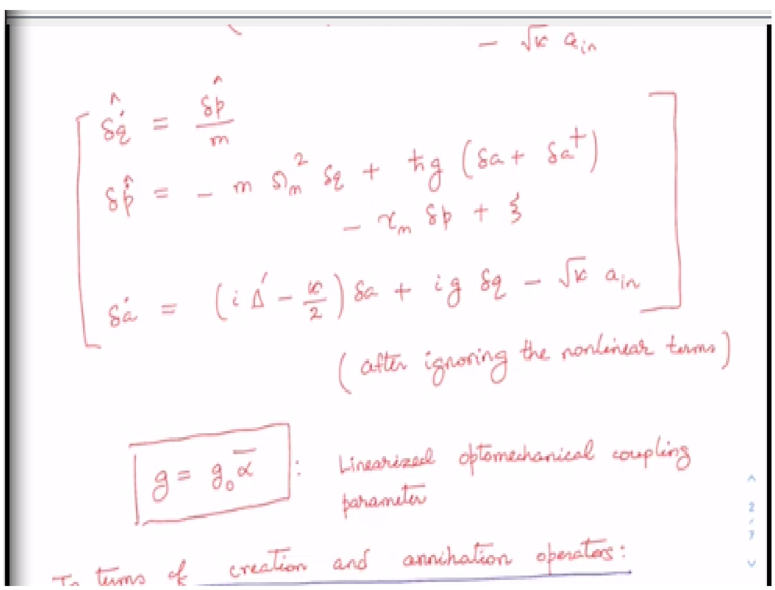

We get the time evolution equations for the quantum fluctuation part and ignoring the nonlinear parts thereby we write down the linearized version of the quantum Langevin equation for the quantum fluctuations. And here this parameter g is known as the linearized optomechanical coupling parameter.

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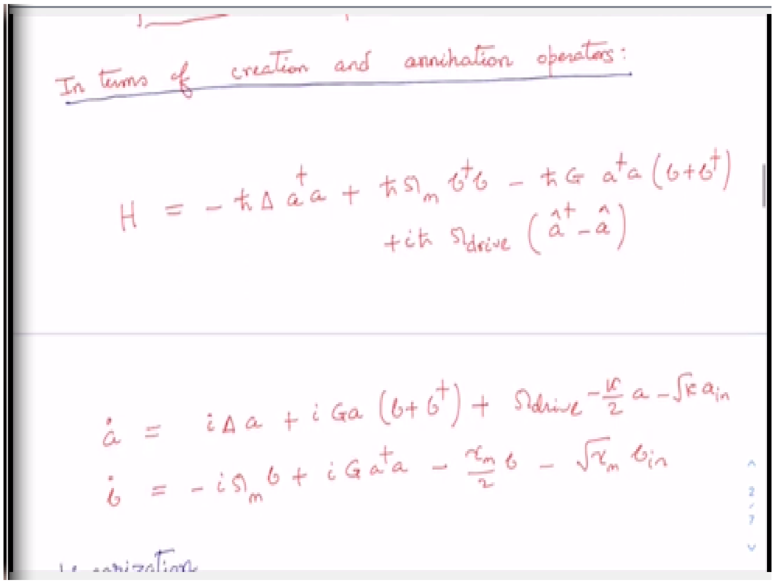

In terms of creation and annihilation operator also we can do the linearization.

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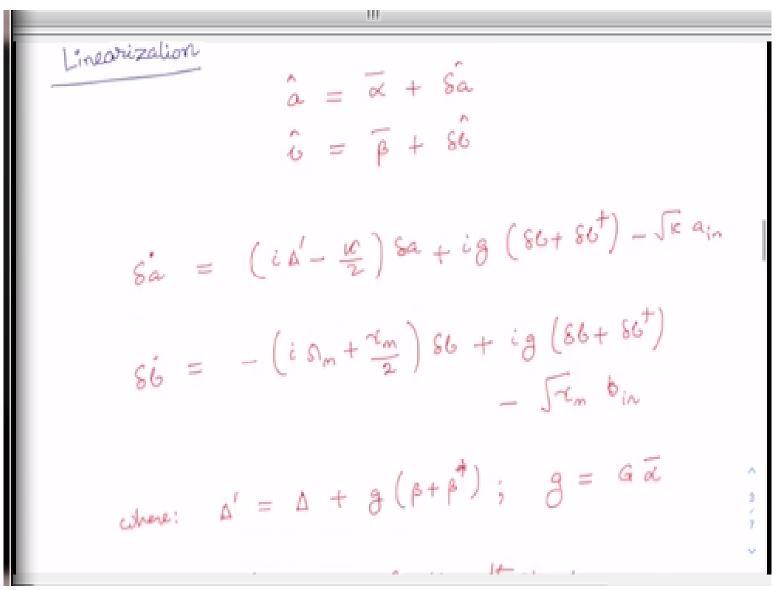

And that is exactly following the similar procedure.

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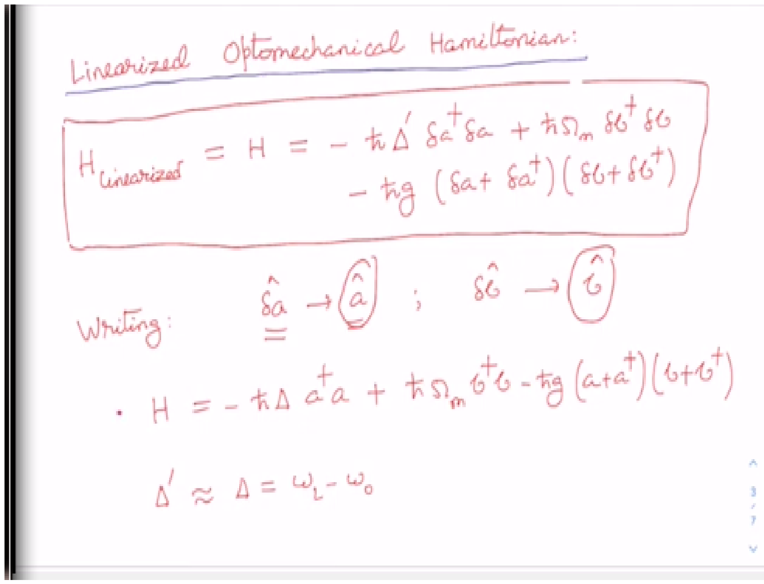

We exclusively write the time evaluation equations for the annihilation operator for the optical mode and the annihilation operator for the mechanical mode. And this will result in a linearized optomechanical Hamiltonian. Here when we have written down the linearized optomechanical Hamiltonian, we have ignored the quantum noise and the damping. In literature or in many places as it is customary to write delta a again as a, this is actually now the completely quantum and b cap. So, the linearized Hamiltonian can be written in this particular form.

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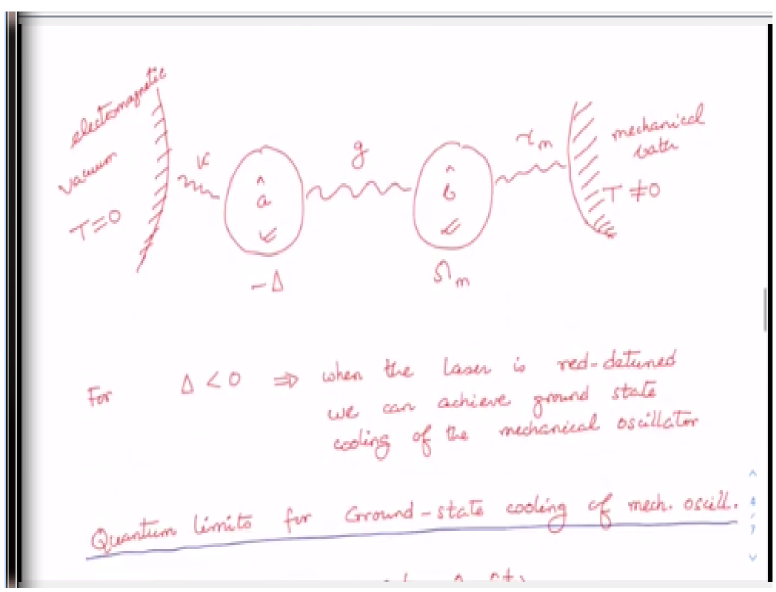

And in our treatment what we have taken is that? We have in the next rest of the treatment we have taken this modified detuning parameter to be simply as the detuning parameter delta because the deviation from this delta would be very small. And then this is also represented in a schematic diagram which represents that we have 2 oscillators, one is due to the optical oscillator and one is due to the mechanical oscillator.

Optical oscillator is oscillating at frequency minus delta as you can see from here and the mechanical oscillator is oscillating at frequency omega m, they are coupled by this parameter g. And optical cavity has this detuning kappa and the mechanical oscillator also has a damping that is gamma m.

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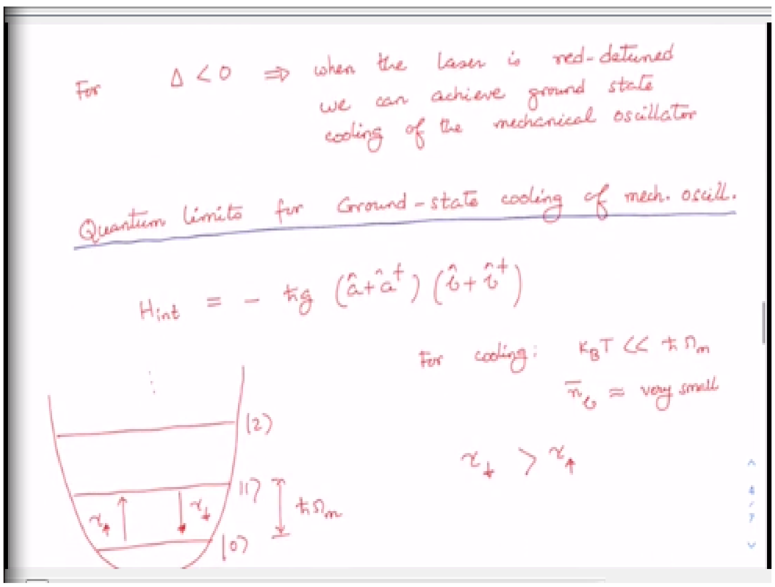

And we find that when delta is less than 0 which means the laser is red detuned we can achieve the ground state cooling of the mechanical oscillator. Then we went on to study the quantum limits for ground state cooling of the mechanical oscillator.

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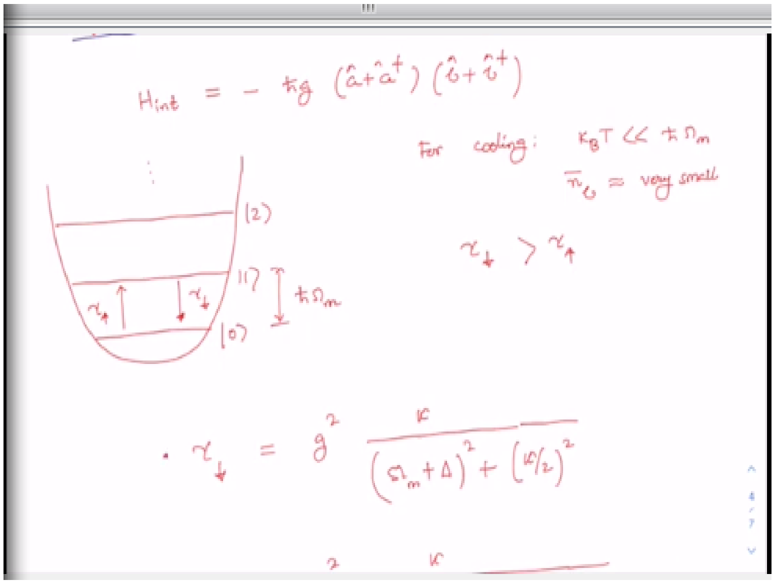

And we confine our discussion to the 1 phonon's state and the 0 phonon state and we worked out the damping for the downward transition when the mechanical oscillator goes from the 1 phonon state to the ground state 0 phonon state.

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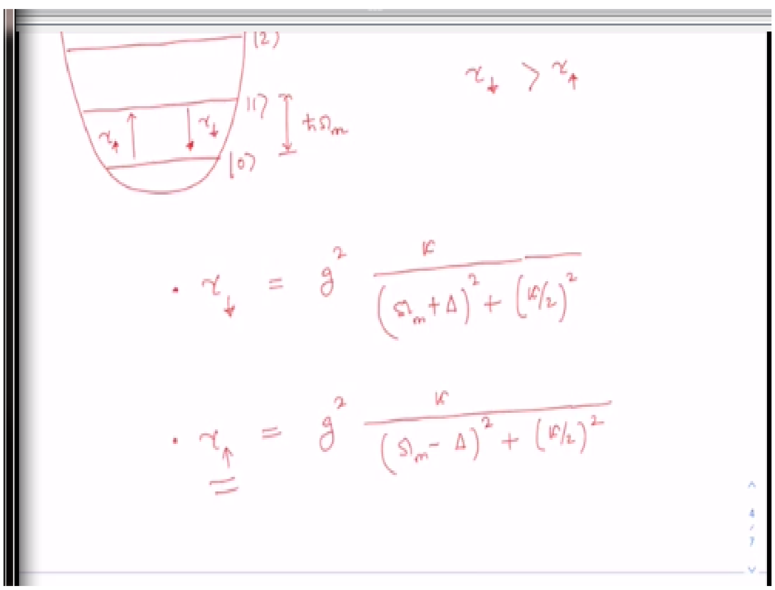

Then the damping we have calculated using the Fermi golden principle. And similarly when the system mechanical oscillator goes from the downward state that is the 0 phonon state to the 1 phonon state this damping rate is denoted by gamma up, we have calculated it in the similar way using the Fermi golden rule.

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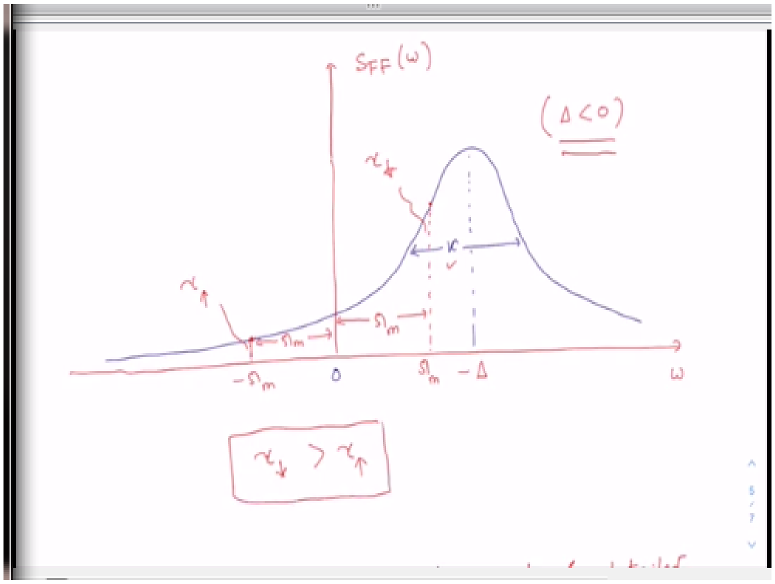

And all these things is plotted for delta less than 0, that is we are in the cooling regime when the laser is red-detuned and it is evident from this plot as well. That in this case the rate of downward transition is pretty high than that of the rate of upward transition, so this is going to lead us to the cooling.

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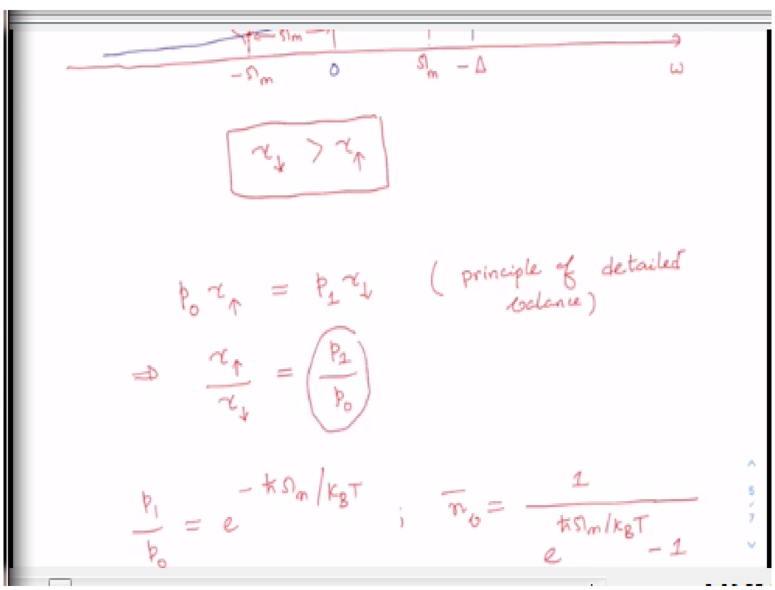

And to then we invoke the principle of detailed balance. Basically that says that in the steady state the rate of downward flow must be balanced by the rate of upward flow. From this we get a ratio between the upward transition and the downward transition in terms of the corresponding occupation probabilities.

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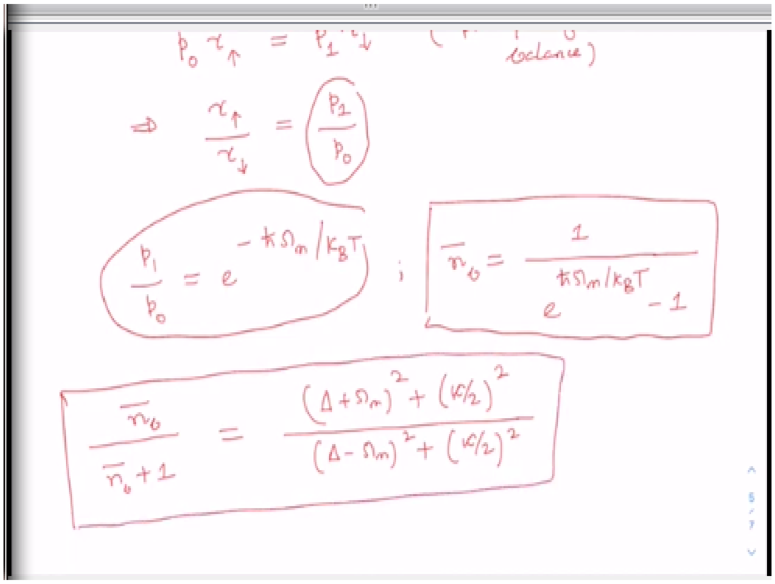

Which are again related to each other by the so-called Boltzmann distribution? So, invoking all these things and knowing that the average phonon number is given by this expression. We can work out a expression for the average phonon number with this relation. And next what we did was to optimize it because we wanted to minimize the average phonon number. Because as we are interested in cooling of the mechanical oscillator.

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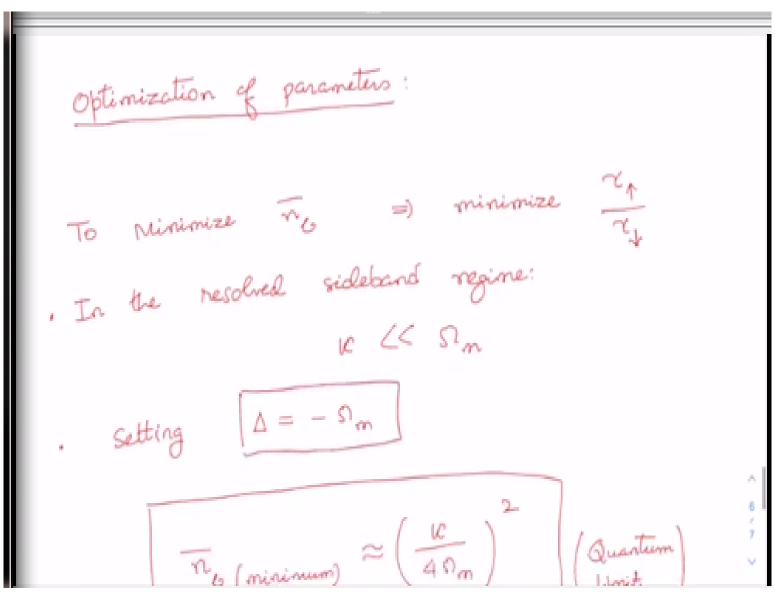

And that can be done if we work in the resolved sideband regime and also if we set the detuning parameter at the negative optomechanical frequency. Then it turns out that the minimum number of phonon one can achieve would be given by this expression kappa by 4 omega m whole square. And this **is** you can consider it as a quantum limit and what it says is that to get into the ground state if we want to make the average number of phonon to be nearly 0, so that we can attain the ground state of the mechanical oscillator.

The cavity decay rate has to be very, very small and which in other words means that we need to have a very high quality optical cavity. Please note that in our treatment we have not considered the effect of coupling of the mechanical oscillator to the intrinsic mechanical damping and external optical drive.

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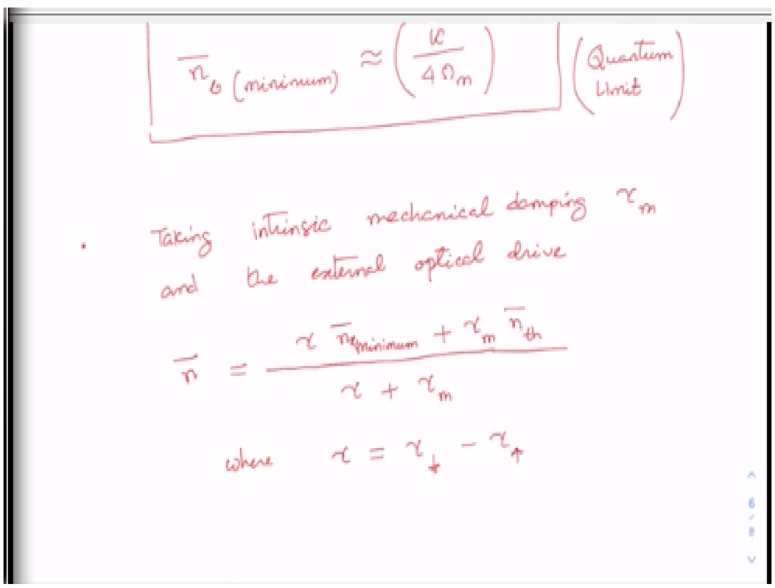

If these are taken into account that means taking intrinsic mechanical damping who is we denote by the red gamma m. And the external optical drive, the expression for the average number of phonon would get modified and it would be given by this expression that would be n bar = gamma, I will define what this gamma is. And n minimum which already we derived say this one let me put m b also here.

And this intrinsic mechanical damping gamma m and the thermal average number of thermal phonons divided by this gamma and the mechanical damping rate. Here this gamma is equal to the difference between the downward transition rate and the upward transition rate.

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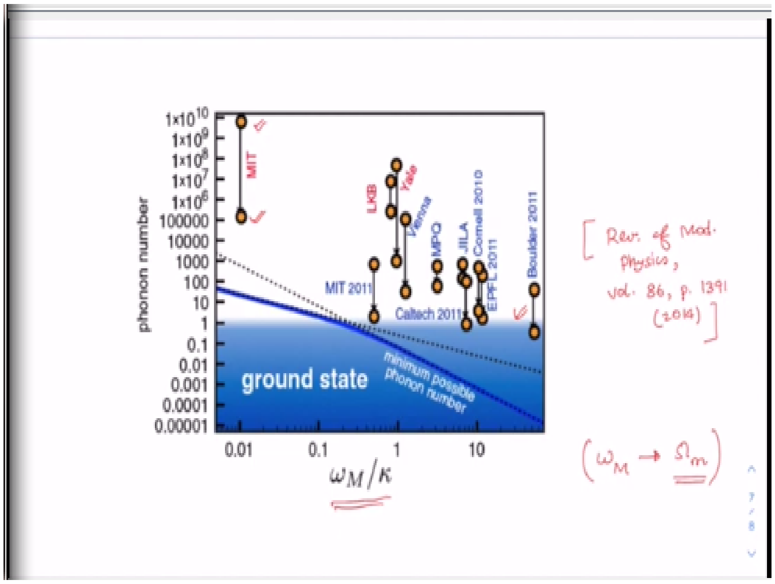

As ground state cooling of mechanical oscillator is of tremendous significance and importance for realizing quantum mechanics in macroscopic objects. Numerous experimental groups around the world have carried our various laser cooling experiments. As you can see from this particular plot, here the initial and final phonon number versus the sideband resolution parameter omega m by kappa which determines the minimum phonon number is plotted. And in our notation omega m is represented by this symbol.

Here as you can see the blue curve shows the quantum limit for the minimum achievable phonon number. And one group from MIT they have started initial phonon number around 10 to the power 10 in the logarithmic scale and they were able to suppress it to this number. On the other hand another group at Boulder at university of Colorado at boulder I am talking about the last one. Here they have started with phonon number around 100, 10 square and in the logarithmic scale and they suppress it to 1 in the log scale which is 10 in the linear scale.

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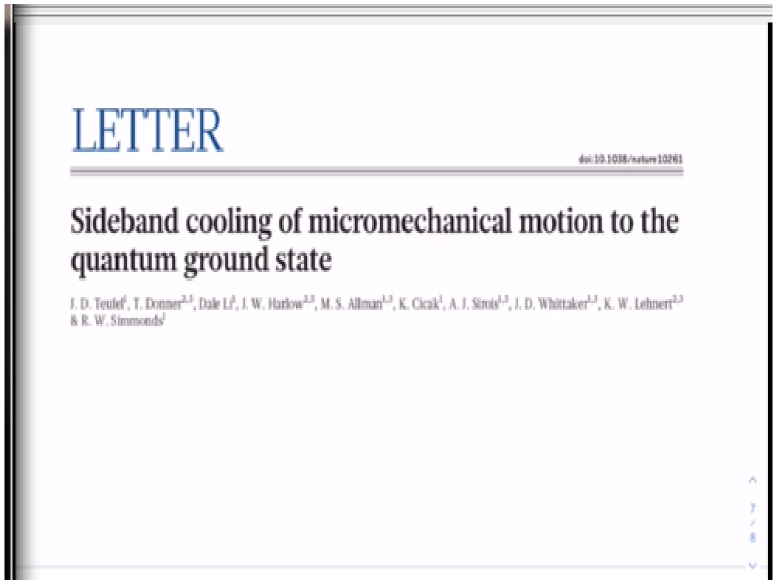

So, this is a landmark experiment and this was reported in this nature physics journal, actually it is nature journal not physics one. So, I encourage all of you to go through it, so you will have an idea about how the experiments were carried out in this laser cooling experiment and how they were able to suppress the phonon number and nearly achieve the ground state of the mechanical oscillator. One phenomenon well known in the context of 2 couple oscillator is the so-called normal mode splitting.

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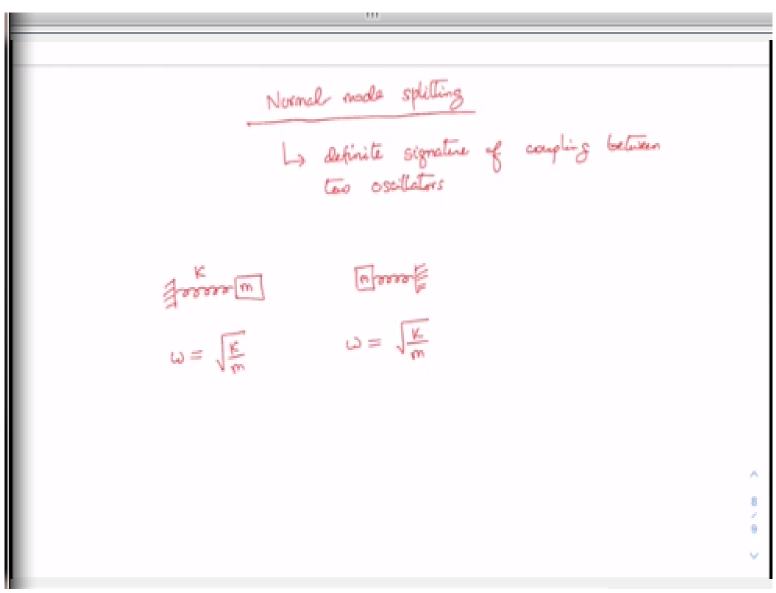

And it is an important phenomena because it is a definite signature of coupling between 2 oscillators. So, normal mode splitting is a definite signature of coupling between 2 oscillators.

Now some of you may ask what is a normal mode? Well, you know that if we have a single oscillator like this, let us consider a mass spring system. Suppose this mass m is attached to a spring of spring constant K then it is natural frequency of this mass spring system is square root of K by m, K is the spring constant.

And suppose we have another spring, same identical mass and here also this has the natural frequency is given by square root of K by m. However, if we have 2 or more coupled oscillators the system may have several natural or normal frequencies. And the general motion is a combination of vibration at all different frequencies.

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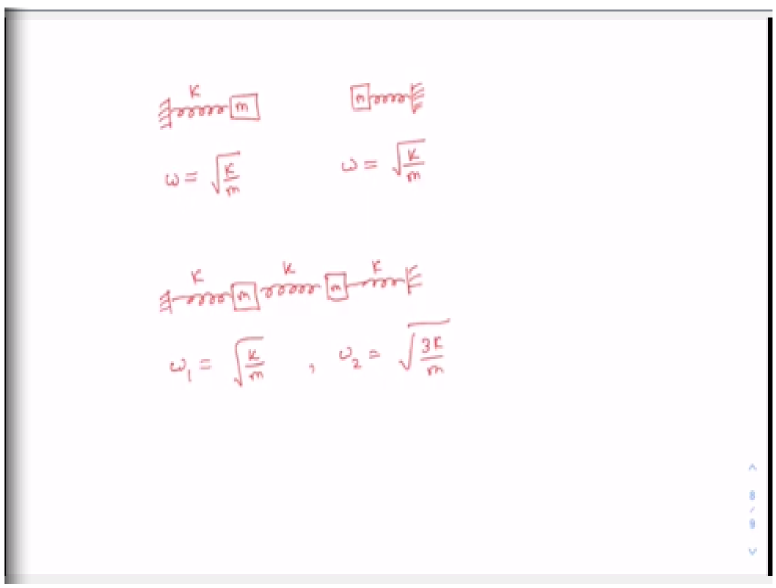

Now here if we couple this identical mass spring system by a, ok I will show you if I couple them by a spring of spring constant K. In that case you know that the natural frequency or the normal mod frequency would be 2 normal mode frequencies one can have. One is say omega 1 = square root of K by m and another one would be square root of 3K by m, all right. Now as we are having in cavity optomechanical system, 2 oscillators.

One is due to the optical oscillator and another one is the mechanical oscillator. So, we can expect normal mode splitting phenomena here also. However, before I go on to discuss it in the case of optomechanical system let me give you a general idea about what I mean by normal mode splitting.

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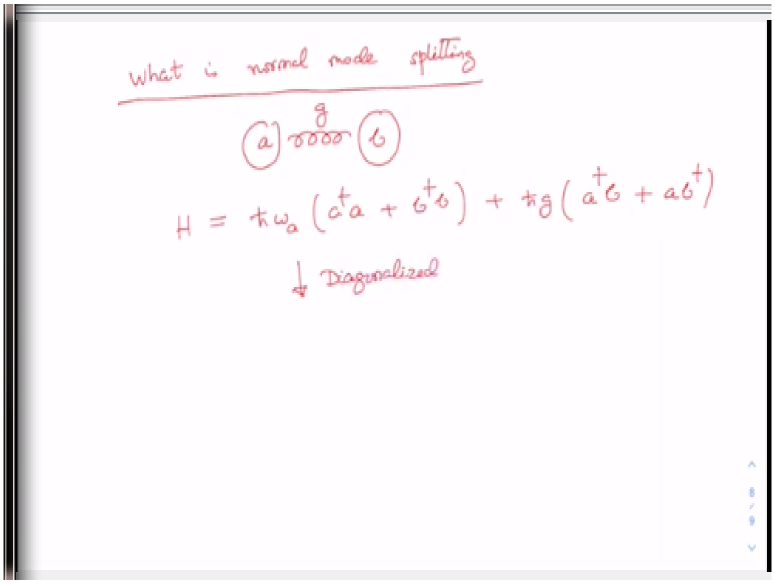

So, what is normal mode splitting? For simplicity let us consider 2 oscillator, oscillator a and oscillator b and they are coupled by this coupling parameters as g. And the Hamiltonian of the system let me say these 2 oscillators are identical, that means they have or degenerate at frequencies. So, they have the frequency say omega a and oscillator a is a dagger a, oscillator b is represented by b dagger b, these are the 2 harmonic oscillators.

And they are coupling and the coupling is such that the quantized accents between the 2 oscillators if the quanta a is created that is at the cost of the quanta in b and so on, and we have this process.

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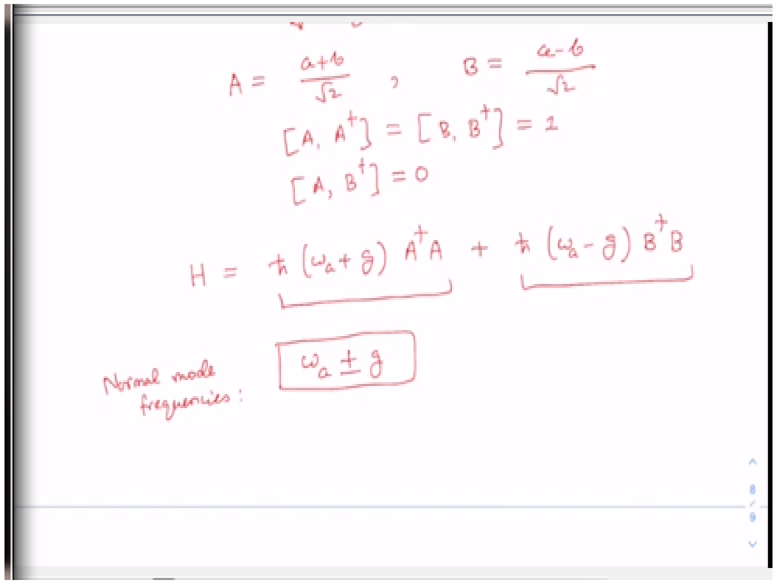

This Hamiltonian can be diagonalized if we take the transformation say if I write A = a + b root over 2, all these are operators and B let me write it as a - b by square root of 2. Then you can show that A, A dagger = B, B dagger that would be equal to 1. On the other hand A and B are independent oscillators, so they will not commute. And if we apply this transformation the Hamiltonian that we are going to have would be h cross omega a + g A dagger A + h cross omega a - g B dagger B.

So, what you see that now we are getting 2 independent harmonic oscillators with different frequencies and the more frequencies are here omega a +- g as you can see. These are the normal mode frequencies one is omega a + g and another one is omega a - g.

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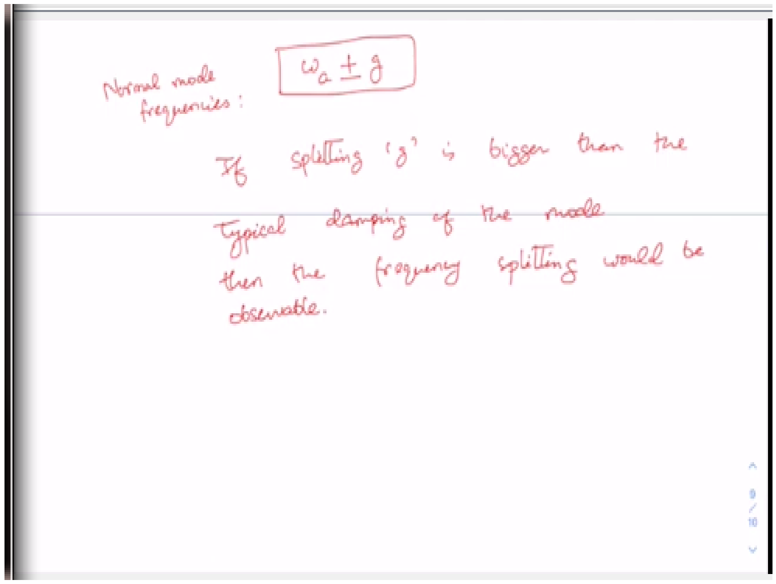

So, the frequency splitting would be observable if the splitting g is, if splitting g is bigger than the typical damping of the mode then the frequency splitting would be observable. I think this will be more clearer because now I am going to discuss the phenomena of normal mode splitting in cavity optomechanics.

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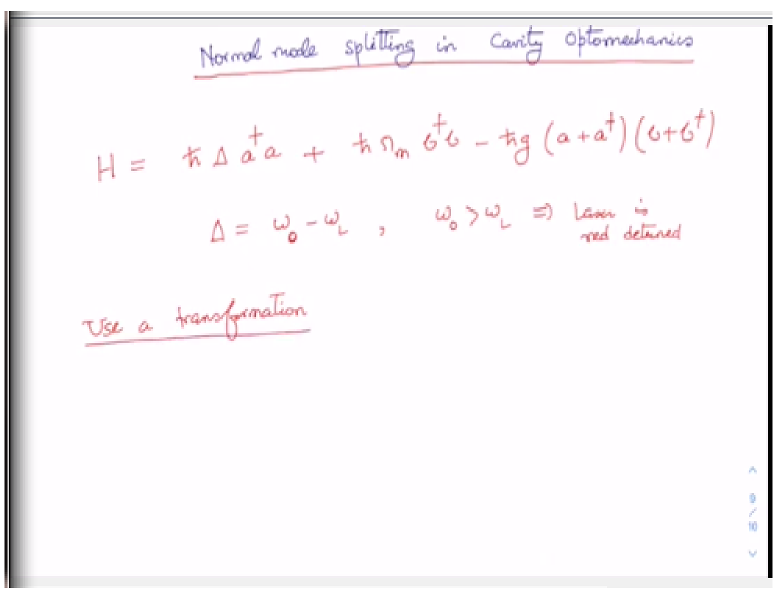

Let us consider the linearized optomechanical Hamiltonian without noise and damping. So, the Hamiltonian is let me write it as h cross delta, delta is the detuning parameter that I am going to define again, a dagger a this is the optical oscillator. Then we have h cross omega m b dagger b, that is the mechanical oscillator and they are coupled by this coupling parameter g and we have these terms a + a dagger into b + b dagger.

Here I define delta as omega o - omega L omega 0 or omega o is the resonance frequency of the optical cavity and omega L is the laser frequency. And we are going to consider that omega 0 is greater than omega L that means the laser is red detuned. Now let us use a transformation, I want to write the Hamiltonian in a convenient form, so that we can diagonalize it.

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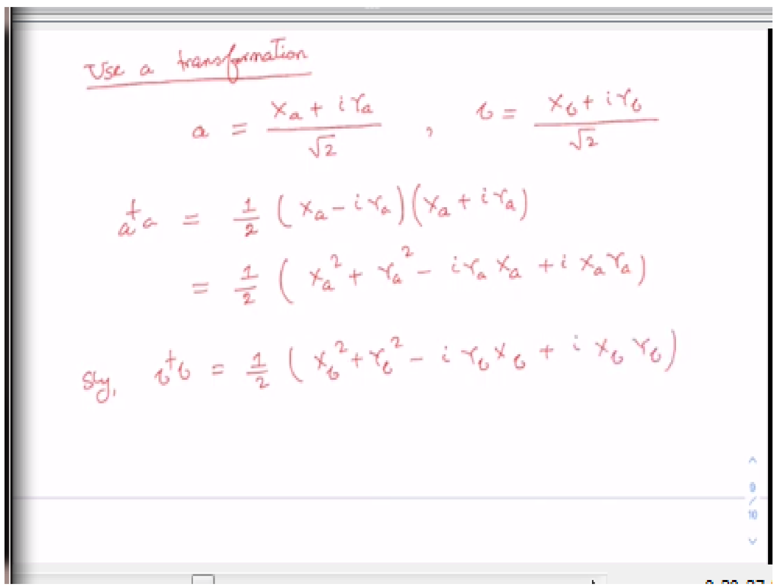

So, let us follow the procedure, let us do it this way. So, let me write the annihilation operator of the optical cavity in terms of the quadratures. So, all these are operators X a + i Y a divided by square root of 2. And for the mechanical oscillator part we have X b + i Y b divided by root 2. Then let me find out a degree a, b dagger b first. So, a dagger a if you do the straight forward calculation, so it will be half X a - i Y a, that is the a dagger part and then we have a is X a + i Y a.

Because these are operators, so this will lead us to will get X a square + Y a square - i Y a X a + i X a Y a. Similarly we can get b dagger b = half into X b square + Y b square - i Y b X b + i X b Y b.

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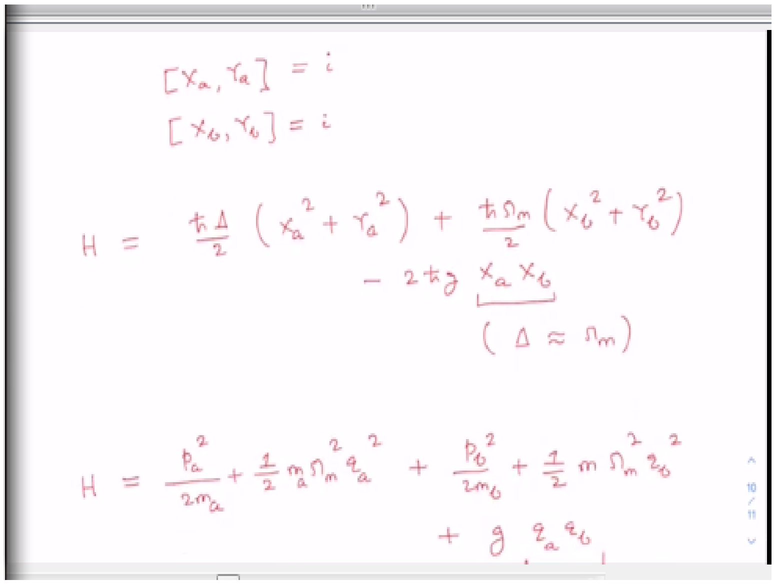

And a + a dagger would be square root of 2 into X a and b + b dagger would be square root of 2 X b. Now as you can see this relation or this one, this is commutation. So, if I use the relation say X a, this commutation relation between X a, Y a and similarly between X b, Y b. So, that the commutation relation between X a and Y a would be equal to i and similarly X b Y b = i. So, if we utilize this then the Hamiltonian can be rewritten and this would be h cross delta by 2 X a square + Y a square + h cross omega m by 2 X b square + Y b square - twice h cross g X a X b.

While I have written it I am assuming that delta the detuning parameter is almost equal to omega m that is the resonance frequency of the mechanical oscillator. This should remind you about the degenerate case that I discussed while explaining what we mean by normal mode splitting. Now if you look at this Hamiltonian this should remind you about the another case that we have discussed in an earlier class about coupled harmonic oscillator, where we have written the Hamiltonian as this.

Suppose we have 2 oscillators a and b then the Hamiltonian for the first oscillator has this kind of a part, this is for the oscillator a. And for the oscillator b this is the kinetic energy and this is the potential energy omega m square q b square. And if you remember then the coupling between them was considered as g into q a, q b, this particular issue I want to point out that here the coupling is of the position-position type.

And similarly if you look at it this structure is similar to this one because here also the coupling is a position-position type. Because X a corresponds to the position and Y a corresponds to the momentum of the optical oscillator similarly for the mechanical oscillator. So, the coupling is a position-position type coupling.

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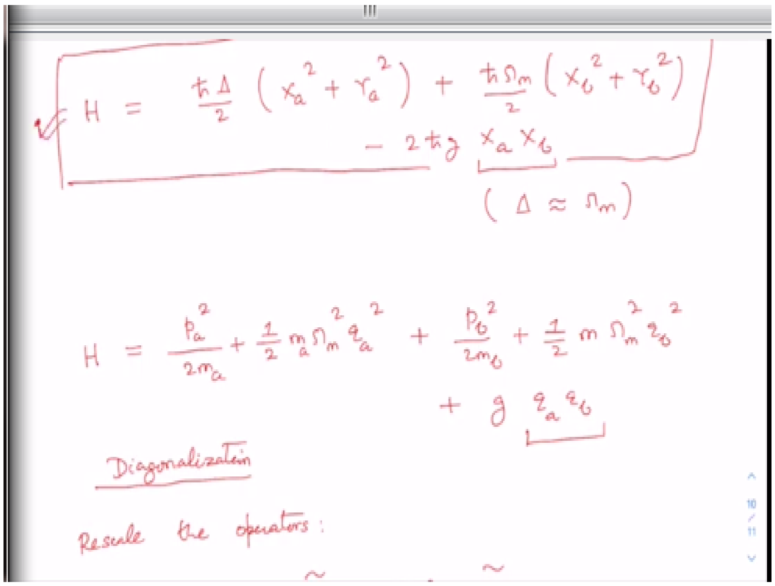

Now we can diagonalize this Hamiltonian.

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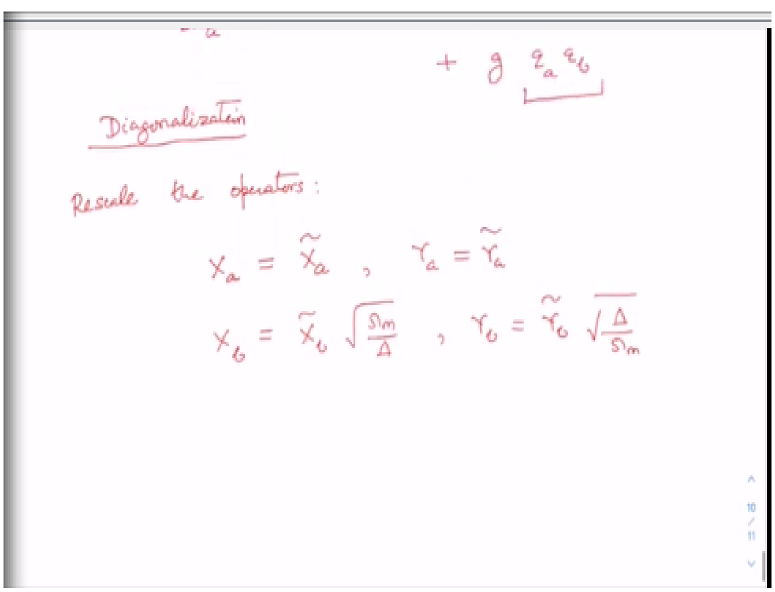

To diagonalize this Hamiltonian let me write or rescale. Let us to rescale certain parameters, rescale the operators actually what I am going to do is the diagonalization. So, I take X a to be equal to X a tilde, so I am not going to disturb the optical oscillator, it would be remain the same. But for a mechanical oscillator I take X b = X b tilde square root of omega m by delta and Y b = Y b tilde square root of delta by omega m. So, now if I put this in this expression in this Hamiltonian here and then what I am going to get?

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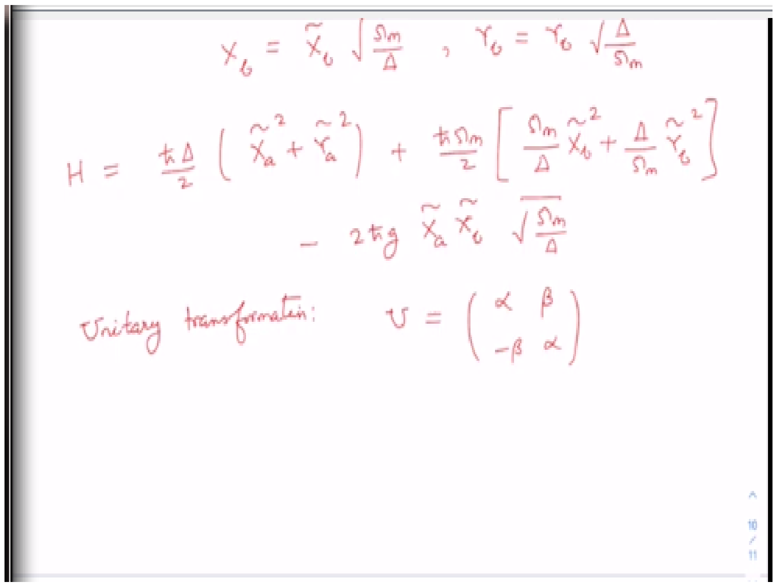

This is simple algebra, so let me write it. So, we will have h cross delta by 2 X a tilde square + Y a tilde square + h cross omega m by 2. Here I will have omega m by delta X b tilde square + delta by omega m Y b tilde square - twice h cross g X a tilde into X b tilde square root of omega m by delta. So, now this Hamiltonian as you can see is a Hermitian Hamiltonian and it can be diagonalized using a unitary transformation.

So, we are going to apply a unitary transformation to diagonalize it, unitary transformation let me say that unitary transformation operator is say alpha beta, - beta alpha where alpha beta are considered to be real quantity with the conditions that alpha square + beta square = 1.

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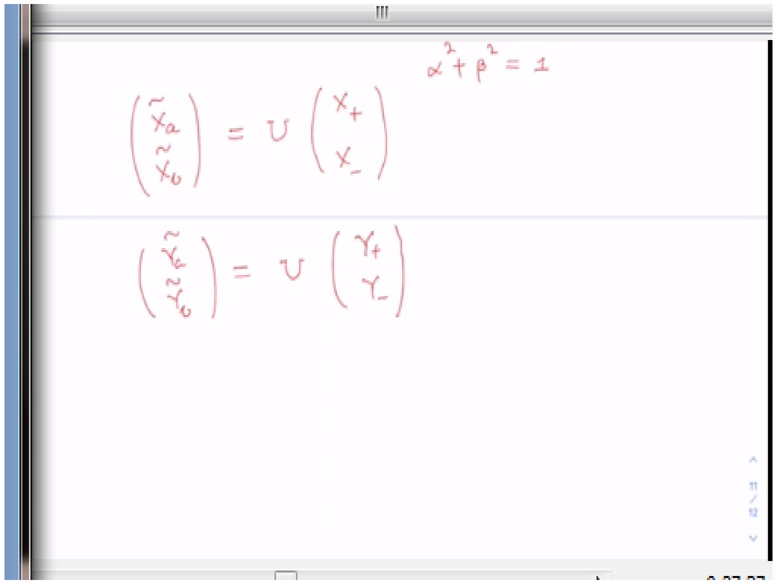

Because this will going to ends your unitarity. And what I mean by transformation is that I am going from the variable say X a tilde, X b tilde and that would be equal to I apply the unitary transformation **i** and I get the variable X + and X -. Similarly I have Y a tilde, Y b tilde, I apply the unitary transformation to get Y + and Y - + Y + and Y -, so sorry this would be Y -. Now putting this transformation into the Hamiltonian, so actually I will encourage you to do these calculations yourself.

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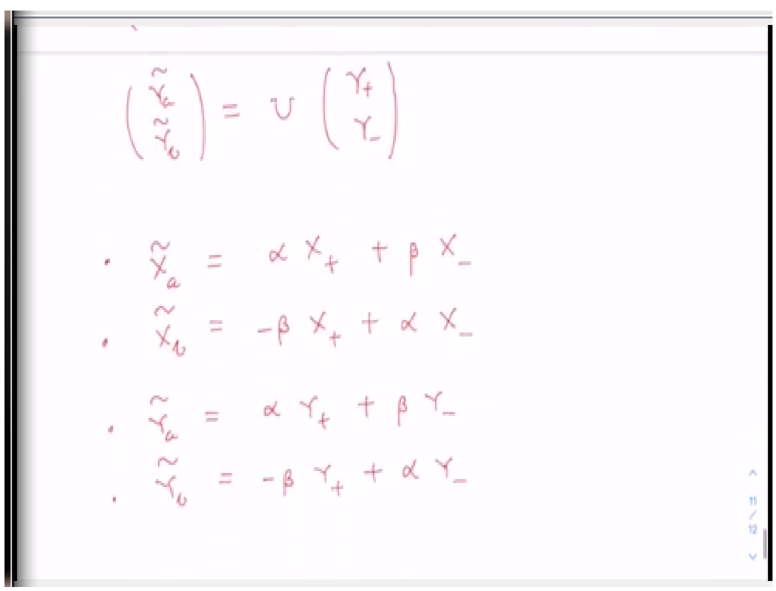

And then you can write for example let me just write X a tilde and X b tilde, then you have to put it X a tilde if I apply the unitary transformation I will get it as alpha X + plus beta X - and X b tilde = - beta X+ plus alpha X -. Similarly you will get Y a tilde = alpha Y + plus beta Y - and Y b tilde = - beta Y + plus alpha Y - and if I put all these variables into this particular Hamiltonian simple algebra you have to do, it may appear to be tedious but actually this is very straightforward.

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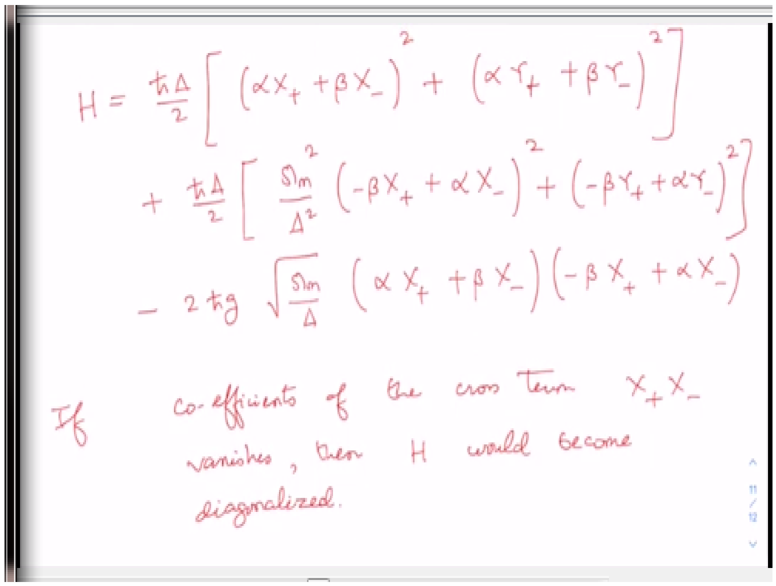

And then you will get the Hamiltonian in this forms, it has h cross delta by 2, so you will have alpha X + let me just put it in and I will explain X - whole square + alpha Y + beta Y - whole square. Then I will have h cross delta, I encourage you to do these things yourself because this is very simple, I am just putting up the terms only here. And I will get if I take h cross delta s common then I will get next expression would be omega m square by delta square - beta X + plus alpha X - whole square + - beta X, actually it would be Y + now.

We will have Y + plus alpha Y - whole square and then I have - twice h cross g square root of omega m by delta and I have this alpha X + beta X - into - beta X + plus alpha X -. Now this Hamiltonian would become diagonal. If the coefficients of all the cross term X + and X - should vanish. Let me write, if coefficient of the cross term X + X - vanishes then it is would become diagonalized.

That means it will not have any off diagonal elements in the Hamiltonian. I am not talking about Y + Y - because if you do the calculations because of the fact that alpha square + beta square = 1. The coefficients of that particular cross term is anyway going to vanish.

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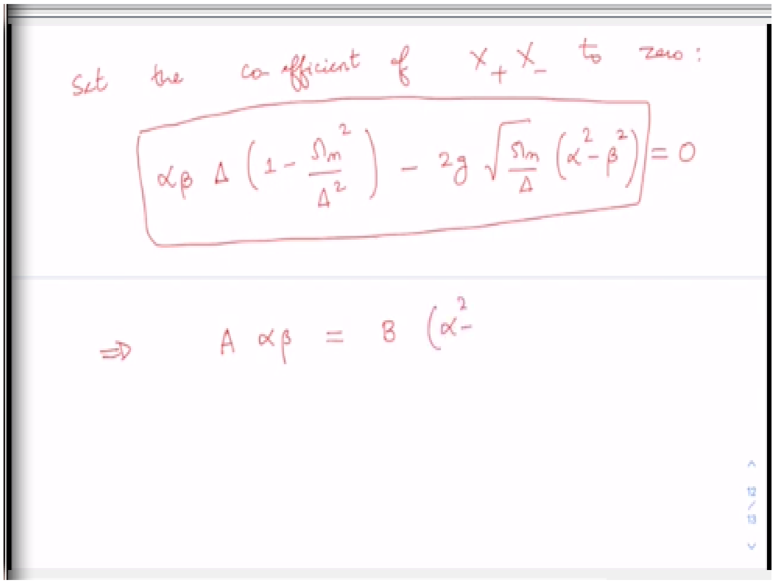

So, now because of these conditions we have to set the coefficient of X + X - to 0, this is going to give us a condition because of which the Hamiltonian would become diagonalized. And if I do it and if you just look at this, you have to open it up and then it is very easy to see the conditions that you are going to get is this.

You will get alpha beta into delta 1 - omega m square by delta square - 2g square root of omega m by delta alpha square - beta square = 0. So, this is the coefficient which I am making it to be 0. And to make my life little bit simpler let me write it in this form. Let me write A alpha beta = B into alpha square - beta square.

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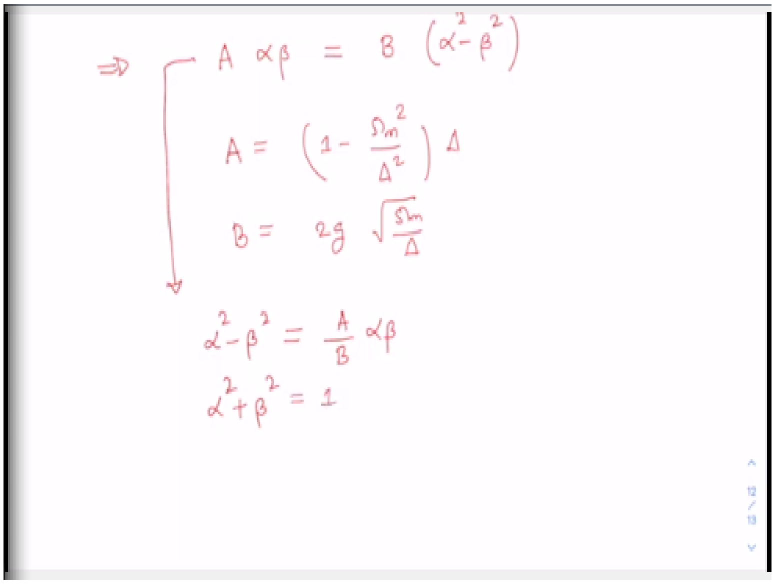

Where A = 1 - omega m square by delta square into delta and B = 2g square root of omega m by delta. So, we have now 2 equation, one equation is alpha square - beta square = A by B alpha beta which I am getting from this. And another one is alpha square + beta square = 1, these 2 equation can be solved to get the value of alpha and beta.

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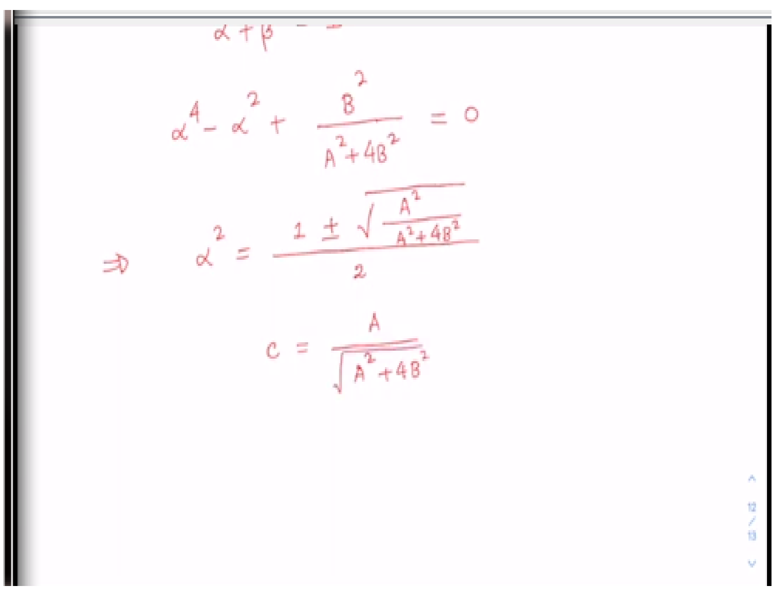

If we do it a little algebra will lead us to this equation that is alpha to the power 4 - alpha square + B square divided by A square into 4B square = 0. And from here we get alpha square = 1 +- square root of A square divided by A square + 4B square. Now if I take C = A divided by A square + 4B square, square root.

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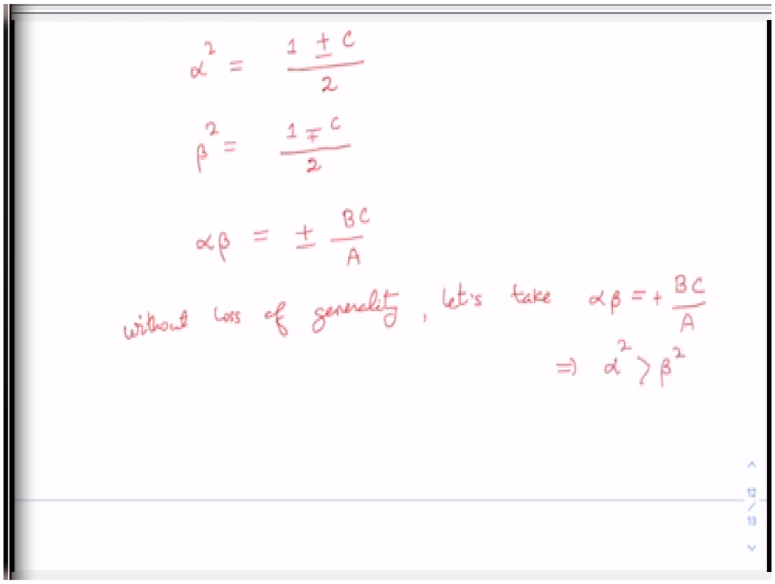

Then I can write alpha square = 1 +- C by 2 and clearly from here I can write beta square = 1 - C -+, so this is what I will get. And also from here I can have alpha beta = +- BC divided by A and without loss of generality let us take alpha beta = +BC by A. And actually this implies that I am taking alpha square is greater than beta square.

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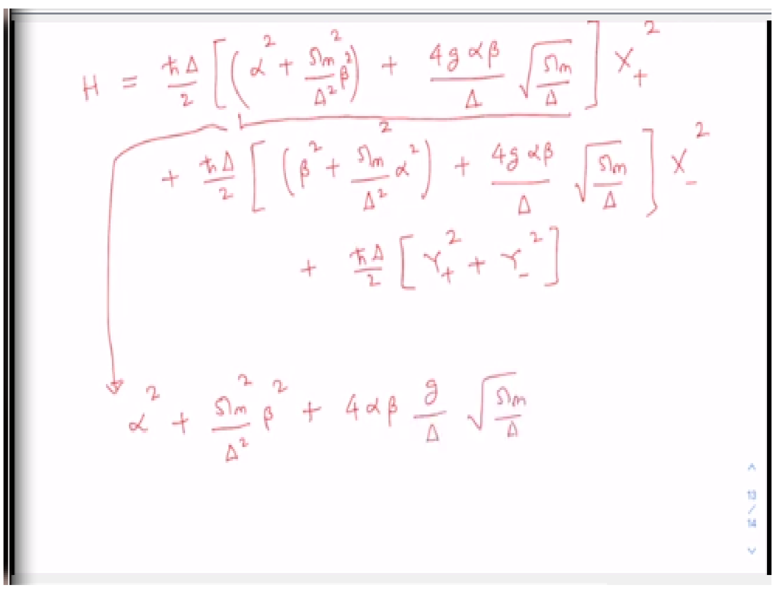

So, using this I can rewrite my Hamiltonian, it is very straightforward I can put the value of alpha beta and everything. I will finally get my Hamiltonian in this form that would be h cross delta by 2 alpha square + omega m square by delta square. It would be beta square as well here +

4g alpha beta divided by delta square root of omega m by delta. Then I have here term X + square and I have h cross delta by 2 beta square + omega m square by delta square alpha square + 4g alpha beta.

Let me write it as 4g alpha beta divided by delta square root of omega m by delta X - whole square and I have h cross delta by 2, Y + square + Y - square. Now what I can do is that, you see I can simplify this expression further because I know the value of alpha beta now, alpha square beta square. So, let me simplify this, this one let me write it as alpha square + omega m square by delta square beta square + 4 alpha beta g divided by delta square root of omega m by delta.

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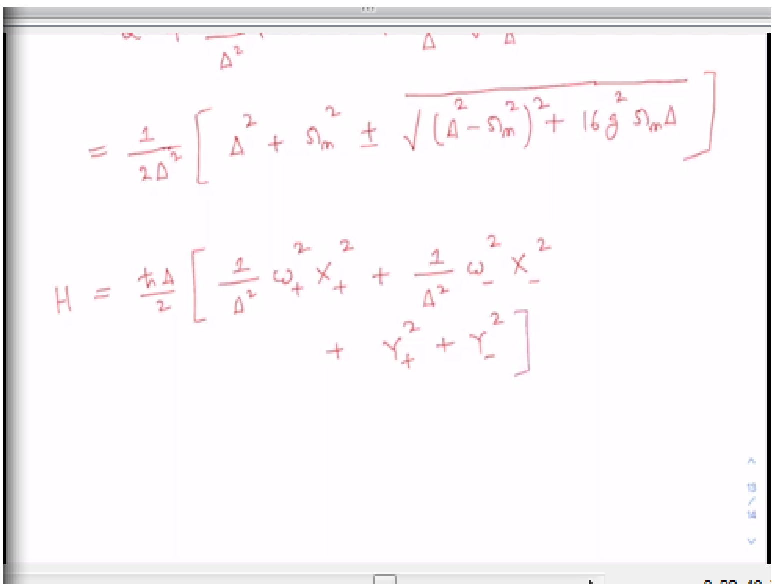

And if I put the value of alpha beta then I have 1 by delta square, these are algebra and you can do it and verify it whether I am doing it correctly. You have delta square + omega m square +- square root of delta square - omega m square whole square + 16g square omega m delta. So, this is what we will get.

And using this, this Hamiltonian I can rewrite in this form and that would be h cross delta by 2, 1 by delta square omega + whole square I will write what is omega + square later. Let me first write it, I will have another term 1 by delta square omega - square X - square. And I will have you see I do not have any cross term. So, these are the terms I have.

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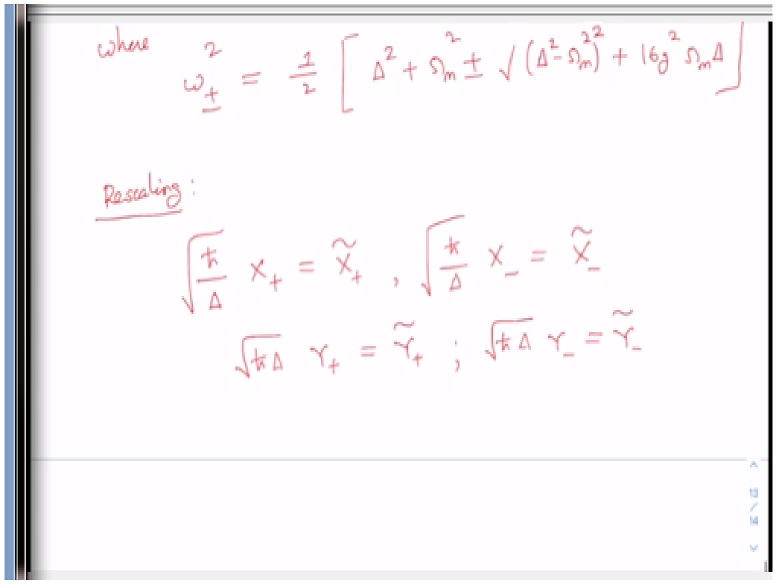

Where this omega+ as well as this omega - square is defined as a half of delta square + omega m square +- square root of delta square - omega m square whole square + 16g square omega m delta, so this is what I will have. Now if I rescale further, rescaling we will get let me rescale by taking h cross by delta square root X + as X + tilde and square root of h cross delta X - as X tilde -. And h cross delta square root Y + let me take it as Y + delta and h cross delta square root Y- as Y- tilde.

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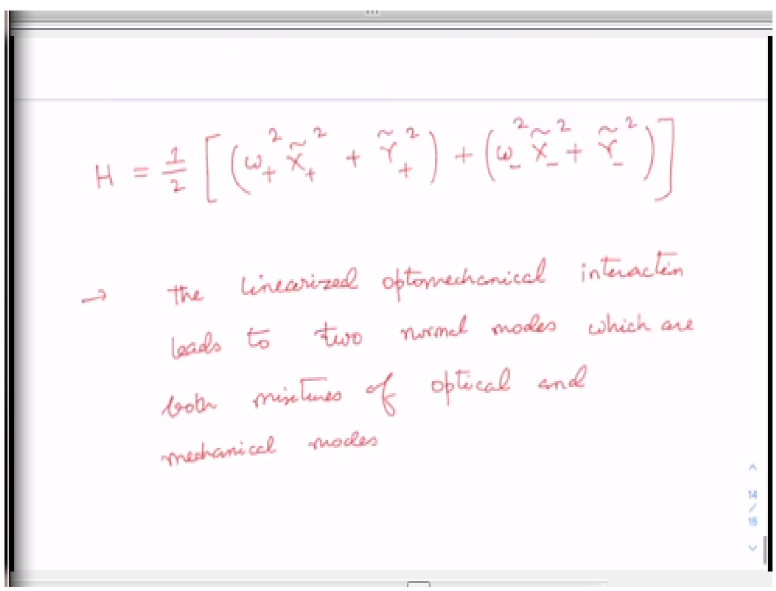

Then if I do it, then we can write the Hamiltonian in a simplified form and that would be half of omega+ square X tilde square + Y tilde square, this is one term I have. And another term I will have is omega - square X tilde - square + Y tilde square; this is what I will have. So, it is clear that the linearized optomechanical interaction; let me write here. The linearized optomechanical interaction leads to 2 normal modes which are both mixtures of optical and mechanical modes.

It is clear because you see this frequency omega +- square, now we have this optical frequency related to the optical delta is related to the optical oscillator and omega m is related to the mechanical oscillator, so it is a mixtures of both optical and mechanical modes and normal mode frequency, let me write it once again.

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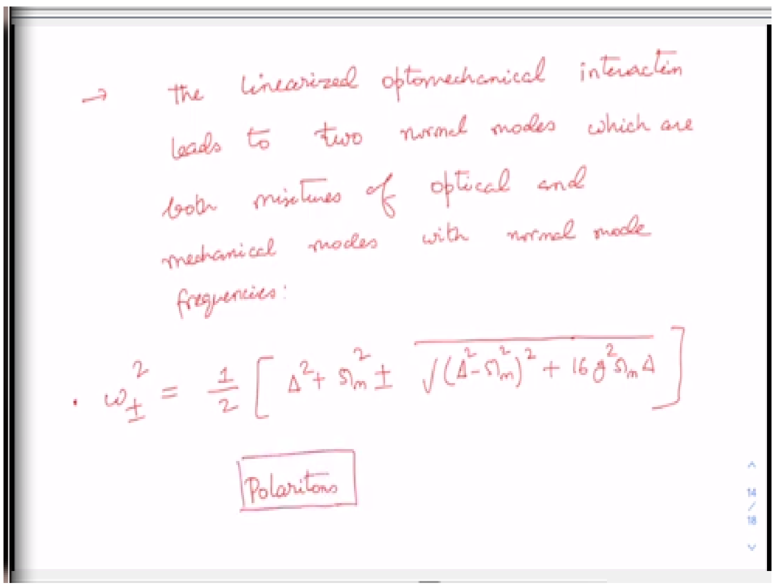

With normal mode frequencies omega +- square = half of delta square + omega m square +- square root of delta - omega m whole square plus, ok, I think this is delta square omega m square + 16g square omega m delta, so this is what I have. These normal modes which are combinations of optical and mechanical mode are frequently referred to as polaritons. We can plot the mode frequencies as a function of the detuning parameter delta for some fixed mechanical frequency omega m and the linearized optomechanical coupling parameter g.

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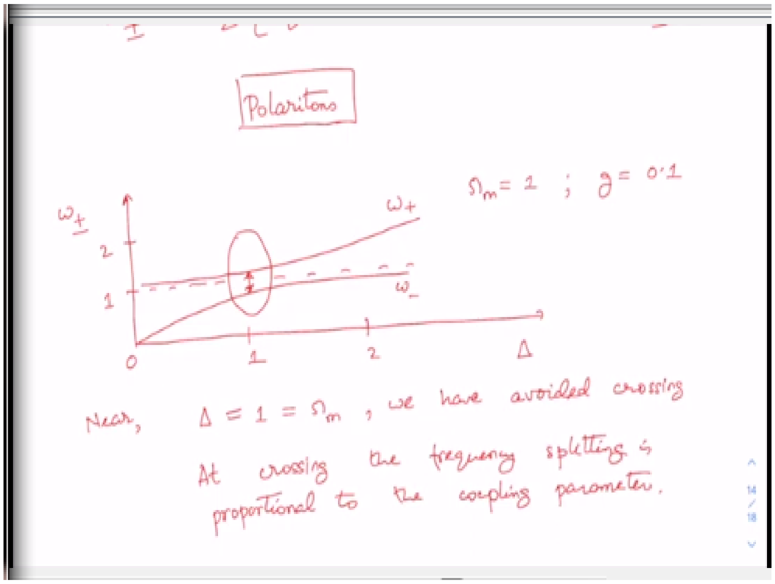

For example we can have a plot like this, if I plot say delta in the x axis and the mod frequencies omega +- in the y axis then I will have a typical plot for say omega m = 1, it may be 1 megahertz and g as 0.1. Then we will get a typical plot of this type, we will have say plot like this, here the upper branch referred to as omega + and the lower one corresponds to omega -.

Now as you can see from this plot that = near delta = 1 which is equal to omega m. We can observe avoided crossing. And the splitting at crossing the frequency splitting is proportional to the coupling parameter. In fact this is the reason why one can observe normal mode splitting only in the strong coupling regime.

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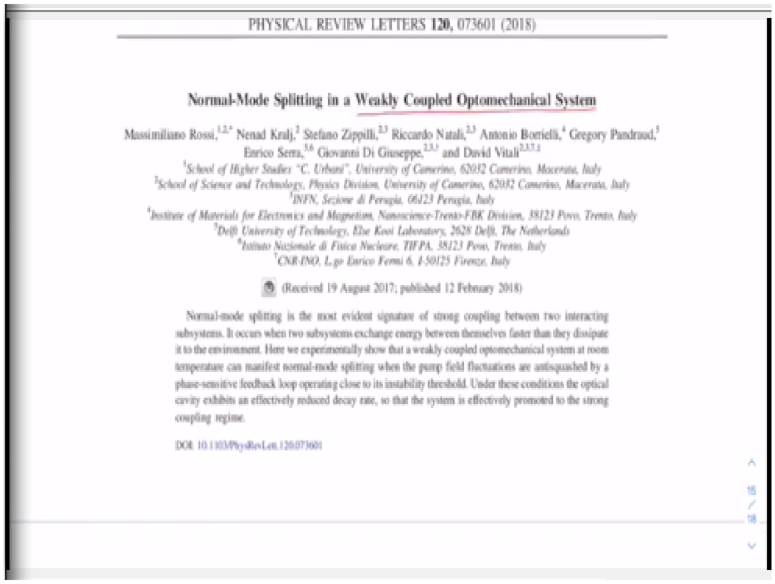

PHYSICAL REVIEW LETTERS **120**, 073601 (2018)

Normal-Mode Splitting in a Weakly Coupled Optomechanical System

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Normal-mode splitting is the most evident signature of strong coupling between two interacting subsystems. It occurs when two subsystems exchange energy between themselves faster than they dissipate it to the environment. Here we experimentally show that a weakly coupled optomechanical system at room temperature can manifest normal-mode splitting when the pump field fluctuations are antisquashed by a phase-sensitive feedback loop operating close to its instability threshold. Under these conditions the optical cavity exhibits an effectively reduced decay rate, so that the system is effectively promoted to the strong coupling regime.

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But recently even in the weak coupling regime, normal mode splitting is observed in a cavity optomechanical system.

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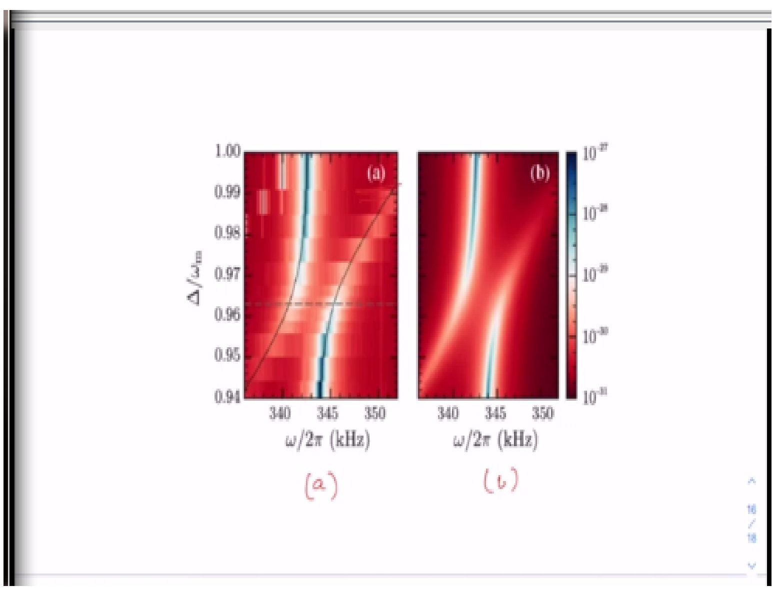

Here, for example in this plot one can observe avoided crossing phenomena that we have just discussed. And here this plot a refers to the experimental one and the plot b is the one given by theoretical calculations. And you can see the excellent agreement between theory and experiment and this is really amazing.

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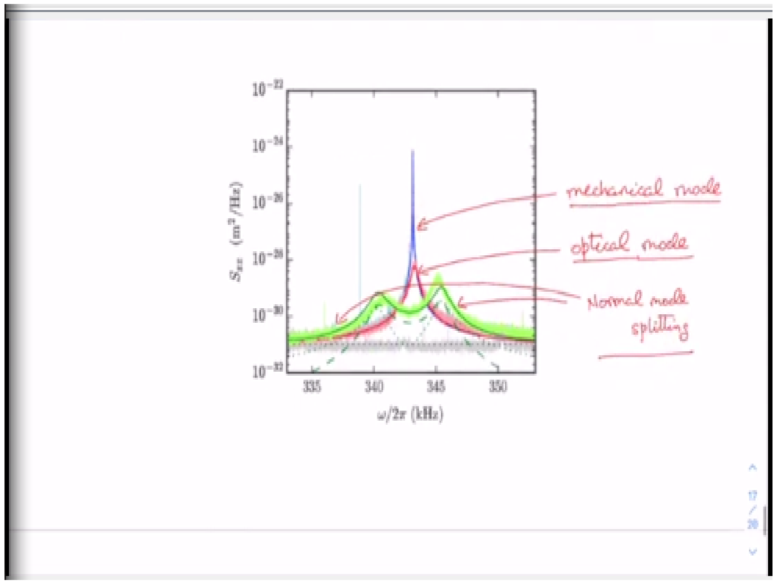

Now this plot depicts the displacement spectral noise, the blue trace here refers to the uncoupled mechanical mode. So, this one refers to the mechanical mode, in their experiment they have used a membrane, a vibrating membrane which gives the mechanical mode. And this one, the red one refers to the optical mode when they are not coupled. And when these 2 oscillators the optical one and the mechanical oscillators are coupled they get the green trace.

The solid line refers to this solid one line refers to the theoretical and the other traces refers to the experimental. And this particular as you can see from here there are appearances of 2 normal modes and this is what is the so-called normal mode splitting.

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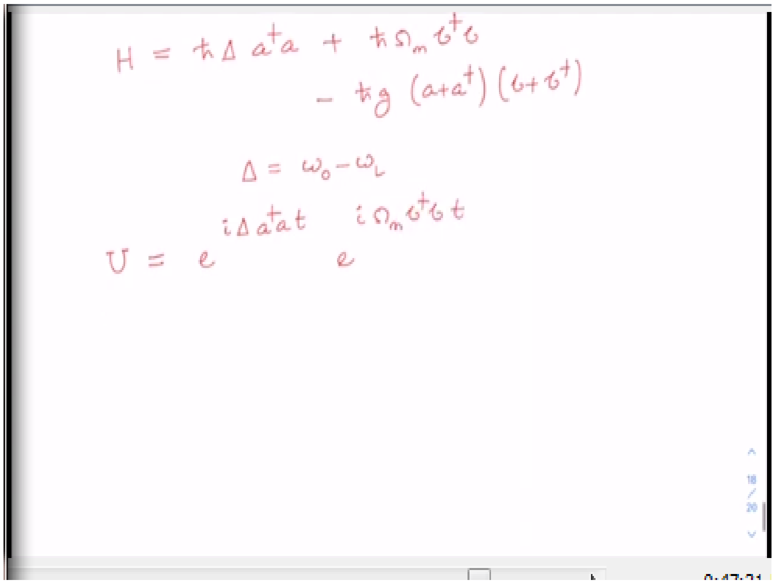

Now finally to look into the linearized system dynamics in the absence of noise and damping, let us transform the linearized optomechanical Hamiltonian which we wrote as h cross delta a dagger a + h cross omega m b dagger b - h cross g a + a dagger into b + b dagger, where we have taken delta = omega 0 - omega L. And now this Hamiltonian we can transform by using this unitary transformation which I take as e to the power i delta a dagger a t e to the power i omega m b dagger b t.

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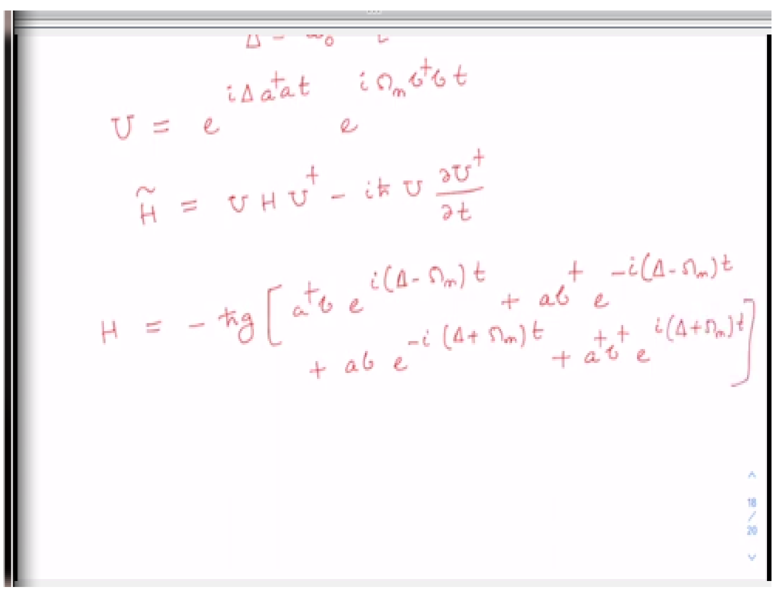

As you know under united transformation the Hamiltonian will transform to a new Hamiltonian which we derived earlier, this relation we have worked out earlier. Now if I put my unitary transformation to this relation then we can obtain our Hamiltonian as this. I am not writing h tilde, so what I am going to write is the transform Hamiltonian.

That would be - h cross g a dagger b e to the power i delta - omega m t + ab dagger e to the power - i delta - omega m t. And we will have terms ab e to the power - i delta + omega m t and a dagger b dagger e to the power i delta + omega m t.

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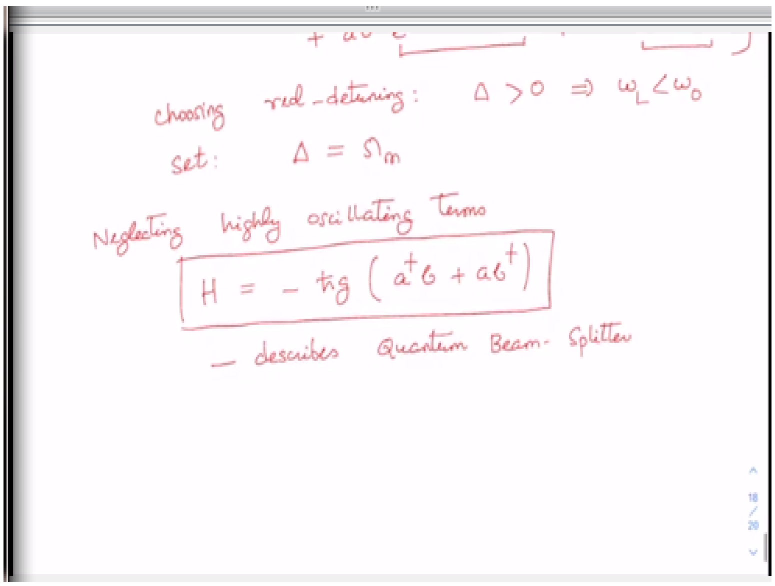

Now let us choosing red-detuning that means the laser frequency is less than the optical frequency delta is greater than 0 which implies omega L is less than omega 0, omega 0 is the cavity resonance frequency. And if we set the detuning parameter delta = omega m, then you will see that this term will get cancelled, it would give you 1 and this is also going to give 1 but this term will oscillate at double the frequency of omega m and similarly this term.

So, all these highly oscillating terms can be neglected, so neglecting highly oscillating terms I get the Hamiltonian in this form that would be - h cross g a dagger b + ab dagger and this is an important Hamiltonian. And these Hamiltonian describe the quantum beam splitter and the corresponding Heisenberg equation of motion for a and b can be very easily worked out.

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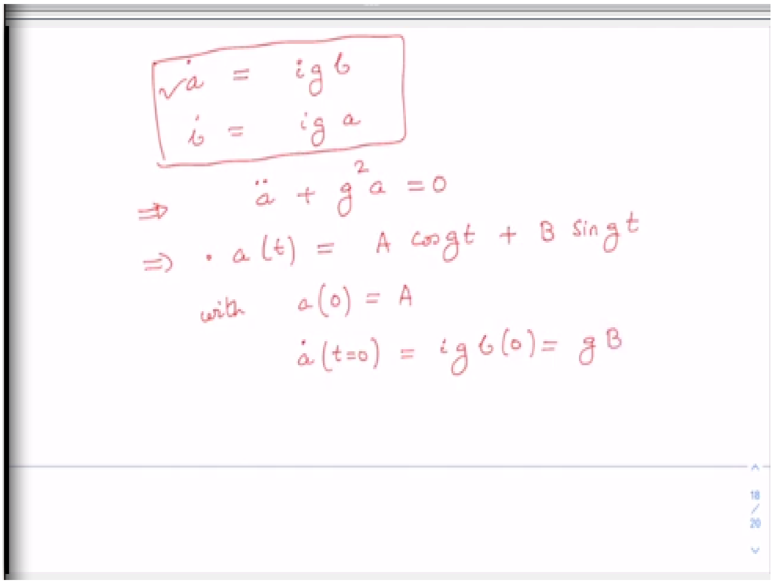

And that would be a dot = igb and b dot = iga and from this couple equation you can immediately get this differential equation for a double dot + g square a = 0. And you know the general solution of this differential equation a of t would be equal to a cos of gt + b sine of g into t. Now with the condition and obviously you will see that at time t = 0, we will get A. And a dot time derivative of a at time t = 0 will give me i into gb of 0, you can check it from here, from the second equation, from this equation.

In fact you will get it from the first equation, you will get it from this equation that a dot at t = 0 would be ig into b of 0. And from this equation immediately you can see that this will result in g into B.

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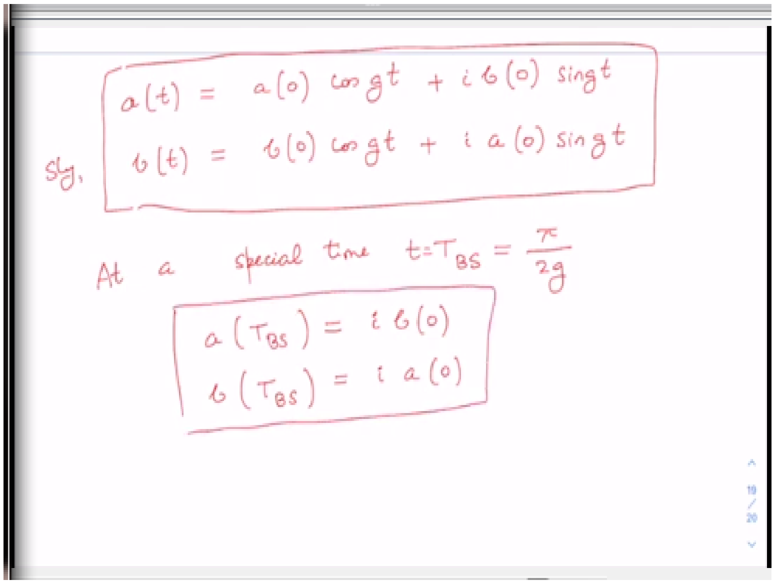

So, therefore the solution I can write as a of t = a of 0 cos of gt + i into b of 0 sine of gt. And similarly I can get the solutions for b and b of t would be equal to b of 0 cos of gt + i a of 0 sine of gt, please verify that yourself. Now clearly at any arbitrary time the modes a and b are the mixtures of their initial values which is very clear from these expressions.

For example this is going to be an important case at a special time say T BS, t = T BS = pi by 2g, you will see at that time T BS it is equal to i b of 0. On the other hand at that time b, the value of b would be i of a 0. So, this show something very interesting, what it shows is that the modes have exchanged their values.

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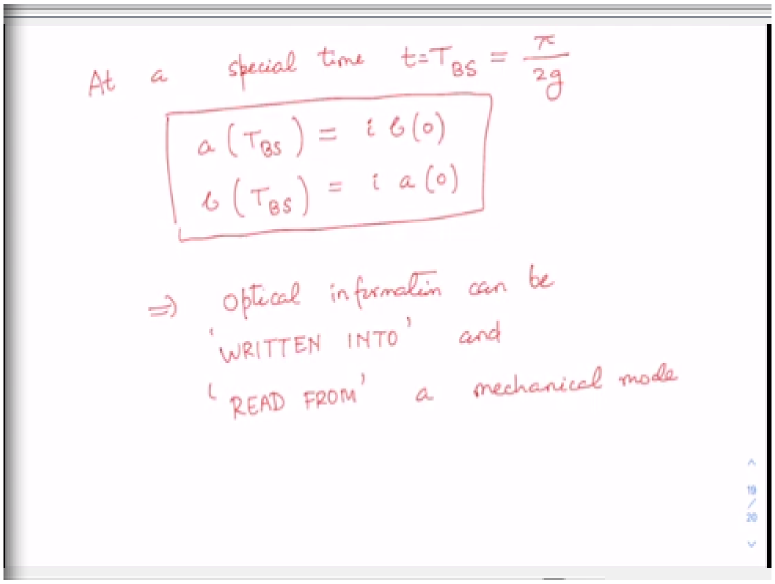

And this feature implies that the optical information because a contains the optical information, b contains the information from the mechanics. So, what it says is that optical information can be 'WRITTEN INTO' and 'READ FROM' a mechanical mode. So, optomechanical system therefore can act like a transducer and this is or state can be transferred from one mode to the another mode.

And in fact this is the principle behind information storage and retrieval in optomechanical system. And this theoretical prediction that we got from this calculation is experimentally verified.

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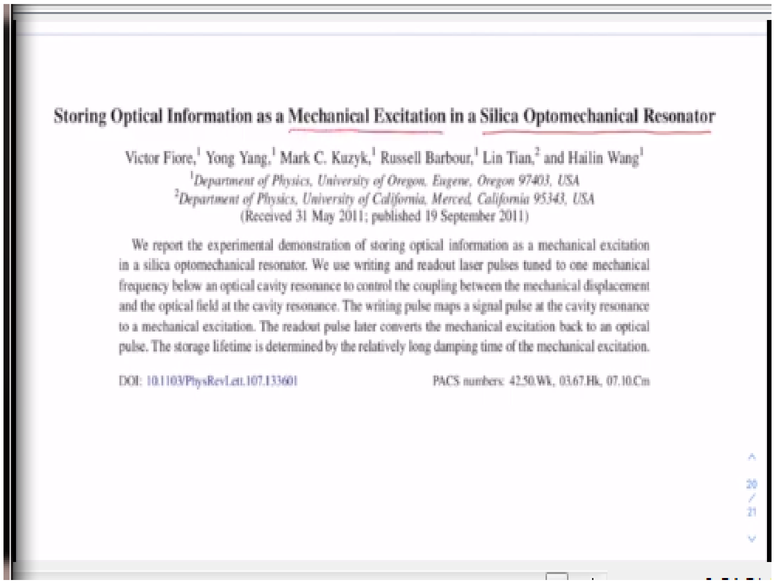

Storing Optical Information as a Mechanical Excitation in a Silica Optomechanical Resonator

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We report the experimental demonstration of storing optical information as a mechanical excitation in a silica optomechanical resonator. We use writing and readout laser pulses tuned to one mechanical frequency below an optical cavity resonance to control the coupling between the mechanical displacement and the optical field at the cavity resonance. The writing pulse maps a signal pulse at the cavity resonance to a mechanical excitation. The readout pulse later converts the mechanical excitation back to an optical pulse. The storage lifetime is determined by the relatively long damping time of the mechanical excitation.

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And realized experimentally in this very interesting work, where they have used silica optomechanical resonator as their cavity optomechanical system. And they have stored optical information as mechanical excitation. Let me stop here for today, in this lecture we discussed the phenomena of normal mode splitting in some more details. We also learned the principle or physics behind how an cavity optomechanical system act like a transducer.

In the next lecture we are going to discuss the phenomena of squeezing and also we will conclude this module, so see you in the next lecture, thank you.